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First determination of the Planck constant using the LNE watt balance

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Abstract

After separate developments of the different elements with continuous characterizations and improvements, the LNE watt balance has been assembled. This paper describes the system in detail and gives its first measurements of the Planck’s constant h. The value determined in air is \( h = 6.626 \, 068 \, 8(20) \times 10^{-34} \) Js which differs in relative terms by \(-0.05 \times 10^{-7}\) from the \( h_{90} \) value and by \(-1.1 \times 10^{-7}\) from that of the 2010 CODATA adjustment of \( h \). The relative standard uncertainty associated is \( 3.1 \times 10^{-7} \).

Keywords: watt balance, Planck’s constant, kilogram

(Some figures may appear in colour only in the online journal)

1. Introduction

The kilogram is the last unit of the Système International of units (SI) still based on an artifact. It is defined by the mass of the International Prototype of the Kilogram (IPK) made from platinum-iridium alloy (Pt90%Ir10%) and stored at the BIPM in Sèvres [1, 2]. It was machined in 1878 as a number of other prototypes designated as official copies a few years later. Four of them are stored in the same conditions as the IPK. Thirteen others are the national prototypes used by the signatory countries of the Convention du mètre.

Since the 1880s, only three comparisons have been organized to survey the evolution of the mass of the official copies and national prototypes relative to IPK. The results display a relative drift of about \( 3 \times 10^{-8} \) over the course of a century [3] which makes the current definition of the kilogram unsatisfactory. This situation is no longer acceptable in a time of ever-increasing measurement precision and for this reason, there is a strong desire among international laboratories to move to a new, more robust definition of the unit of mass [4–7].

For the past few years, significant efforts have been made to link the unit of mass to a fundamental constant of physics with high accuracy. National metrology institutes have essentially developed two experiments—a silicon sphere [8] and watt balances [9–16]—to link the unit of mass respectively to the Avogadro and Planck constants. In 2001, the Laboratoire national de métrologie et d’essais (LNE) decided to develop a watt balance experiment in order to contribute to the international effort to redefine the kilogram. Twelve years later, after separate developments of the different elements with continuous characterizations and improvements, the entire system has been assembled.

The paper presents an overall description of the LNE watt balance, gives a first measurement of the Planck constant and analyses the main contributions to the uncertainty budget.

2. Principle of the watt balance

The watt balance principle proposed by Kibble in 1976 [17] consists in the comparison between virtual electromagnetic and mechanical powers measured during two steps (called static and dynamic phases) by means of a beam balance.

In the static phase (or force mode), the weight \( m \cdot \vec{g} \) of a standard mass \( m \) subject to a gravitational acceleration \( \vec{g} \) is balanced by the Laplace force \( \vec{F} \) exerted on a coil of length \( l \) in which there flows a current \( I \) when the coil is immersed in a magnetic induction field \( \vec{B} \). For perfect alignment of
the system (horizontal magnetic field, horizontal coil and vertical Laplace force), this equilibrium is described by the relation:

\[ mg = (B \cdot l)_{\text{stat}} \cdot I \]  

(1)

where \( I \) may be measured by the voltage drop \( U \) it produces at the terminals of a known resistance \( R \).

In the dynamic phase (or velocity mode), the coil is moved at a vertical velocity \( V_z \) in the same magnetic induction field \( B \). Again for perfect alignment of the system, Lenz’s law of induction leads to a voltage drop at the coil terminals given by:

\[ U = (B \cdot l)_{\text{dyn}} \cdot V_z \]  

(2)

If the magnetic flux in both phases is the same (i.e. the same coil in the same magnetic induction), the combination of relations (1) and (2) can be re-expressed as the equality of virtual mechanical and electrical powers in SI units leading to:

\[ (B \cdot l)_{\text{dyn}} = (B \cdot l)_{\text{stat}} \]  

(3)

In practice the voltages are linked to a Josephson effect voltage standard (via the Josephson constant \( K_J \)) and the resistance is linked to the quantum Hall effect resistance standard (via the Von Klitzing constant \( R_K \)). In this way, the mass \( m \) can be expressed in terms of the Planck’s constant \( h \), provided it is accepted that the product \( K_J^2 \cdot R_K \) is equal to \( 4h \) [18].

