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REVIEW ARTICLE

Watt balance experiments for the determination of the Planck constant and the redefinition of the kilogram*

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Abstract

Since 1889 the international prototype of the kilogram has served as the definition of the unit of mass in the International System of Units (SI). It is the last material artefact to define a base unit of the SI, and it influences several other base units. This situation is no longer acceptable in a time of ever increasing measurement precision.

It is therefore planned to redefine the unit of mass by fixing the numerical value of the Planck constant. At the same time three other base units, the ampere, the kelvin and the mole, will be redefined. As a first step, the kilogram redefinition requires a highly accurate determination of the Planck constant in the present SI system, with a relative uncertainty of the order of 1 part in 10^8.

The most promising experiment for this purpose, and for the future realization of the kilogram, is the watt balance. It compares mechanical and electrical power and makes use of two macroscopic quantum effects, thus creating a relationship between a macroscopic mass and the Planck constant.

In this paper the background for the choice of the Planck constant for the kilogram redefinition is discussed and the role of the Planck constant in physics is briefly reviewed. The operating principle of watt balance experiments is explained and the existing experiments are reviewed. An overview is given of all presently available experimental determinations of the Planck constant, and it is shown that further investigation is needed before the redefinition of the kilogram can take place.

1. Introduction

The International System of Units (SI) is the most widely used system of units for measurements in commerce and science. The SI is based on seven base units (metre, kilogram, second, ampere, kelvin, mole and candela) from which other units are derived [1]. The SI was officially adopted by the General Conference on Weights and Measures (CGPM) in 1960, but has its origins in the Metre Convention of 1875.

The kilogram, the unit of mass, is nowadays the last base unit which is still defined by a manmade object, the international prototype of the kilogram. This is a cylinder made of an alloy of 90% platinum and 10% iridium, cast by Johnson Matthey in 1879, and kept at the International Bureau of Weights and Measures (BIPM) since then. It was ratified as the international prototype of the kilogram in 1889 during the first meeting of the General Conference and still serves today to define the unit kilogram.

Every measurement in the world expressed using the kilogram unit is ultimately traceable to the international prototype of the kilogram. Most Member States of the BIPM hold national prototypes which are compared from time to time against the working standards of the BIPM, which are traceable to the international prototype. Although this system has worked quite well until now, and ensures uniform mass measurements throughout the world, unit definitions which can be realized anywhere are preferable. The international

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prototype as a material object could also be damaged with obvious negative consequences for mass metrology.

Three comparisons were carried out between the international prototype, its official copies at the BIPM and national prototypes in the 1880s, in 1946 and in 1989 and these comparisons indicated a trend towards larger mass values for most of the prototypes with respect to the international prototype, of approximately 50 µg over 100 years [2]. In relative terms this corresponds to a mass change of five parts in 10^8 over 100 years. This observation might be interpreted as an indication that the international prototype loses mass. However, there is no clear explanation of the situation, because over the last century a more stable mass reference did not exist. In the past, these changes did not lead to a problem, but due to much improved measurement accuracy these changes would have noticeable consequences in the future. It cannot be excluded that all prototypes show a common mass drift in addition to this relative drift, which cannot be detected by comparisons between the prototypes, and the magnitude of which is completely unknown at present.

As a material object, the international prototype is subject to contamination. During the last verification, it was cleaned and washed following the standard BIPM technique. The international prototype is the reference for one kilogram immediately after cleaning and washing. Although it had been kept under three glass bells, in air, since the previous verification about 40 years before, the related mass loss was about 60 µg [3]. Similar changes were observed for the other kilogram prototypes. After cleaning, the mass slowly increases again, stabilizing at a rate of about 1 µg/year. Surface contamination therefore limits the achievable uncertainty in mass dissemination at the highest level.

The definitions of several other base units depend on the kilogram. This is the case for the ampere, the mole and the candela. Typical measurement uncertainties in chemistry and photometry are such that a possible slight drift of the kilogram, and consequently of the mole and the candela, would go unnoticed. Practical electrical metrology has since 1990 been based on the use of the Josephson effect and the quantum Hall effect, together with conventional values of the Josephson constant, $K_{\mathrm{J}\text{-}90}$, and the von Klitzing constant, $R_{\mathrm{K}\text{-}90}$ [4]. Conventional values were chosen because the reproducibility of both effects was better than the knowledge of the Josephson constant K_J and the von Klitzing constant R_K in SI units. The use of conventional values allows us to benefit from the very high reproducibility of the Josephson effect (parts in 10^{10}) and the quantum Hall effect (parts in 10^9) but, strictly speaking, takes electrical metrology outside of the SI. Realizations of electrical units based directly on the SI definition of the ampere require complex electromechanical instruments and suffer from comparatively large uncertainties: the ampere can be realized with a current balance with an uncertainty of 4 parts in 10^6 [5], the voltage balance allows realization of the volt to within 3 parts in 10^7 [6], and the calculable capacitor realizes the farad to within 2 parts in 10^8 [7].

The main shortcomings of the present SI system are the use of an artefact to define the unit of mass and the fact that the practical realization of electrical units is not based directly on the SI definition of the ampere but on conventional values for the Josephson and the quantum Hall effects. These are the main drivers for the planned redefinition of the kilogram and the ampere, which will remedy both problems. It is expected that the unit of thermodynamic temperature, the kelvin, and the unit of amount of substance, the mole, will be redefined at the same time [8, 9].

This paper describes the possible improvements resulting from a redefinition of the kilogram with reference to a fixed numerical value of a fundamental constant (section 2). To understand why the particular constant, the Planck constant, was chosen requires some understanding of the realization of electrical units (section 3). A short review of the introduction of the Planck constant into physics, its role and early measurements is given in section 4. One way to link the Planck constant to a macroscopic mass is via the watt balance experiment, the principle of which is described in section 5. An overview of existing experiments is given in section 6. Section 7 discusses some considerations for the design of watt balances. The following section provides an overview of the published results of watt balance experiments and section 9 draws conclusions on the status of the redefinition of the kilogram. Section 10 summarizes this paper.

2. Definition of the kilogram based on a fundamental constant

Fundamental constants are, to the best of our present knowledge, invariant in time and space and therefore well suited as a basis for a system of measurement units. The definition of the second has, since 1967, been based on the frequency of a transition of the caesium atom and the metre has been defined, since 1983, by a fixed numerical value of the speed of light in vacuum [1].

Since the international prototype of the kilogram might drift by about 50 µg per century, a definition based on a fundamental constant should allow a realization of the kilogram with a relative uncertainty of not more than a few parts in 10^8, equivalent to several tens of micrograms, so presenting an advantage over periods significantly shorter than a century.

The definition of the kilogram can be based on several different fundamental constants. Fixing the numerical value of a fundamental constant only allows us to define the kilogram if the dimension of the constant contains the quantity mass. For example, if the present definition of the kilogram is abolished, it could be redefined by the assignment

$$h = 6.626\,06X\ \mathrm{J\,s} = 6.626\,06X\ \mathrm{kg\,m^2\,s^{-1}}, \qquad (1)$$

where the symbol X stands for one or more digits to be added to the numerical value at the time when the definition will be adopted. The value or the 'size' of the Planck constant is given by nature, the numerical value is fixed by this assignment, the metre and the second are defined in the SI. This equation therefore defines the unit kilogram. To guarantee that the redefinition does not lead to a step change in the size of the unit, the assigned value needs to be the correct value (within the uncertainty of its determination) of the constant in the present SI system.

In addition, an experiment is needed to establish a relationship between a macroscopic mass and the relevant constant. From the point of view of the existing experimental techniques, the most interesting are the Planck constant h, the Avogadro constant N_A or an atomic mass m_x. A direct link between a macroscopic mass and the Planck constant can be established with a watt balance, as will be shown in section 5, and between a macroscopic mass and the Avogadro constant or an atomic mass by counting atoms in a nearly perfect silicon sphere [10]. The latter two options can be easily seen as equivalent since they are linked by the relationship

$$m_x N_A = A_r(x) M_u \tag{2}$$

with $A_r(x)$ being the relative atomic mass (with a relative uncertainty typically below 1 part in 10^9) and $M_u = 0.001\,\mathrm{kg\,mol^{-1}}$ being the molar mass constant (with no uncertainty). If the particular choice of ^{12}C is made, which serves to define the mole and the atomic mass unit, the uncertainty of its relative atomic mass is zero and

$$m_{^{12}\mathrm{C}} N_A = 0.012\,\mathrm{kg\,mol^{-1}}. \tag{3}$$

The numerical values of the atomic mass of ^{12}C and of N_A can be fixed to be strictly equivalent, since fixing $m_{^{12}\mathrm{C}}$ fixes N_A and vice versa. In practice, any other atomic species can be chosen, because the uncertainties of the relative atomic masses in equation (2) are significantly smaller than the uncertainties of experimental determinations of N_A.

The Planck constant and the Avogadro constant are linked to each other through the definition of the Rydberg constant by the following equation:

$$N_A h = \frac{A_r(e) c \alpha^2}{2R_\infty} M_u. \tag{4}$$

Since the relative uncertainty of the relative atomic mass of the electron $A_r(e)$ is 4.0×10^{-10}, that of the Rydberg constant R_∞ is 5.0×10^{-12}, that of the fine-structure constant α is 3.2×10^{-10} [11] and the speed of light c and the molar mass constant M_u have no uncertainty, the Avogadro constant can be deduced from the Planck constant, and vice versa, with a negligible additional uncertainty which is about twice the relative uncertainty of the fine-structure constant, that is 7×10^{-10}. Therefore the choice of the constant which will serve as the basis for the new definition of the mass unit has no practical implications on the choice of the experimental method used for the realization of the kilogram. Independently of the choice of h or N_A, the realization of the new kilogram could be carried out with either a watt balance or by counting atoms in a silicon sphere.

Considering only mass metrology, it appears to be most appropriate to base the new definition of the kilogram on a fixed numerical value of an atomic mass, because it is a quantity of the same type as the mass of the international prototype of the kilogram. One kilogram would then be specified as the mass of a large number of atoms of a certain type. Such a definition would also have the advantage of being conceptually very simple and easy to teach. However, fixing the numerical value of the Planck constant presents great advantages for electrical metrology as will be shown in the next section. Therefore a general consensus has been reached that the kilogram shall be redefined based on a fixed numerical value of the Planck constant.

The new definition of the kilogram could thus have the following form, proposed by the Consultative Committee for Units (CCU):

'The kilogram, kg, is the unit of mass; its magnitude is set by fixing the numerical value of the Planck constant to be equal to exactly $6.626\,06X \times 10^{-34}$ when it is expressed in the unit $\mathrm{s^{-1}\,m^2\,kg}$, which is equal to J s.'

The symbol X represents one or more digits to be added to the numerical value at the time when the definition will be adopted. This definition is equivalent to the exact relation $h = 6.626\,06X \times 10^{-34}\,\mathrm{s^{-1}\,m^2\,kg}$. As explained for equation (1), such an assignment has the effect of defining the unit kilogram.

An important aspect for the redefinition of any unit is its continuity. The newly defined unit shall be of the same size as the previous unit so that the results of past measurements need not be changed. The unavoidable discontinuity shall be smaller or at least comparable to the uncertainty with which the unit can be realized. In the case of the redefinition of the kilogram this requires a measurement of the Planck constant in the present SI system to determine the numerical value to be fixed. Another important aspect is that the future definition can be practically realized with a sufficiently small uncertainty. Experts in mass metrology who participate in the Consultative Committee for Mass and Related Quantities (CCM) estimate that the uncertainty for the realization of the kilogram should not be larger than 2 parts in 10^8, mainly because of the requirements of legal metrology (see section 9).

The target uncertainty for the determination of the Planck constant is therefore 2 parts in 10^8. As already stated above, the Planck constant can be directly determined with a watt balance or indirectly, via the definition of the Rydberg constant, by counting atoms in a silicon sphere. The watt balance approach presents an advantage in that it can be carried out by a single laboratory. This allows for the development of several watt balances and the possibility of their comparison. The Avogadro approach had required a large international collaboration [10] and it would require considerable resources to repeat it.

3. Realization of the electrical SI units

This section presents the advantages for electrical metrology of basing the new kilogram definition on a defined numerical value for the Planck constant. The electrical SI base unit is the ampere, with the following definition:

'The ampere is that constant current which, if maintained in two straight parallel conductors of infinite length, of negligible circular cross section, and placed 1 m apart in vacuum, would produce between these conductors a force equal to 2×10^{-7} newton per metre of length' [1].

The equation for the force per length between two parallel wires with the same current I and at the distance d:

$$\frac{F}{l} = \frac{\mu_0}{2\pi} \frac{I^2}{d} \tag{5}$$

shows that the ampere definition has the effect of defining the numerical value of the magnetic constant $\mu_0 = 4\pi \times 10^{-7}\,\mathrm{N\,A^{-2}}$. Therefore each experimental method linking a current to a force and using this value for the magnetic constant, in principle, allows us to realize the SI ampere.

The experimental situation described in the definition cannot be applied in practice, not even as an approximation, due to the requirements of infinitely long straight conductors of negligible cross section. The experiment becomes feasible, however, if the conductors are not straight, but have the form of two coils. One coil can be fixed, the other suspended from a balance and the electromagnetic force between them can be compared with the weight of a mass. Such 'current balance' experiments have been carried out at the National Physical Laboratory (NPL) in the UK [5] and the NBS (now the National Institute of Standards and Technology (NIST)) in the USA [12]. If the same current I flows through the fixed and the suspended coils, the equation for force balance is

$$F = \frac{\partial M}{\partial z} I^2 = mg, \tag{6}$$

where F is the force, $\partial M/\partial z$ is the vertical gradient of the mutual inductance between the two coils, m the mass and g the gravitational acceleration. Determination of the current requires knowledge of the mutual inductance, which is a geometric factor, which also includes the magnetic constant μ_0. Although the geometry of the coils was determined at the micrometre level, it dominated the total uncertainty at about 4 ppm. Another limiting factor was the very small force, 0.04 N for the NPL current balance at a current of 1.02 A. Since no significant improvement appeared possible at the time, the last current balance experiments were carried out in the 1960s.

An SI realization of the unit of electric potential difference, the volt, has to be based on the definition of the magnetic constant by the ampere definition and the relationship $1\,\mathrm{V} \times 1\,\mathrm{A} = 1\,\mathrm{W}$. An experimental realization is possible in the form of a voltage balance, which compares an electrostatic force with a mechanical force [6, 13]. In one experiment, carried out by the CSIRO (Australia) [6], a high voltage was applied to a horizontal electrode placed closely above the surface of a mercury pool, which formed the opposing electrode of a plate capacitor. The electrostatic force lifted the mercury surface, and the height change was measured with an interferometer. For an applied voltage of several kilovolts, the height change was of the order of 1 mm. The advantage of this experiment is that it replaces the measurement of very small forces, as in the current balance, by a very sensitive interferometric measurement. The achievable uncertainty was mainly limited by the density determination of the mercury and by the interferometry to 0.27 ppm.

The electrical unit which can be realized according to its SI definition with the smallest relative uncertainty is the unit of capacitance, the farad. In 1956 Thompson and Lampard showed [14] that the capacitances C_1 and C_2 between opposite electrodes, per unit length, of a four-electrode system, the intersection of which with a perpendicular plane encloses a two-dimensional area (with gaps of negligible size), and which does not change its shape over an infinitely long distance perpendicular to this plane, are related by

$$\exp(-\pi C_1/\varepsilon_0) + \exp(-\pi C_2/\varepsilon_0) = 1, \tag{7}$$

where ε_0 is the electric constant. If the system is symmetric, the two capacitances are identical and can be derived from a single length measurement, which can be carried out by interferometry. The requirement of infinitely long electrodes can be circumvented by a difference measurement for two electrode lengths, which eliminates edge effects. The effective capacitor length is defined by grounded guard electrodes in the space between the main electrodes. All existing calculable capacitors, also called Thompson–Lampard cross-capacitors, employ electrodes of cylindrical shape [7, 15]. The smallest uncertainty in the realization of the farad, 2.4 parts in 10^8, was obtained at the NIST (USA) in 1998 [7]. This experiment was limited by the unavoidable geometric imperfections of the electrode bars. At present several National Metrology Institutes (NMIs) are developing new calculable capacitors with high quality electrodes, with the objective of reducing the uncertainty by a factor of at least 2.

To conclude, there is no known technique which allows us to realize an electrical unit according to its SI definition to better than 2 parts in 10^8. It should also be noted that all the electromechanical experiments necessary for an SI realization of electrical units place very high demands on the quality of the mechanical realization of the apparatus. On the other hand, a class of experiments exists, which allow an extremely *reproducible* realization of electrical quantities without requiring precision mechanics. These are macroscopic quantum effects, which directly link electrical quantities to fundamental physical constants. Of particular interest in this respect are the Josephson effect and the quantum Hall effect.

In 1962 Brian Josephson predicted three closely related effects associated with the tunnelling of Cooper pairs through a thin isolating junction between two superconductors [16]. The effects were observed first in the following year by Sidney Shapiro [17]. A Cooper pair current can flow through the junction, up to a certain critical value, without a related voltage drop (dc Josephson effect). If a constant voltage U (up to a maximum value) is applied to the junction, the supercurrent in the junction oscillates at the frequency

$$f_\mathrm{J} = \frac{2e}{h} U = K_\mathrm{J} U \tag{8}$$

(ac Josephson effect) where $K_\mathrm{J} = 2e/h$ is the Josephson constant, e is the elementary charge and h the Planck constant. If an oscillatory current is induced by irradiating the junction with microwaves of frequency f_m, constant voltage steps of value

$$U_n = n \frac{h}{2e} f_\mathrm{m} = \frac{n f_\mathrm{m}}{K_\mathrm{J}} \tag{9}$$

appear (inverse ac Josephson effect). In this experiment, the oscillating supercurrent related to U_n via equation (8) synchronizes with the externally applied frequency or its harmonics. The exact value of n and the specific voltage level are selected by choosing the value of a dc current through the junction.

As far as is known, the value of the Josephson constant K_J is independent of all specific junction properties, such as material, geometry, size and temperature. Therefore, the Josephson effect allows the production of voltage values, which depend only on two fundamental constants (h and e) and on an accurately measurable frequency. A single junction, driven at 70 GHz, produces voltage steps of 150 µV. Nowadays, junction series arrays are available, which allow output voltages up to 10 V to be achieved. For a review of Josephson voltage standards, see [18].