Since the Josephson effect and quantum Hall effect are used with the values conventionally defined in 1990, respectively \( K_{J,90} \) and \( R_{K,90} \), it follows that:

\[ h = \frac{4}{K_{J,90}^2 \cdot R_{K,90}} \cdot \frac{mg}{U_{\text{stat}}} \cdot \frac{1}{V_z} \]  

(4)

Following the formalism [18], one can then write:

\[ \frac{h}{h_{90}} - 1 = \frac{(B \cdot l)_{\text{stat}}}{(B \cdot l)_{\text{dyn}}} - 1 \]  

(5)

The relative difference between the value of the Planck’s constant \( h \) in the SI and its value \( h_{90} \) expressed in terms of \( R_{K,90} \) and \( K_{J,90} \) is equal to the relative difference between the measured values of the product \( (B \cdot l) \) in the static and dynamic phases.

3. LNE watt balance description

One of the characteristics of the LNE watt balance is that, during the dynamic phase, the force comparator (balance beam and its suspension) is moved as a single element by means of a translation stage actuated by a stepper motor, in order to avoid one’s using the balance beam as the element generating the movement. Figures 1 and 2 show respectively a picture of the LNE watt balance and a schematic of the core of the system.

The following subsections describe the experimental set-up that was used to obtain the first results of measurements \( h \).

3.1. Translation stage

The translation stage is represented in the upper section 1 in figure 1. Its moving part is linked to the support structure of the watt balance by means of six sets of flexure hinges that constrain its movement to take place along only one direction [19, 20]. The adjustment of the parallelism of the hinge axes allows one to reduce the directional deviations of movement and
unwanted rotations around the horizontal axis. The total stroke of the guiding stage is 72 mm. Measurements are made in the central 40 mm portion of this. In this region, we have established prior to measurement of $h$ that lateral deviations of the guiding stage from the vertical trajectory are below 0.5 $\mu$m and unwanted rotations in the horizontal plane are below 5 $\mu$rad.

Once unidirectional motion has been obtained, the direction of the stroke is adjusted to lie along a vertical reference axis defined by a precision tiltmeter. The procedure, based on the determination of the position of the centre of a metallic sphere fixed to the guiding stage by means of capacitive sensors, enables one to adjust the verticality to better than 10 $\mu$rad.

The main elements of the force comparator (balance beam and its suspensions) are fixed to the lower plate of the guiding stage. It is shown in the lower section 2 in figure 1.

3.2. Force comparator

The force comparator (3) is based on a 200 g homemade aluminium alloy beam [21–23], with two symmetrical 100 mm length arms. The three pivots are constituted by 20 $\mu$m thick stainless steel flexure strips clamped at both ends.

The end flexure strips support on one side a simple suspension for the tare mass (5) and on the other side two suspensions (4) on the same vertical axis: one is to receive a 500 g mass standard and the other is to fix the coil. These suspensions are articulated in their middle by the mean of clamped flexure strips gimbals. The suspensions of the mass and the coil are linked together with an electro-machined double monolithic gimbal [24] specially designed to reduce the effect of mutual static and dynamic coupling and to merge the points of application of the gravitational and Laplace forces.

The force comparator has a total mass of 6.2 kg (0.5 kg of which is due to the standard mass) hanging on the central flexure strip of the beam.

3.3. Coil and magnetic circuit assembly

The coil (6), made up of eight superimposed layers of a total of 684 windings of mean diameter 268 mm, secured with epoxy resin, is wound on a Delrin polymer support. The copper wire, insulated by a 15 $\mu$m polyimide layer has a diameter of 250 $\mu$m. Its resistance of 200 $\Omega$ leads to dissipated power of only 5 mW for a 5 mA nominal current.

The magnetic circuit (7) is of a loud-speaker type. The central ring made up of sixty samarium cobalt Sm$_2$Co$_{17}$ permanent magnets is inserted between two XC48 steel plates. This assembly is surrounded by two yokes composed of two soft magnetic materials: pure iron and iron-cobalt alloy. The geometry of the yokes defines a 9 mm thick and 90 mm long air gap where the magnetic induction field, reaching 0.94 T, presents a minimum value. This minimum defines the position at which the coil is placed during the static phase in order to minimize the influence of its positioning error.