It has been found that voltages of two single junctions agree to within 3×10^{-19} [19], whereas two complete Josephson voltage standards agree typically within 1×10^{-10}. On the other hand, the Josephson constant is known in SI units, $\mathrm{Hz\,V^{-1}}$, only to within 2.2×10^{-8} [11], principally because the electrical SI units *themselves* cannot be realized with smaller uncertainty, as discussed previously. To remedy this situation and to allow the Josephson effect to be used for voltage metrology, the International Committee for Weights and Measures (CIPM) adopted in 1990 a conventional value of the Josephson constant, $K_{J\text{-}90} = 483\,597.9\,\mathrm{GHz\,V^{-1}}$ [4]. Nearly all NMIs now use the Josephson effect together with $K_{J\text{-}90}$ to represent the volt. The word 'representation' is used, because the unit volt derived in this way is strictly speaking not an SI realization, which would have to be based on the value of μ_0 defined in the ampere definition as explained at the beginning of this section. Voltages derived from the Josephson effect and the use of $K_{J\text{-}90}$ are often identified by the symbol U_{90} to distinguish them from voltages in the SI.

The quantum Hall effect was observed experimentally in 1980 by Klaus von Klitzing [20]. It is observed in samples which contain two-dimensional electron gases, such as GaAs heterostructures, Si-MOSFETs and graphene. When such structures are subjected to very low temperature and a strong magnetic field perpendicular to the electron plane, the electronic states are grouped into separated Landau levels. The structure then exhibits a quantized Hall resistance

$$R_H = \frac{1}{i}\frac{h}{e^2} = \frac{R_K}{i}, \qquad (10)$$

where i is an integer quantum number and R_K is the von Klitzing constant. As far as is known, this constant does not depend on the particular structure of the device, its material, the size etc. The quantum Hall effect therefore is a universal resistance standard, which derives a resistance from the two fundamental constants h and e. The resistance value for the commonly used quantum number $i = 2$ is 12 906 Ω. For a review of quantum Hall resistance standards, see [21].

Different quantum Hall resistance standards have been compared and found to agree typically to within parts in 10^9 and sometimes even better [22, 23]. In 1990, when the conventional values for the Josephson constant and the von Klitzing constant were introduced, the latter was known in the SI only to within several parts in 10^8, the typical uncertainty of calculable capacitors. The fact that R_K is related to the fine-structure constant α by

$$\alpha = \frac{\mu_0 c e^2}{2h} = \frac{\mu_0 c}{2R_K} \qquad (11)$$

allows us in principle to derive R_K from very accurate determinations of α (since the speed of light c and the magnetic constant μ_0 have no uncertainty). These are, however, indirect determinations of R_K, which are based on the validity of the equation $R_K = h/e^2$, which is generally supposed, but not yet strictly proven. In analogy to the Josephson effect, a conventional value for the von Klitzing constant, $R_{K\text{-}90} = 25\,812.807\,\Omega$, was introduced in 1990 [4]. Many NMIs use the quantum Hall effect together with the conventional value $R_{K\text{-}90}$ to represent the ohm. As in the case of the volt, the word 'representation' indicates that this is not strictly speaking a realization of the SI unit ohm. Resistances derived from the quantum Hall effect and the conventional value $R_{K\text{-}90}$ are often written as R_{90} to distinguish them from resistances in the SI.

The common property of both effects is that they establish a relationship between a macroscopic measurand, a voltage and a resistance, and fundamental constants, the elementary charge and the Planck constant. Both effects are nowadays widely used as standards for resistance and voltage metrology [18, 21] but they do not realize the electrical units in accordance with their SI definition. As shown above, the SI definition of the ampere requires that the magnetic constant $\mu_0 = 4\pi \times 10^{-7}\,\mathrm{N\,A^{-2}}$, whereas the quantum-based units are derived from conventional values $R_{K\text{-}90}$ and $K_{J\text{-}90}$ because their 'true' SI values are not known exactly. If the elementary charge e and the Planck constant h were to have exactly known numerical values in the SI, then the conventional values could be abandoned and quantum-based electrical units would become direct SI realizations.

We now come to the main advantage of fixing the numerical value of the Planck constant in the kilogram definition instead of fixing the value of the Avogadro constant. When h is fixed to define the kilogram and the elementary charge e is fixed to define the ampere, both the Josephson constant $K_J = 2e/h$ and the von Klitzing constant $R_K = h/e^2$ will become exactly known. Therefore, the need for the conventional constants $K_{J\text{-}90}$ and $R_{K\text{-}90}$, as discussed above, will cease and the Josephson and quantum Hall effects will become direct realizations of the SI. The plan for the new SI is therefore to redefine the kilogram by fixing the numerical value of h and to redefine the ampere by fixing the numerical value of e. In addition, the redefinition the kelvin is foreseen by fixing the numerical value of the Boltzmann constant k and the redefinition of the mole by fixing the numerical value of the Avogadro constant N_A, both of which are not within the scope of this paper [8, 9].

4. The Planck constant in physics

This section reviews briefly the role of the Planck constant in physics and some early experimental determinations.

At the end of the 19th century physics appeared to be largely understood and described by the theories of classical mechanics, classical electrodynamics and classical and statistical thermodynamics. It is reported that Philipp von Jolly, professor at the University of Munich, explained in 1874 to the 16 year old Max Planck that 'in this science almost everything is already discovered, and all that remains is to

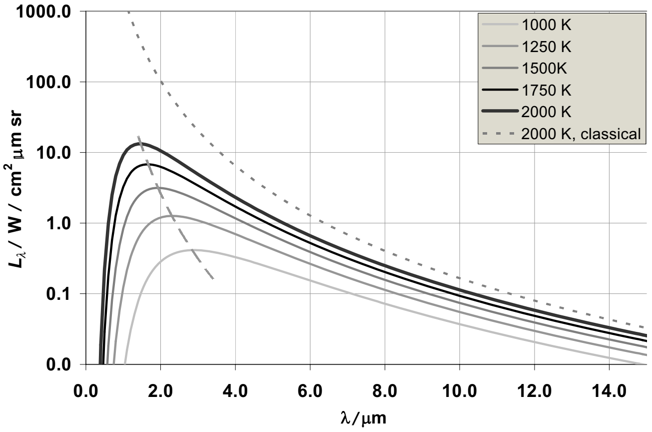

Figure 1. Calculated spectral radiance distributions of blackbody radiators at different temperatures. The dotted curve shows the classical expectation according to the Rayleigh–Jeans law (equation (13)).

fill a few unimportant holes'. Planck replied that he did not wish to discover new things, but only to understand the known fundamentals of the field. As we know, he became one of the founders of quantum mechanics.

One of the unsolved problems at this time was the theoretical description of the thermal radiation of a blackbody. A blackbody is an idealized body, which absorbs all incident radiation so that there is no reflection and no transmission. According to Kirchhoff's law of thermal radiation, a blackbody, which has the highest possible absorption coefficient $\alpha = 1$, also has the highest possible emission of thermal radiation, at a given temperature and wavelength. The emission can be described by the quantity spectral radiance

$$L_\lambda = \frac{\mathrm{d}^3\Phi}{\cos\varepsilon\,\mathrm{d}A\,\mathrm{d}\Omega\,\mathrm{d}\lambda} \tag{12}$$

which is the radiant flux $\mathrm{d}^3\Phi$ (in watts) divided by the wavelength interval dλ, by the emitting surface element dA and the solid angle dΩ forming the angle ε with the direction normal to the surface. At a given temperature and wavelength a blackbody has the highest possible spectral radiance of all bodies. Thermal radiation emitted by a blackbody is called blackbody radiation. The theoretical description of the spectrum of blackbody radiation was one of the remaining physical problems to be solved at the end of the 19th century (figure 1).

In practice, a good approximation to a blackbody can be achieved by a closed cavity with uniform temperature walls. The blackbody radiation inside the cavity can be observed by a small hole in the wall, which emits the blackbody spectral radiance corresponding to the temperature of the walls.

A classical description had to be based on electromagnetic theory and statistical thermodynamics. The laws of electrodynamics allowed the determination of the density of oscillator states in the cavity and the equipartition theorem states that each degree of freedom (each oscillator) carries the energy of $kT/2$, where k is the Boltzmann constant and T the temperature. The result for the spectral energy density and the spectral radiance is then

$$u_\lambda(T,\lambda) = \frac{8\pi}{\lambda^4}kT, \qquad L_\lambda(T,\lambda) = \frac{2c}{\lambda^4}kT, \tag{13}$$

which is known as the Rayleigh–Jeans law. This equation diverges towards infinity for short wavelengths (figure 1). Classical theories could therefore not describe blackbody radiation correctly. Interestingly, the total *spectrally integrated* emission can be derived correctly from classical electromagnetism and thermodynamics in the form of the Stefan–Boltzmann law for the energy density

$$u(T) = \frac{4\sigma}{c}T^4, \qquad L = \frac{\sigma}{\pi}T^4, \tag{14}$$

where σ is the Stefan–Boltzmann constant, which had to be determined experimentally.