The geometry of the magnetic circuit, the choice of the different materials, the influence of magnetostriction, the machining and the mounting of the circuit have already been described extensively [25].

During mounting, the orientation of the radial magnetic field was made horizontal in a two-step procedure [26]. The first one, performed outside the watt balance structure, consists in tilting the magnet to make the field horizontal. To this end the Faraday effect probe is used to measure the orientation of the magnetic field with respect to the upper surface of the magnetic circuit. The second step, after the transfer of the magnetic circuit in the enclosure, consists in reproducing the upper surface orientation by means of a precision tiltmeter and a capacitive probe. At final, the mean magnetic plane is horizontal with an uncertainty of 10 $\mu$rad. The horizontality has been checked before this determination of the Planck constant.

3.4. Mass exchanger

The last element we describe is the mass exchanger (8). It has been developed to bring the standard mass onto the mass pan during the static phase (horizontal translation of 170 mm) and to lift it (maximum vertical translation of 6 mm) if necessary. In this way, the mass is removed during the dynamic phase.

The horizontal movement is guided by means of a commercial slide while the straightness of the vertical movement results from the use of a pantograph. For both movements, linear piezo-electric motors are used in a closed loop configuration with high-resolution linear encoders as position sensors. The horizontality of the mass exchanger is adjusted to allow the self centring of the mass on the pan. In addition to minimize mass pan oscillations the vertical velocity is kept at 10 $\mu$m s$^{-1}$.

4. Measurement considerations

4.1. Static phase measurement

After the two arms of the beam have been balanced in terms of mass, a tare mass equal to half the standard mass is added to the counterweight of the tare side. During the static phase, when the 500 g mass standard is placed on the pan (step ‘on’), there is an excess weight of 250 g on the double suspension side and a stabilized current produces an electromagnetic force to compensate it. When the mass is removed (step ‘off’), the same excess weight is on the tare suspension side and the current going through the coil is reversed to produce a force in the opposite direction: in both steps, the beam is balanced.

This procedure was adopted to cancel the effect of the relative difference length of the two arms (by using only one arm to compare forces) and also to reduce the parasitic electromotive force in the electrical circuit (by reversing the current in the coil). By combining these two steps, one can evaluate the ratio mg/I which corresponds to the product $(B \cdot I)_{stat}$ of the static phase.

The different parameters to be evaluated are the mass $m$, the acceleration of gravity $g$ and the current $I$:

- All measurements have been performed with a 500 g mass standard $m$ made from XSH Alacrite which has been calibrated by the department of mass and derived quantities at the LNE.
4.2. Dynamic phase measurement

During the dynamic phase, the force comparator is moved vertically (see performance of the translation stage in 3.1) on a 14 mm range travel. At the beginning and the end of the movement, the acceleration is first increased linearly with time and then decreased to zero. Outside the period of acceleration, the velocity is maintained at \( v_0 = 2 \text{ mm s}^{-1} \). This movement induces a voltage \( U \) around 1 V at the terminals of the coil (which is entirely immersed in the air gap of the magnetic circuit).

The quantities \( U \) and \( v_0 \) are measured: the ratio \( U/v_0 \) corresponds to the product \( (B \cdot I)_{\text{dyn}} \) of the dynamic phase.

The voltage \( U \) is measured by means of three Agilent 3458A voltmeters in the same way as for the static phase and \( v_0 \) is measured by three heterodyne Michelson interferometers [32] lighted by a frequency doubled stabilized Nd:YAG laser. The moving arms of the interferometers are comprised by three corner-cube reflectors located at the periphery of the coil support (placed at 120° from each other). The interferometers determine not only the vertical velocity \( v_z \) but also the angular velocities around the horizontal axis.

Three other position sensors are used to measure positions and velocities in the horizontal plane as well as the angular position and velocity around the vertical axis. They are based on vertical propagating gaussian beams intercepted by screens located at the periphery of the coil support.

Attention has been paid to measure simultaneously \( U \) and \( v_0 \) so as to achieve maximal noise rejection: indeed the rejection ratio obtained is close to 100. To do this, the triggering signals issued from the voltmeters are also shaped in a sequence used to trigger the velocity measurements during the dynamic phase. In this case, there is a maximum delay of 1 μs between voltage and velocity measurements due to time resolution of the FPGA used to determine the interferometric phase which is more than five orders of magnitude smaller than the duration of one measurement of velocity and voltage (200 ms).