In 1896 Wilhelm Wien empirically derived the so-called Wien radiation law, in analogy to the Maxwell–Boltzmann velocity distribution:

$$L_\lambda(\lambda,T) = \frac{c_1}{\pi}\frac{1}{\lambda^5}\frac{1}{\mathrm{e}^{c_2/\lambda T}} \tag{15}$$

with two empirical constants c_1 and c_2. Until mid-1900 this law was compatible with the existing measurements but could not be theoretically derived. In autumn 1900 new results of precision measurements of the blackbody spectrum at the PTR (now PTB, Germany) showed, however, systematic deviations in the long-wavelength range. To obtain an equation which would describe the experimental data, Max Planck proposed intuitively the slightly modified formula

$$L_\lambda(\lambda,T) = \frac{c_1}{\pi}\frac{1}{\lambda^5}\frac{1}{\mathrm{e}^{c_2/\lambda T}-1}, \tag{16}$$

which indeed agreed very well with the observations. The next task was to derive this equation from the laws of electromagnetism and thermodynamics. To achieve this goal Planck had to make the assumption that energy could be exchanged between matter and the electromagnetic radiation field not continuously but only in the form of small 'energy elements' ε, proportional to the frequency ν such that $\varepsilon = h\nu$ [24]. This was the first time that the quantity h we now call the Planck constant appeared in an equation of physics. The Planck equation then takes the form

$$L_\lambda(\lambda,T) = \frac{2hc^2}{\lambda^5}\frac{1}{\mathrm{e}^{hc/\lambda kT}-1}, \tag{17}$$

where k is the Boltzmann constant which Planck also introduced in the same paper.

In Planck's understanding, it was not the electromagnetic field itself which is quantized, but the quantization was a consequence of the complicated, but not yet understood, interactions between matter and radiation. It was Albert Einstein in 1905 [25] who proposed that electromagnetic energy itself is distributed discontinuously in the form of energy quanta, nowadays called photons.

As we now know, the Planck constant appears in many other equations of quantum mechanics, such as for example in those describing the energy levels of the hydrogen atom and the quantization of particle spin. It also is central for the Heisenberg uncertainty principle which limits the possibility of simultaneous exact determinations of position and momentum to

$$\Delta x \Delta p_x \geqslant \frac{\hbar}{2}, \tag{18}$$

where $\hbar$ is defined as $h/2\pi$. It appears in the Schrödinger equation which describes the temporal evolution of quantum states and in the formula for the de Broglie wavelength. As observed earlier, it also occurs in the description of macroscopic quantum effects, as the Josephson effect (equations (8) and (9)) and the quantum Hall effect (equation (10)).

One of the first precision measurements of the Planck constant, which supported Einstein's quantum hypothesis, was carried out in 1916 by Millikan using the photoelectric effect [26]. In this experiment UV and visible light between 253 nm and 546 nm from a mercury lamp liberated electrons from sodium and lithium surfaces in a vacuum. The energy of the photoelectrons was determined by applying an increasing negative voltage to the collecting electrode until the photocurrent dropped to zero. To control the influence of the work function (the energy needed for the electron to leave the metal surface) he designed an apparatus which allowed the production of a new clean surface under vacuum before each measurement. The relationship between the stopping potential U, the frequency ν and the work function W, based on the quantum hypothesis, is

$$U = \frac{h}{e}\nu - \frac{W}{e}. \tag{19}$$

Millikan's results confirmed the linear relationship between frequency and stopping potential which strongly supported the quantum hypothesis. From the slope of the curve he determined a value for h/e, which together with his result for the elementary charge from the oil-drop experiment, resulted in $h = 6.57 \times 10^{-34}$ J s with a 'precision' of 0.5%. This result deviates 0.8% from the value of the 2010 CODATA adjustment of fundamental constants, mainly because Millikan's value for the elementary charge was too small by 0.7%.

Nowadays, the most accurate experiments for the determination of the Planck constant are the silicon sphere experiment and the watt balance. The next section describes the operating principle of the watt balance experiment.

5. The principle of the watt balance experiment

A watt balance establishes a relationship between a macroscopic mass m and the Planck constant h. Whereas m is the mass of a macroscopic object, the Planck constant is the fundamental constant of quantum physics which describes the behaviour of the microscopic world. The watt balance therefore needs to establish a link between the very different domains of the macroscopic and the microscopic worlds. This link is provided by the two macroscopic electrical quantum effects described in section 3: the Josephson effect and the quantum Hall effect.

The common property of both effects is that they establish a relationship between a macroscopic measurand, a voltage (equation (9)) and a resistance (equation (10)), and fundamental constants, the elementary charge e and the Planck constant h. The watt balance experiment takes advantage of this property to relate an electrical power to the Planck constant. An electrical power P_{el} takes the form

$$P_{\text{el}} = UI = UU'/R, \tag{20}$$

where the current I is measured as the voltage drop U' over a resistance R. The value of the resistance can be determined with respect to the quantized Hall resistance (indicated by R_{90}) and the voltages can be measured with respect to a Josephson voltage standard (U_{90}), using the conventional constants $R_{\text{K-90}}$ and $K_{\text{J-90}}$:

$$R = R_{90}\frac{R_{\text{K}}}{R_{\text{K-90}}} \qquad \text{with } R_{\text{K}} = \frac{h}{e^2}, \tag{21}$$

$$U = U_{90}\frac{K_{\text{J-90}}}{K_{\text{J}}} \qquad \text{with } K_{\text{J}} = \frac{2e}{h}.$$

Therefore the electrical power can be expressed as

$$P_{\text{el}} = \frac{U_{90}U'_{90}}{R_{90}}\frac{K_{\text{J-90}}^2 R_{\text{K-90}}}{4}h. \tag{22}$$

The electrical power is then linked to the Planck constant. The elementary charge drops out from the equations.

Mechanical and electrical powers are of the same kind, they are expressed with the same unit, they can be compared with each other and be transformed into each other. The equations for mechanical power take different forms depending on the physical phenomenon being described, but are always dependent on a mass m, and other quantities such as velocity and acceleration. One special form is $P_{\text{m}} = mgv$, which describes the vertical motion of a mass m with the velocity v against the attraction of gravitational acceleration g.

A watt balance compares an electrical power in the form derived above with a mechanical power, which leads to

$$mgv = UI = \frac{U_{90}U'_{90}}{R_{90}}\frac{K_{\text{J-90}}^2 R_{\text{K-90}}}{4}h. \tag{23}$$

In principle every experiment which converts electrical power into mechanical power could establish a link between a mass and the Planck constant, for example an electric motor lifting a mass. However, every direct energy conversion suffers from energy losses, which would need to be quantified at the level of several parts in 10^9, which is very demanding. Experiments of this type, in the form of magnetic levitation of a superconducting body, have been carried out but were subsequently abandoned for this reason [27]. Direct energy conversion should therefore be avoided.

The three necessities for the watt balance are therefore (1) the use of the Josephson and quantum Hall effects, (2) the equivalence of electrical and mechanical power and (3) a clever

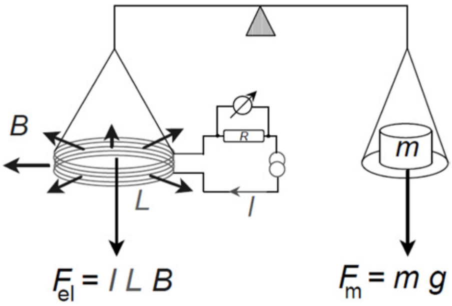

Figure 2. Static phase of the watt balance measurement. The weight of the test mass m is balanced against the Lorentz force on a coil with wire length L and current I hanging in a magnetic field with a radial flux density B.

measurement scheme that avoids energy losses entering into the measurement equation.

The watt balance experiment realizes these three principles. To avoid direct energy conversion, Bryan Kibble from the NPL proposed in 1976 that the experiment is carried out in two separate phases, the static phase and the dynamic phase [28]. In 1976 the quantum Hall effect had not yet been discovered and therefore the watt balance experiment could not establish a relationship between a mass and the Planck constant. It was originally a means to realize electrical power. Together with a calculable capacitor, providing a realization of the farad and the ohm, the SI unit ampere could be derived. It only became possible to determine the Planck constant with a watt balance after the discovery of the quantum Hall effect by Klaus von Klitzing in 1980.

In the static phase (figure 2) the weight of a mass m subjected to gravitational acceleration g is balanced by the Lorentz force on a coil with current I, hanging in a magnetic field such that the flux Φ passes through it:

$$mg = -I\frac{\partial\Phi}{\partial z}. \tag{24}$$

The direction of the gravitational acceleration defines the vertical direction z. Since a balance is only sensitive to vertical forces, even if small horizontal components of the Lorentz force exist, only the vertical flux gradient contributes to the force balance. In the case of a circular coil with wire length L placed in a horizontal, purely radial magnetic field B, this equation becomes

$$mg = IBL. \tag{25}$$

In reality the situation is more complex, because the mass m of the test mass needs to be isolated from the mass of the suspension which typically is several times larger than m. Therefore, a substitution principle is used. First a measurement is made with the test mass on the weighing platform and a current I is passed through the coil, which gives an upwards force. The test mass is then removed and the direction of the current reversed, so that the Lorentz force is now oriented downwards. A counterweight is used such that the balance is in equilibrium in both situations. Therefore the change in the mechanical force mg corresponds to a change in the Lorentz force of $2ILB$. The use of two currents of the same value but in opposite directions is advantageous, because the Joule heating is the same in both cases.

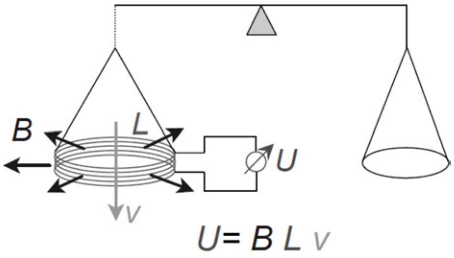

Figure 3. Dynamic phase of the watt balance experiment. The coil is moved vertically with velocity v through a magnetic field with the flux density B, which leads to an induced voltage U.