4.3. Environment condition measurements

All measurements were performed in air after the enclosure of the watt balance has been sealed. Temperature, pressure, humidity and carbon dioxide content were measured continuously during the experiment. The air density and refractive index were calculated using [33, 34] in order to apply corrections to the results. An example of the determination of these relative corrections is given in figure 3 which corresponds to a one week period.

5. Results

Software has been developed with the ability to construct a full measurement sequence from a set of elementary steps corresponding to the configuration of each device. Environmental conditions, calibration and measurement data are then stored and exploited a posteriori.

5.1. Dynamic phase: determination of the product \((B \cdot I)_{\text{dyn}}\)

A typical sequence of the dynamic phase consists of 100 pairs of up and down trajectories. Every trajectory is described by 40 data points and takes 8 s. A spatial profile of the product \((B \cdot I)_{\text{dyn}}\) is then extracted and averaged over the set of the 200 steps. An example is shown in figure 4. As expected, the resulting shape goes through a minimum at mid-height of the
magnet gap. By fitting a second order polynomial curve to the data (red curve) we determine the value of the height at which the weighing phase takes place.

5.2. Static phase: determination of the product $B \cdot I_{\text{dyn}}$

A typical sequence of the static phase consists of ten pairs of ‘mass on’ and ‘mass off’ steps.

For both kind of step, the coil current is continuously adjusted by a programmable current source to oppose any change in the forces acting on the beam. However, from one step to another step, the current changes in response to force changes are not fast enough to maintain the beam in a horizontal position. It takes 100 s for the beam position to reach the steady state. Once the beam has reached its equilibrium position, 250 voltage readings taken with a 200 ms integration time are averaged to give the current $I_{\text{on}}$ value in the configuration ‘mass on’. After this measurement, the mass is raised by 1 mm (‘mass off’), the vertical translation being made at a velocity of $10 \mu m \ s^{-1}$. In the same way, the value of the current $I_{\text{off}}$ in the configuration ‘mass off’ is determined. In practice, the process ‘mass on’ / ‘mass off’ is repeated 10 times and the final value of the current for the sequence is given by the relation:

$$ I = \frac{1}{9} \sum_{i=1}^{9} \left( \frac{I_{\text{on}}^{i} + I_{\text{on}}^{i+1}}{2} - I_{\text{off}} \right) $$

Figure 3. Evolution of the relative corrections in terms of $h$ determinations due to air index and air buoyancy during one week. The gray areas stand for the uncertainty associated.

Figure 4. Product $(B \cdot I)_{\text{dyn}} = U/I_z$ obtained during one sequence of 100 up and down trajectories. The red curve is the 2nd order polynomial fit, together with the estimation of $(B \cdot I)_{\text{dy}n}$ at weighing altitude. Uncertainties shown are an evaluation of type A.
The knowledge of the mass, the acceleration of gravity and the measurements of the current \(I\) flowing through the coil allow one to evaluate the product \(B \cdot I\). Figure 5 shows how this product varies over the course of one week (black circles). As expected, because of the temperature coefficient of the samarium-cobalt magnet \((\text{Sm}_2\text{Co}_{17})\), approximately \(-3 \times 10^{-4}\text{K}^{-1}\), the observed change is correlated with the change in temperature (red curve). Note that two successive static determinations are separated by a dynamic phase.

5.3. Determination of the Planck constant from electrical and mechanical factors

Two successive values of \(h\) are determined in two different ways by considering:

- a first dynamic phase with one hundred pairs of up and down trajectories, a static phase with ten pairs of ‘mass on’ and ‘mass off’ and a second dynamic phase with one hundred pairs of up and down trajectories;
- or a first static phase with ten pairs of ‘mass on’ and ‘mass off’, a dynamic phase and a second static phase with ten pairs of ‘mass on’ and ‘mass off’.

Each determination is then composed of three steps which are not processed at the same time and to take into account the drift due to the temperature changes of the magnet, a linear interpolation is performed between two successive values of \((B \cdot I)_{\text{dyn}}\) which is combined with one value of \((B \cdot I)_{\text{stat}}\). The same applies for two successive values of \((B \cdot I)_{\text{stat}}\) combined with one value of \((B \cdot I)_{\text{dyn}}\). More than 420 values were calculated during the summer of 2014. Results are presented in figure 6 where each point (black
full dot) corresponds to an independent value of $h$ with its standard deviation. The gaps observed between some data were used for checks.