In the dynamic phase the coil is moved vertically through the same magnetic field as in the static phase. A voltage is induced which is given by

$$U = -\frac{\partial\Phi}{\partial t} = -v_z\frac{\partial\Phi}{\partial z}. \tag{26}$$

When the velocity v is not purely vertical, analogous contributions to the induced voltage from the horizontal movement exist. For a horizontal coil in a radial magnetic field (figure 3) this simplifies to

$$U = v_z BL, \tag{27}$$

where L is again the wire length on the coil.

If the velocity at the position where the static measurement is made is purely vertical, and if the magnetic field and the alignment of the coil with respect to the magnet do not change between the static and the dynamic measurements, equations (24) and (26) or (25) and (27) can be combined by eliminating the terms $\partial\Phi/\partial z$ or BL. The resulting equation

$$UI = mgv \tag{28}$$

is in the form of equation (23). The left side of this equation is an electrical power, the right side is a mechanical power, which explains the name of the watt balance experiment. Since the two phases are carried out separately, the current I and the voltage U are not present at the same time, and the power in this sense is a virtual power; the same holds true for the mechanical power. There is no direct energy conversion, which is of importance as discussed above.

The electrical quantities are measured using the Josephson effect and the quantum Hall effect, as described above, which leads to the measurement equation

$$mgv = UI = \frac{U_{90}U'_{90}}{R_{90}}\,\frac{K^2_{\mathrm{J}\text{-}90}R_{\mathrm{K}\text{-}90}}{4}h, \tag{29}$$

which shows that a relationship between a mass m and the Planck h constant has been established.

Watt balance experiments can be interpreted in another way, by making use of the definition of the Rydberg constant R_∞:

$$R_\infty = \frac{\alpha^2 m_{\mathrm{e}} c}{2h}, \tag{30}$$

where α is the fine-structure constant, known with a relative uncertainty of 3.2 parts in 10^{10} [11] and c is the speed of light, with no uncertainty because its numerical value is fixed in the definition of the metre [1]. The Rydberg constant is known with a relative uncertainty of 5.0 parts in 10^{12} [11]. Since the uncertainties of α and of R_∞ are much smaller than that of h, the mass of the electron m_e can be obtained from this equation with the same relative uncertainty as that of h. A watt balance can therefore be considered as a true balance which determines the mass of the electron by comparing it with a macroscopic test mass. In fact, this is the most accurate method available to determine the electron mass, which is therefore known with the same relative uncertainty as the Planck constant, that is 4.4 parts in 10^8 [11].

A watt balance needs to include the following components: (1) a magnet to create the magnetic flux density, (2) a force comparator to measure the vertical force, (3) a coil suspended from the force comparator exposed to the magnetic field of the magnet, (4) a mechanism to displace the coil vertically, (5) an interferometer to measure the coil velocity, (6) a current source to deliver the coil current, (7) a Josephson voltage standard to measure the induced voltage, (8) a quantum Hall resistance standard to calibrate the standard resistor, (9) a calibrated test mass, (10) a gravimeter to determine the gravitational acceleration and (11) devices to control the coil and magnet alignment. To avoid problems related to air buoyancy, convection and changes of refractive index, watt balance experiments are in general carried out in vacuum.

Detailed descriptions of existing experiments can be found in review articles [29, 30]. The measurement equation (equation (29)) shows which quantities need to be measured in a watt balance experiment. The electrical measurements require the determination of the induced voltage and the current-related voltage drop using a Josephson voltage standard, and the calibration of the resistor needed for the current measurement against the quantized Hall resistance. The resistance calibration needs to be carried out only occasionally because high quality resistors are very stable in time. The Josephson voltage standard forms an integral part of the experiment because both the induced voltage in the dynamic phase and the current-related voltage drop in the static phase are time-dependent and need to be measured in real-time. A quantized and accurately known Josephson voltage is opposed to the voltage to be measured and the small difference is determined with a voltmeter. The coil velocity v is obtained by interferometry. It is important that the measurements of velocity and induced voltage are well synchronized, because this leads to a high rejection of vibration-induced correlated noise in both signals. The value of the gravitational acceleration g needs to be known at the centre of mass of the test mass, which is inaccessible once the experiment is set up. One technique to achieve this is to establish a map of the variation of g in the laboratory with a relative gravimeter before the watt balance is installed. In addition, the absolute value needs to be known at least at one point. The absolute value of the gravitational acceleration at the centre of mass of the test mass can then be obtained by interpolation. The gravitational acceleration also varies in time by as much as 2.5 parts in 10^7 due to tidal forces from external bodies. This needs to be taken into account either by permanent g measurements or by modelling of the tidal effects. A correction needs to be applied for the gravitational effect of the watt balance itself. To determine a value for the Planck constant, the mass needs to be calibrated with respect to the present SI. Later, after the redefinition, the fixed numerical value of the Planck constant will become the basis for the determination of the mass.

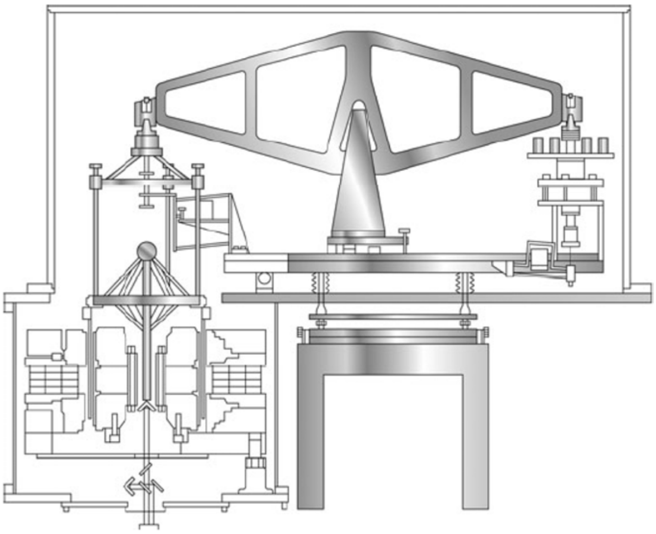

Figure 4. The NPL Mark II watt balance (courtesy of NPL). Top: the balance beam, left: the suspension with the coil hanging in the air gap of the magnet (bottom), right: an auxiliary magnet and a coil, used to tilt the balance beam.

6. The existing watt balance experiments

In this section, the main characteristics and distinctive features of the existing watt balance experiments are described. More details can be found in the review papers [29, 30]. The published results of the experiments are reviewed and compared in the following section.

The development of a watt balance at the NPL in the UK started soon after the proposal of the two-phase operation, described above, by Bryan Kibble in 1976 [28]. This experiment used a very heavy permanent magnet (6 t) to produce a uniform magnetic field, into which a figure-of-eight-shaped moving coil was placed. A large balance beam resting on a knife-edge was used for the force measurement. The final result of this experiment, with a relative uncertainty of 2 parts in 10^7, was published in 1990 [31]. At the same time, plans for an improved apparatus, the NPL Mark II watt balance, were presented (figure 4). This apparatus still used the large balance beam, but with a new magnet, which produced a radial magnetic field of 0.42 T inside a circular air gap, into which the circular coil was placed. A Michelson interferometer was used for the measurement of the velocity of about 1.3 mm s^{-1}. The movement of the coil was generated by tilting the balance beam. The test masses were made of gold-plated copper (1 kg and 0.5 kg) and of silicon (0.5 kg). A result was published in 2007 [32] with a relative uncertainty of 6.6 parts in 10^8.

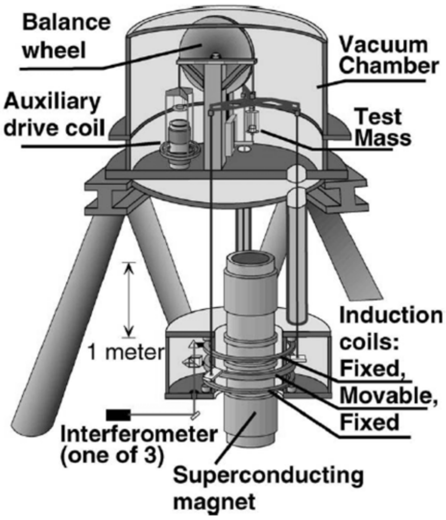

Figure 5. The NIST watt balance (courtesy of NIST).

The NPL then decided to stop this project and the experiment was transferred to the National Research Council (NRC) in Canada. Two possible systematic errors, related to the effect of the weight of the test mass on the structure of the apparatus, were discovered just before the experiment was shut down at the NPL, and the limited time available did not allow an in-depth analysis. As a consequence, the uncertainty of the final NPL result was increased to 2 parts in 10^7 [33]. It was believed that after analysis and elimination of this effect, it should be possible to approach a relative uncertainty of about 3 to 4 parts in 10^8 with this apparatus. The experiment was shipped to the NRC in mid-2009 [34]. After re-assembly measurements were made under similar conditions to those at the NPL and the mass exchange errors were investigated. It was found that a correction of -3.98×10^{-7} needed to be applied. The result [35] with an uncertainty of 6.5 parts in 10^8 was consistent with the previous NPL result. The experiment has now been modified to eliminate the mass exchange error and to reduce the uncertainty further.

Briefly after the proposal of the watt balance concept, the National Bureau of Standards (NBS), now the NIST in the USA, began to construct a watt balance. This apparatus used a conventional electromagnet to generate the magnetic flux. A result for the realization of the electrical watt was published in 1989 with a relative uncertainty of 1.3 parts in 10^6 [36]. This was followed by the development of a second apparatus which used two large superconducting solenoids wired in opposition to create a radial magnetic field of 0.1 T (figure 5). As a consequence of the size of the solenoids the whole apparatus is about 6 m high. Instead of a balance beam, a large balance wheel of 0.61 m diameter was chosen. Rotation of the wheel leads to a purely vertical movement of the coil, which is placed around the superconducting solenoids. The test masses of 1 kg are made of Au, PtIr and steel. A first result of this apparatus was published in 1998 with a relative uncertainty of 8.7 parts in 10^8 [37, 38]. The experiment was then largely rebuilt to eliminate many of the previous error sources, but the basic concept stayed the same. Further results were published in 2005 [39] and in 2007 [40] with relative uncertainties of 5.2 parts in 10^8 and 3.6 parts in 10^8, respectively, all results were consistent, and differed significantly from the NPL and NRC results, see figure 7. Since then, many tests for possible systematic errors have been made, to increase confidence in the correct operation of the instrument. A cooperation with the NRC has been started to find the origin of the discrepancy between the two instruments. Recently, NIST began a project to build a new watt balance for future mass dissemination.

The Federal Office of Metrology (METAS) in Switzerland began the development of a watt balance in 1997, characterized by two original ideas [30]. Instead of a 1 kg test mass a 100 g mass is used, which reduces the forces by a factor of ten and leads to a significant size reduction of the apparatus, in particular of the magnet. The second distinctive feature is that in the dynamic phase, the coil is disconnected from the balance and moved by a separate mechanical system. This allows the balance to always operate in an equilibrium position and avoids problems of hysteresis, but it requires a coil transfer between the balance suspension, for the static measurement, and the mechanical translation ‘seesaw’ system, for the dynamic phase. The METAS experiment is the only recent watt balance experiment to use a uniform magnetic field between two flat pole pieces, all others use radial fields. The work on the METAS watt balance has now led to a published result [41] with a relative uncertainty of 2.9 parts in 10^7. This uncertainty is dominated by alignment issues and the present apparatus has reached its limits. A new project, based on the experience gained from the first experiment, has started with the objective to reach a relative uncertainty close to 1 part in 10^8. A new magnet with radial symmetry, similar to one used at the BIPM, will be developed. To ensure purely vertical movement, a highly constrained mechanical guiding system (13 hinge stage) will be used. The new experiment shall be operational in 2013.

The Laboratoire National de Métrologie et d’Essais (LNE) in France commenced a watt balance project in the year 2000 and development started in 2002 [30, 42]. A distinctive feature of this experiment is that the force comparator is moved together with the coil by a precision guiding stage. The guiding stage is mechanically very rigid to ensure a close to vertical movement of the coil. A motorized translation stage moves the guiding stage at a velocity of 2 mm s^{-1}. A two-stage velocity control system is used to control the velocity of the coil very precisely. The permanent magnet produces a radial field of about 0.9 T in the centre of the air gap. The pole faces have been machined at micrometre accuracy to ensure that the variation of the magnetic field in the vertical direction is within 1 part in 10^4. The assembly of the watt balance is now almost complete and first measurements of the Planck constant will be carried out in mid-2012. The objective is to obtain a result in 2014, which can be taken into account in the fixing of the numerical value of the Planck constant.

At the BIPM, development of a watt balance began in 2003. The BIPM approach is to carry out the static and dynamic phases simultaneously [43, 44]. The derivation of the watt balance equation (equation (27)) depends on the term BL being constant between the two phases, otherwise corrections are necessary. To achieve this requires a constant magnetic field (B is a function of temperature) and the same alignment and position of the coil in both phases. When both phases are carried out at the same time, these requirements are relaxed. Although in this case conversion of electromagnetic and mechanical energy exists, energy losses due to friction or magnetic hysteresis are compensated for by the motor generating the coil displacement. Separation of the induced voltage from the resistive voltage drop, which results from the current flow, is required. One way to achieve this would be to use a superconducting coil, in which the resistive voltage drop does not exist. The BIPM is therefore carrying out a feasibility study for a future cryogenic experiment. Another possibility is to use a coil with two windings, one carrying the current and the other used to measure the induced voltage [45]. This work is focused on the development of a room-temperature experiment. In this experiment movement of the coil is driven by an electrostatic motor which is part of the coil suspension. A closed magnetic circuit is being developed which will screen the coil at its position in the air gap from external electromagnetic perturbations. A mechanical system for automatic correction of deviations of the coil trajectory from purely vertical has already been developed. Since early 2010, measurements of the Planck constant have been carried out, at present with a repeatability of the order of 1 part in 10^6. In 2012 the experiment will be moved to a new laboratory, where it will be placed in a vacuum system installed on a concrete block to reduce vibration. It is expected that the combined uncertainty will then be at the level of 1 part in 10^6. The long-term goal is to reach a level of several parts in 10^8 around 2015.

The experiment conducted by the National Institute of Metrology (NIM), China, follows a different approach which does not require a dynamic measurement phase [46, 47]. The electromagnetic force on the coil is created by the magnetic field of a second coil, aligned parallel to the coil which is suspended from the balance. The force equation, equivalent to equations (24) and (25), is given by

$$mg = \frac{\partial M}{\partial z} I_1 I_2, \tag{31}$$

where M is the mutual inductance between the two coils, $\partial M/\partial z$ is the variation of the mutual inductance with the coil separation z and I_1 and I_2 are the currents in the two coils. Using a special multi-coil system, it is possible that over a certain range of coil separations the force, or $\partial M/\partial z$, is nearly constant. Equation (31) can then be integrated between two coil separations z_1 and z_2 which lie within the range of nearly constant force:

$$[M(z_1) - M(z_2)]I_1 I_2 + mg(z_2 - z_1) = \int_1^2 \Delta f_z(z)\,\mathrm{d}z. \tag{32}$$

The first term on the left of equation (32) is the change in magnetic energy between the two positions; the second is the change in potential energy of the test mass. The term on the right corresponds to the small change in force with coil separation. Because this experiment compares energies instead of power, as is the case for watt balances, this experiment is called a joule balance. Up to now the focus has been on the determination of the dc mutual inductance between the two coils. Different techniques have been investigated, the most promising being the dc square wave compensation method. While the current is linearly increased in one coil, a constant voltage is induced in the second coil. This is compensated by an opposed well-known constant voltage, so that only the small difference needs to be measured accurately [48]. Other aspects of the experiment are also being developed, for example, the balance necessary for the force measurement and an optical system to determine the vertical position of the moving coil. In the future it is planned to use a superconducting fixed coil to increase the magnetic flux density and to reduce the heating. First measurements of the Planck constant with an uncertainty of 2.5 parts in 10^5 have been carried out. It is expected to reach the level of several ppm in 2013. At that time the development of an improved apparatus, capable of reaching the level of 1 part in 10^7 or below in 2020 will be started.

The Measurement Standards Laboratory (MSL) in New Zealand is pursuing the idea of using a twin pressure balance as the force transducer for the static mode and an oscillatory coil movement at about 1 Hz for the dynamic mode [49]. The coil and the test mass will both be supported by the piston of a pressure balance. A second, coupled pressure balance produces a nearly identical reference pressure and the difference between them is measured with a differential pressure sensor. The piston–cylinder assembly with an air gap of less than 1 µm ensures a very precise guiding of the coil during the dynamic phase. At present the performance of the pressure balance used as force transducer is being investigated. Results are expected in 2014.

7. Considerations for future watt balances

Many choices are required during the planning phase of a watt balance experiment. At present it is premature to know which is the best concept. Within a watt balance experiment, many quantities need to be measured close to the state of the art so inevitably compromises have to be made.

The most fundamental decision required is that of the measurement principle. Two examples have been presented in this text: the moving-coil watt balance originally proposed by Kibble and the mutual inductance joule balance built at the NIM in China. The main difference between them resides with the fact that the joule balance does not require a dynamic phase but instead determination of the mutual inductance of two coils is necessary. The dynamic phase of a watt balance experiment is particularly sensitive to noise and requires high quality mechanics to generate a close to purely vertical movement, ideally at constant velocity. It remains to be seen whether the joule balance approach is simpler or capable of achieving smaller uncertainties. Another related technique is superconducting magnetic levitation [27] in which electromagnetic energy is converted into potential energy of a superconducting floating body above a superconducting coil. This experiment determines the flux quantum $\Phi_0 = h/2e$, but from the definition of the fine-structure constant (equation (11)) it follows that

$$h = \frac{8\alpha\Phi_0^2}{\mu_0 c}. \tag{33}$$

The superconducting magnetic levitation experiments have been abandoned, principally because the incomplete Meissner effect in the superconducting floating body led to energy losses, which were too difficult to quantify.

Other measurement schemes linking the Planck constant to macroscopic mass might exist and it remains an open question as to which are the best techniques in terms of uncertainty and simplicity.