The value of $h$ extracted from these data is $6.6260688(20) \times 10^{-34}$ Js which is lower than the $h_{90}$ value by $-0.05 \times 10^{-7}$ in relative terms. The relative type A standard uncertainty is considered equal to the relative Allan deviation and estimated to be $8 \times 10^{-8}$ (figure 7).

6. Uncertainty contributions and discussion

The main uncertainty contributions are summarized in table 1. For this first determination of $h$ with the LNE watt balance, the relative combined standard uncertainty amounts to $3.1 \times 10^{-7}$. Only the major uncertainty contributions are discussed in the following.

The $h$ value reported here does not take into account the correction on the calibration of the French primary mass standard (copy no 35) due to the deviation of 35 µg on the unit of mass maintained at the BIPM discovered from the extraordinary calibrations carried out in 2014. The correction on the LNE $h$ value amounts to $-3.7$ parts in $10^9$ with an uncertainty of 3 parts in $10^9$, so negligible in respect with the total uncertainty [35].

6.1. Major contributions for the $(B \cdot I)_{\text{stat}}$ determination

The product $(B \cdot I)_{\text{stat}}$ is given by the ratio of the weight of the standard mass to the total current injected in the coil. The measurements is made in two steps, with either a direct or a reversed current flowing through the coil for the configurations ‘mass on’ and ‘mass off’. In both cases, a voltage drop is measured at the terminals of a 200Ω resistor.

The uncertainties associated with the $(B \cdot I)_{\text{stat}}$ determination without taking into account the effects of the parasitic forces (see 5.3 and 5.4) are due to voltage measurements, resistance measurements and mass contributions.

6.1.1. Voltage measurements. The voltimeters are used in the 1V range and are calibrated by a Zener voltage standard. The main relative standard uncertainty component is due to the gain stability of the voltimeters estimated to be $1.2 \times 10^{-7}$V. Other contributions are smaller as shown in table 2.

6.1.2. Resistance measurements. The 200Ω resistor placed in a thermo-controlled enclosure (within $\pm 10$ mK) is calibrated

![Figure 7. Relative Allan deviation of $h$ measurements versus time.](image)

Table 1. Main uncertainty contributions.

<table>
<thead>
<tr>
<th>Uncertainty budget for $h$ measurements</th>
<th>$k = 1$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Type A</td>
<td>$8 \times 10^{-8}$</td>
</tr>
<tr>
<td>Type B</td>
<td></td>
</tr>
<tr>
<td>Voltage measurements $U'$ and $U$</td>
<td>$2.4 \times 10^{-7}$</td>
</tr>
<tr>
<td>Resistance $R$</td>
<td>$6 \times 10^{-9}$</td>
</tr>
<tr>
<td>500 g Alacrite mass (including traceability to the national prototype of the kilogram, buoyancy, magnetic susceptibility and alacrite density contributions)</td>
<td>$7.4 \times 10^{-8}$</td>
</tr>
<tr>
<td>Absolute gravity value $g$</td>
<td>$5 \times 10^{-9}$</td>
</tr>
<tr>
<td>Velocity $v$ (including air refractive index and verticality of the laser beams)</td>
<td>$1.2 \times 10^{-7}$</td>
</tr>
<tr>
<td>Parasitic watt ratio term (see equation (7))</td>
<td>$9.1 \times 10^{-8}$</td>
</tr>
<tr>
<td>Force comparator contribution</td>
<td>$3.3 \times 10^{-8}$</td>
</tr>
<tr>
<td>Other contributions (switch effect(^a), polynomial fitting(^b), trigger delay, hysteresis of the flexure strips)</td>
<td>$5 \times 10^{-8}$</td>
</tr>
<tr>
<td>Combined relative uncertainty</td>
<td>$3.1 \times 10^{-7}$</td>
</tr>
</tbody>
</table>

\(^a\) The switch effect term concerns the performance of the commercial Data Proof low thermal scanner used for automating measurements of the static and dynamic phases.

\(^b\) The 3rd and 4th order polynomial fits have been also tested and the differences obtained compared to the second order are lower than $5 \times 10^{-8}$. Moreover, the histogram of the residual of the second order adjustment is gaussian contrary to the 3rd and 4th. Considering that the relative standard uncertainty associated with $h$ measurements is $3.1 \times 10^{-7}$, these differences have been considered, for the moment, as negligible contributions.
against a quantum Hall resistance standard (QHRS). The total relative standard uncertainty associated with the resistance determination is $6 \times 10^{-9}$. The uncertainty budget is given in table 3. The main uncertainty component, which arises from the instability of the resistor due to its being moved to and from the QHRS room located 20 m away from the watt balance room does not exceed $5 \times 10^{-9}$.

To take into account the buoyancy contribution and apply the associated correction to the mass, the air density has been calculated with an uncertainty of $5.4 \times 10^{-4}$ kg m$^{-3}$ by measuring temperature, pressure, relative humidity and mole fraction of carbon dioxide.

In addition, due to the presence of a residual magnetic field of $1.5$ mT at the standard mass location, the magnetic susceptibility of the mass introduces a significant undesired magnetic force of $3.7 \times 10^{-7}$ N. The resulting error on the mass determination is 37 μg. This error has been corrected and its associated relative uncertainty is estimated to be 30%, i.e. 11 μg.

Finally, the relative standard uncertainty of the 500 g Alacrite mass (including traceability to the national prototype of the kilogram, buoyancy, magnetic susceptibility and alacrite density contributions) is $7.4 \times 10^{-8}$.

### 6.2. Major contributions for the $(B \cdot l)_{\text{dyn}}$ determination

The product $(B \cdot l)_{\text{dyn}}$ is given by the ratio of the induced voltage at the terminals of the coil to the velocity of the coil. Measurements are made when the coil is translated vertically (up and down) through the air gap of the magnetic circuit.

The uncertainty associated with the determination of $(B \cdot l)_{\text{dyn}}$ is composed of both voltage and velocity measurements.

The first contribution has already been discussed above. Taken into account the correlation between the two successive voltage measurements (dynamic then static phases), the contribution of the two voltage measurements is added: the total voltage contribution is then equal to $2.4 \times 10^{-7}$ in relative value.

The second contribution is due essentially to the air refractive index $(1 \times 10^{-7})$ and to the verticality of the three laser beam directions. The verticality was determined with a standard uncertainty of $350 \mu$rad which translates into a relative standard uncertainty for the power of $0.6 \times 10^{-7}$.

In total, the relative standard uncertainty associated with the velocity is $1.2 \times 10^{-7}$.

### 6.3. Watt ratio error term

Equation (3) was established for an ideal case free of unwanted parasitic forces and torques and unwanted horizontal and angular velocities. If one takes these parameters into account, the equation becomes:

$$\frac{U \cdot I}{F_v} = 1 + \frac{F_i}{F_v} \cdot \frac{\nu_i}{\nu_v} + \frac{F_i}{F_v} \cdot \frac{\tau_i \cdot \omega_i}{\nu_v} + \frac{F_i}{F_v} \cdot \frac{\tau_i \cdot \omega_v}{\nu_v} + \frac{F_i}{F_v} \cdot \frac{\tau_i \cdot \omega_z}{\nu_v} \quad (7)$$

where $F_i$ is the force along the $i$th axis and $\nu_i$ is the velocity along that axis. $\tau_i \omega_i$ is the torque about the $i$th axis and $\omega_i$ is the angular velocity about that axis. The vertical velocity $\nu_v$ is $2 \text{mm/s}$, whereas the vertical force $F_v$ (nominally equal to the weight of the standard mass $m$) is 4.9 N.

The five latter terms in the equation above are referred to as the parasitic watt ratio error term.

Using the double pendulum mathematical model [36, 37], and the different experiments described in [38], we have estimated the parasitic forces and torques during this campaign of measurements. By using the three Gaussian position detectors and the three heterodyne Michelson interferometers, the horizontal and angular velocities were estimated during measurements of $h$. The watt ratio error was found to lie within $\pm 1.6 \times 10^{-7}$ (table 4). If one assumes a rectangular distribution, the relative standard uncertainty associated is $9.1 \times 10^{-8}$.