One of the key elements of a watt balance is the magnet, which is needed to create the Lorentz force on the coil. The main choice is between an electromagnet and a permanent magnet. Conventional electromagnets are limited to relatively small flux densities, therefore superconducting electromagnets appear to be more appropriate. The NIST watt balance uses two opposed superconducting solenoids to create a radial field with 0.1 T at the position of the coil, and the NIM is developing a superconducting coil system for the joule balance. The advantages of an electromagnet are that the flux density can be varied, set to zero and inverted, which is interesting for studies of systematic errors. On the other hand, electromagnets require a power supply which allows the current to be maintained constant at the required level of about 1 part in 10^8 between the static and the dynamic measurement phases. Superconducting magnets require a complex cryogenic environment. A magnetic circuit using permanent magnets within a yoke structure appears to be simpler to use, because it does not require an external power supply and cryogens. The yoke can be shaped such that it concentrates the magnetic flux in the region of the coil. A disadvantage is that the magnetic flux density in the air gap cannot be easily changed. Care needs to be taken when evaluating the influence of the magnetic field of the moving coil on the magnetization of the yoke and the magnets, because it may lead to a change in the flux density as a function of the coil current. Typically the magnet materials used such as $\mathrm{Sm_2Co_{17}}$ have large temperature coefficients of the order of $-0.03\%\,^\circ\mathrm{C}^{-1}$, although some variants with much lower temperature coefficient exist, but at the expense of a reduced remanent induction. It is also possible to reduce the temperature coefficient using magnetic shunts. Another consideration for watt balance experiments is the geometry of the magnetic field. Most experiments currently in progress use fields with radial symmetry; however, the initial NPL apparatus and the first METAS watt balance both had a uniform field. Both institutes have since changed to a radial field, which is more efficient because the total wire length of the coil contributes to the Lorentz force, which is not possible in a uniform field. A radial field, which varies as $1/r$, has the additional benefit of the Lorentz force being insensitive to dimensional variations in the induction coil, which result from temperature changes. The coil needs to be rigid and the coil–magnet assembly optimized so that the power dissipated in the coil is minimized.

Another important consideration is the test mass, in particular the mass value. Present watt balances operate with masses between 0.1 kg and 1 kg. A larger mass requires a larger electromagnetic force in the static phase, which generally means a larger magnet. The choice of mass has therefore direct consequences for the size of the total apparatus. The mass should have a low magnetic susceptibility to avoid parasitic forces between the magnet and the mass. The material of the international prototype of the kilogram and of the national prototypes, PtIr, is not ideal in this respect, because it is paramagnetic with a susceptibility of 2.5×10^{-4}. A higher density is desirable to minimize surface effects and good hardness is needed to reduce mechanical wear. The ideal material should be chemically stable and be not porous to avoid outgassing in vacuum. To minimize the electrostatic forces, the material should be a good conductor. At present many different materials are used such as gold-plated copper, steel, PtIr, Au and Si.

Existing experiments differ in the way the coil is moved and guided during the dynamic phase. All experiments use a vertical coil movement at more-or-less constant velocity, where the coil moves sequentially up and down. In the NPL and NIST watt balances a rotation of the balance arm (NPL) or wheel (NIST) induces the vertical motion of the suspended coil. The rotation of a balance arm is accompanied by a small horizontal movement. In the LNE experiment, a motorized precision guiding stage moves the force comparator and its suspension. At the BIPM, the force comparator is kept fixed, but an extension system, driven by an electrostatic motor in the suspension, allows the coil to move while it is hanging from the force comparator. In the first METAS watt balance the coil was transferred from the force comparator to a special mechanical translation system for the dynamic phase. Differences also exist in the way the velocity is controlled. In some experiments the velocity is kept constant and the induced voltage is allowed to change (if the magnetic flux density varies along the trajectory). In other experiments, the velocity is controlled so that the induced voltage remains constant. It is essential to synchronize the velocity measurements very accurately with that of the induced voltage to eliminate common noise due to ground vibrations. A proposal has been made for a watt balance with an oscillatory movement [49], where the coil would oscillate at a low frequency with an amplitude of the order of 1 mm. This scheme has the advantage of long measurement times due to the continuous motion and a much smaller magnetic circuit due to the shorter travel range.

A very important topic is alignment because although an imperfectly aligned apparatus can provide very reproducible results, they are systematically wrong. The reference for the alignment of a watt balance is the vertical direction, as defined by the direction of gravitational acceleration. The balance can only detect vertical forces and the interferometer, when correctly aligned, only measures vertical velocities. In reality, the coil has six degrees of freedom and the complete measurement equation contains additional terms:

$$\frac{UI}{F_z v_z} = 1 + \frac{F_x v_x}{F_z v_z} + \frac{F_y v_y}{F_z v_z} + \frac{\tau_x \omega_x}{F_z v_z} + \frac{\tau_y \omega_y}{F_z v_z} + \frac{\tau_z \omega_z}{F_z v_z}, \tag{34}$$

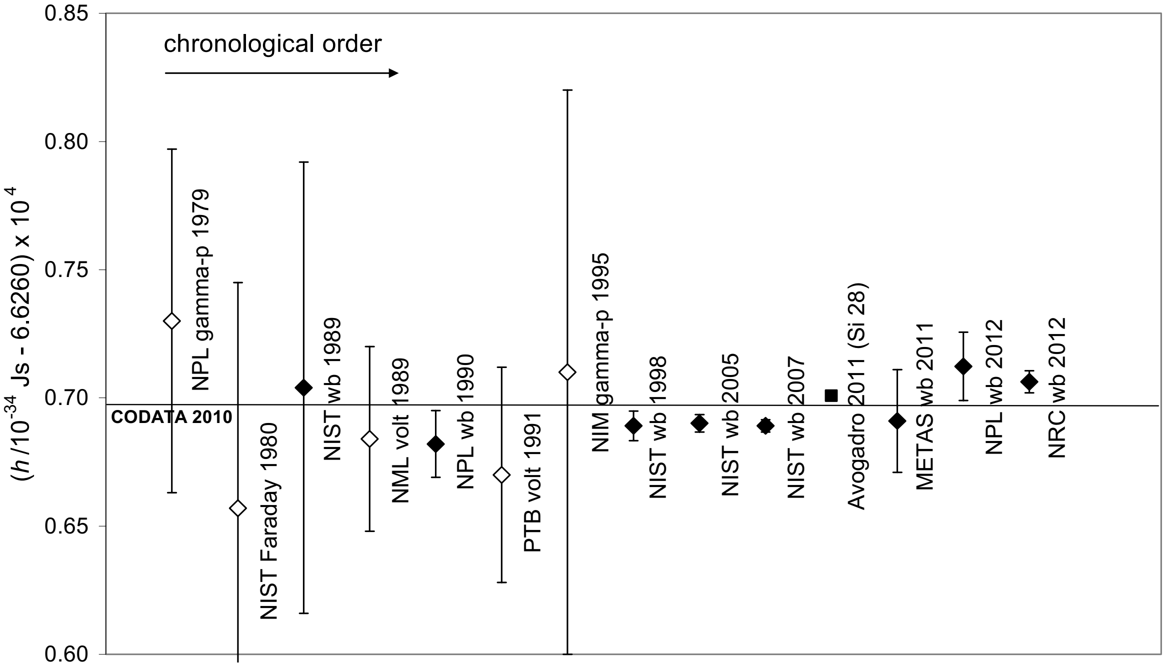

Figure 6. Values for the Planck constant obtained with different techniques. The data available for the 2010 CODATA adjustment of fundamental constants are taken from [11], which also includes individual references. The uncertainties are indicated at the level of one standard deviation. All watt balance results are shown by filled diamonds. Results obtained by counting Si atoms in a Si sphere are shown by filled squares.

where F_i are forces, τ_i are torques, v_i are velocities and ω_i are angular velocities. The five terms on the right side, when they exist, contribute to the induced voltage, but are not measured by the balance and result therefore in a measurement error. Ideally the trajectory of the coil during the dynamic phase would be such that all angular velocities ω_i and all velocities v_i, except v_z, are zero, that is, a purely vertical movement without rotation. In addition, all torques and all forces, except the vertical force F_z, should also vanish. In reality, is it sufficient that each of the five terms is of the order of a few parts in 10^9. The forces and torques are related to flux gradients

$$F_i = I\frac{\partial\Phi}{\partial x_i} \quad \text{and} \quad \tau_i = I\frac{\partial\Phi}{\partial\alpha_i} \qquad \text{for } i = x, y, z \tag{35}$$

and therefore depend on the geometry of the magnetic field and on the coil-to-magnet alignment. To avoid horizontal forces, the electrical plane of the coil needs to be parallel to the magnetic field. The coil needs to be centred with respect to the radial magnetic field, because otherwise torques around horizontal axes will occur. To measure velocity correctly the laser beam of the interferometer also needs to be vertical.

The coil has a centre of mass, an electrical centre (through which the Lorentz force acts) and an optical centre (the velocity of which is determined with the interferometer). All three need to lie on the same vertical axis. If the optical centre is not aligned with the electrical centre and a coil tilt around a horizontal axis occurs during the coil movement, the interferometer does not determine the velocity which is relevant for the induced voltage (Abbe offset error). If the electrical centre and the centre of mass do not coincide, a tilt occurs when current is injected into the coil. In the alignment phase it is helpful if the coil has the freedom to respond to horizontal forces and torques so that they can be detected and eliminated. During measurements, however, it is desirable that the coil is more rigidly connected to the suspension so that its vertical movement is as undisturbed as possible. A system which allows the coil to be clamped and unclamped has been described in [43]. A detailed description of alignment methods is found in [50, 51].