From table 4, it worth noting that:

(a) The ratios $\nu_i/\nu_v$ and $\nu_i/\nu_z$ of the coil should be of the order of $1.2 \times 10^{-5}$, considering the performance of the translation stage (the lateral deviations from the 40 mm vertical trajectory are below 0.5 μm). The values measured are higher by more than two orders of magnitude. This could be due to the fact that the three laser beams which intercept the three screens fixed at the periphery of the coil support (placed at 120° from each other) are not vertical (at better than 1 mrad), however this is currently under investigation. We nevertheless use measured velocities as an overestimation of the parasitic velocities of the coil.

(b) The force ratios are below $50 \times 10^{-6}$.
6.4. Force comparator contribution

Because the comparator is not used as a mass comparator, the horizontality of its pivots is of great importance. Indeed, for a non-horizontal position, parasitic forces introduce a systematic error in the weighing. To make this error negligible, either the comparator is aligned horizontally or the parasitic forces are nulled. At present, it is not possible to adjust the angular position of the pivot axis of the beam to better than 3 mrad relative to the horizontal. For this reason, we must take into account the contribution of the parasitic forces along the mass comparator ($F_x$ estimated smaller than 90 $\mu$N, or 20 ppm in relative term) during the weighing. The contribution of these alignments to the relative standard uncertainty is then evaluated to $3.3 \times 10^{-8}$.

7. Conclusion

Measurements of the Planck constant $h$ were performed in air during the summer of 2014. The value $6.626 068 8(20) \times 10^{-34}$ Js has been extracted from these data. It differs in relative terms by $-0.05 \times 10^{-7}$ from the $h_{90}$ value (figure 8) and by $-1.1 \times 10^{-7}$ from that of the 2010 CODATA adjustment of $h$. The relative standard uncertainty associated with this measurement $3.1 \times 10^{-7}$ is thus larger than these differences.

Currently, the major contributions to the published uncertainty arise from voltage measurements, velocity measurements and suspension alignments. These contributions are not yet a limitation of our experiment and it will certainly be possible to reduce them. To achieve this, several works are planned, and notably:

- Voltage measurements during static and dynamic phases will be made directly against a 1V programmable Josephson voltage standard (PJVS) [39, 40], available in the laboratory.
- Some improvements will be implemented to adjust the verticality of the six gaussian laser beams (going through the position detectors and the corner cubes of the coil) using a mercury pool and a 1.1 m focal length telescope.
- Another 500g mass artifact, made from Pt–Ir alloy provided by the English company Johnson–Matthey, and polished, adjusted and calibrated by the BIPM will be used for the next campaign of measurements, notably because its magnetic susceptibility is lower than that of Alacrite and its density is higher.

Acknowledgments

The authors would like to thank F Bielsa for his contribution to the optical matter in particular and instrumentations

Table 4. Estimation of the watt ratio error term from relative parasitic velocities, parasitic forces and torques.

<table>
<thead>
<tr>
<th>Relative parasitic forces and torques</th>
<th>Relative parasitic linear and angular velocities</th>
<th>Individual component of watt ratio error term</th>
</tr>
</thead>
<tbody>
<tr>
<td>$F_x/F_z$</td>
<td>$v_x/v_z$</td>
<td>$F_xv_x/F_zv_z$</td>
</tr>
<tr>
<td>$F_y/F_z$</td>
<td>$v_y/v_z$</td>
<td>$F_yv_y/F_zv_z$</td>
</tr>
<tr>
<td>$M_x/F_z$</td>
<td>$\omega_x/v_z$</td>
<td>$M_x\omega_x/F_zv_z$</td>
</tr>
<tr>
<td>$M_y/F_z$</td>
<td>$\omega_y/v_z$</td>
<td>$M_y\omega_y/F_zv_z$</td>
</tr>
<tr>
<td>$M_z/F_z$</td>
<td>$\omega_z/v_z$</td>
<td>$M_z\omega_z/F_zv_z$</td>
</tr>
</tbody>
</table>

Figure 8. Summary of recent determinations of the Planck constant with watt balances. A silicon sphere determination is added as well as the last CODATA recommendation. Values are referred to the $h_{90}$ value. (For watt balances results, see [9, 12, 41, 42, 45]; for IAC silicon sphere, see [43]; for CODATA, see [44].)
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