8. Overview of the available results for the Planck constant

An overview of determinations of the Planck constant available for the 2010 CODATA adjustment of fundamental constants, some of which have been officially published as recently as 2012, can be found in [11]. Figure 6 shows these results in chronological order plus the most recent one (NRC watt balance 2012 [35]).

Values for h can be obtained directly from watt balance experiments but also indirectly from measurements of various other constants, as discussed in [11]: the gyromagnetic ratio of the proton (indicated in figure 6 as 'gamma-p'), the Faraday constant, the Josephson constant (indicated as 'volt', because it is obtained with a voltage balance) and the Avogadro constant. In figure 6 all watt balance results are shown by filled diamonds, results obtained by other electrical measurements by open diamonds and results based on the silicon sphere technique by filled squares. The value for the Planck constant obtained in the last 2010 CODATA adjustment of fundamental constants is indicated by the solid line. This value is largely dominated by the 2007 NIST watt balance result and the 2011 Avogadro result and is therefore close to their weighted mean.

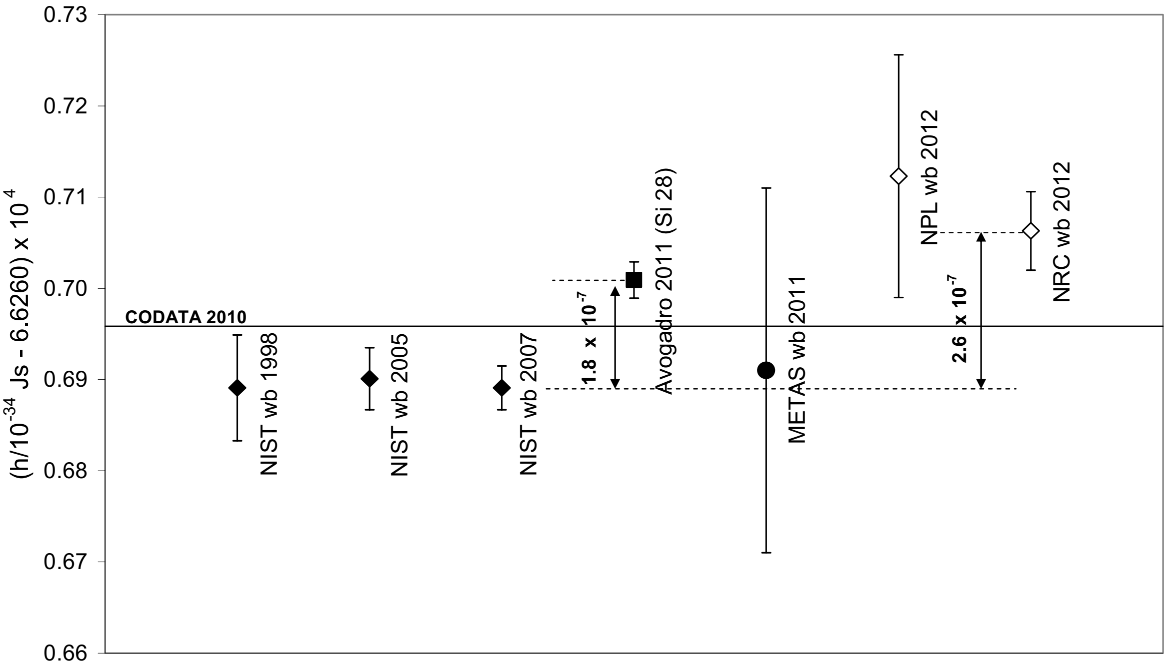

Figure 7. Recent values for the Planck constant obtained with watt balances and the silicon sphere technique, using spheres made of isotopically pure ^{28}Si. The uncertainties are indicated at the level of one standard deviation.

The two techniques which allow the Planck constant to be determined with the smallest uncertainty (several parts in 10^8) are the watt balance and the silicon sphere technique, which has spheres made of isotopically enriched ^{28}Si. Recent results obtained by these two techniques are shown in figure 7.

Due to the small uncertainties, the difference between the Avogadro 2011 result and the latest NIST watt balance result is statistically highly significant. The difference corresponds to nearly four times the combined standard deviation of the two results. The NRC result from 2012 confirms the result obtained at the NPL with the same apparatus, but disagrees significantly with the NIST result.

At the present time the main problem for the redefinition of the kilogram is the significant discrepancy between the result of the silicon sphere approach and those obtained by the NIST watt balance on the one hand, and the discrepancy between the NIST and the NPL/NRC results on the other hand.

9. Status of the redefinition of the kilogram

As described above, it is planned to redefine four of the seven base units of the SI: the kilogram, the ampere, the kelvin and the mole [9]. A decision on this change will be made by the CGPM which meets regularly in Paris, with the next meeting scheduled for November 2014. The Resolutions of the CGPM are prepared by the CIPM, which meets annually at the BIPM. The CIPM receives scientific advice from its Consultative Committees, which comprise world experts in the different fields of metrology.

The Consultative Committee for Mass and Related Quantities (CCM) has discussed the redefinition of the kilogram and recommends that the following conditions are met before the kilogram is redefined:

1. At least three independent experiments, including work both from watt balance and from International Avogadro Coordination projects, yield values of the relevant constants with relative standard uncertainties not larger than 5 parts in 10^8.
2. At least one of these results should have a relative standard uncertainty not larger than 2 parts in 10^8.
3. For each of the relevant constants, values provided by the different experiments should be consistent at the 95% level of confidence.

The first condition is at present fulfilled by the NIST 2007 watt balance result [40] and the Avogadro 2011 [10] result. An interesting question in this respect is how far the NIST 1998 result [37] can be viewed as independent from the later results because the apparatus had been considerably modified. However, its uncertainty of 8.7 parts in 10^8 is too large to fulfil the first condition. The present uncertainty of the NRC watt balance is 6.5 parts in 10^8. If progress continues as planned, it is expected that an uncertainty below 5 parts in 10^8 will be reached relatively soon, which would fulfil the first requirement.

The target uncertainty of the second condition has, so far, not been reached in any experiment. The participants of the Avogadro project will continue their work and plan to reduce the uncertainty of the Avogadro constant to 2 parts in 10^8 in 2013. The third condition is violated by the significant discrepancy between the NIST and NRC watt balances and between the NIST watt balance and the Avogadro result. A collaboration between NIST and NRC shall explain the origin of their discrepancy. From the point of view of mass metrology it is therefore too early to make any decision on the redefinition of the kilogram.

The CCM has also pointed out that it is necessary to have a sufficient number of facilities that realize the new kilogram definition with a relative standard deviation of not larger than 2 parts in 10^8. The CCM is developing a *mise en pratique* which will specify how the new definition can be realized in practice. One of the difficulties to be addressed is how to ensure uniformity of mass calibrations in the future. Even if several watt balances with relative uncertainties as small as 2 parts in 10^8 did exist, it would be necessary to organize, at least during an initial period, periodic comparisons between them.

The CCU is responsible for the development of the SI. In recent years it has discussed the planned redefinitions of four base units and has contacted the relevant Consultative Committees. Although it is too early to proceed with the redefinition, a consensus has been reached on the form of the new SI. This proposal is to redefine

- the kilogram by fixing the numerical value of the Planck constant h,
- the ampere by fixing the numerical value of the elementary charge e,
- the kelvin by fixing the numerical value of the Boltzmann constant k_B and
- the mole by fixing the numerical value of the Avogadro constant N_A.

All definitions will be of a form in which the unit is defined indirectly by specifying explicitly an exact value for the related fundamental constant. An example for the kilogram is provided in section 2. The CGPM (2011) approved the general principle of the planned redefinitions based on fixed numerical values of four fundamental constants [52].

10. Conclusions

The international prototype of the kilogram has fulfilled its role very well since its ratification in 1889. There are, however, indications that its 'absolute' mass has not been perfectly stable, although it is by definition always 1 kg. Because measurements are becoming more and more precise this instability will become a problem in the future. Therefore preparations are being made to redefine the kilogram with respect to a fixed numerical value of a fundamental constant. It is also the intention to redefine at the same time the ampere, the kelvin and the mole.

Several constants could be chosen for the kilogram redefinition but due to the advantages it presents to electrical metrology, the Planck constant was selected. This will bring the units used for electrical measurements, based on the Josephson effect and the quantum Hall effect, into the SI. The first step towards a redefinition is the determination of the numerical value of the Planck constant in the present SI, with an uncertainty of about 2 parts in 10^8.

Watt balances establish an experimental link between the Planck constant and a macroscopic mass. Watt balance experiments rely on the equivalence of mechanical and electrical energy and on the use of two macroscopic quantum effects: the Josephson effect and the quantum Hall effect. Watt balance experiments at different stages of development exist in several NMIs. Until now only two experiments have achieved measurement uncertainties below 1 part in 10^7, at NIST and the NRC, but these results are not in agreement. Another technique for determining the Planck constant, via the Avogadro constant, by 'counting' the number of silicon atoms in a nearly perfect silicon sphere, has led to a value which also differs from the NIST result.

Experts in mass metrology have recommended that several conditions must be met before the kilogram can be redefined, and to date these have not been fulfilled. However, the form of the new SI has already been defined, so that in the future all seven SI base units will be based on fixed numerical values of constants. The exact timing of when the new definitions will be officially adopted by the CGPM will depend on the progress of future work to determine the Planck constant and on its evaluation by the relevant expert committees. Official approval by the CGPM is required prior to acceptance of such redefinitions. The CGPM is scheduled to meet next in 2014.

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