CFTC Centro de Física Teórica e Computacional

Ciências ULisboa

Percolation

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http://www.namaraujo.net

Books on percolation

➢D. Stauffer and A. Aharony, *Introduction to percolation theory*. CRC Press (2000).

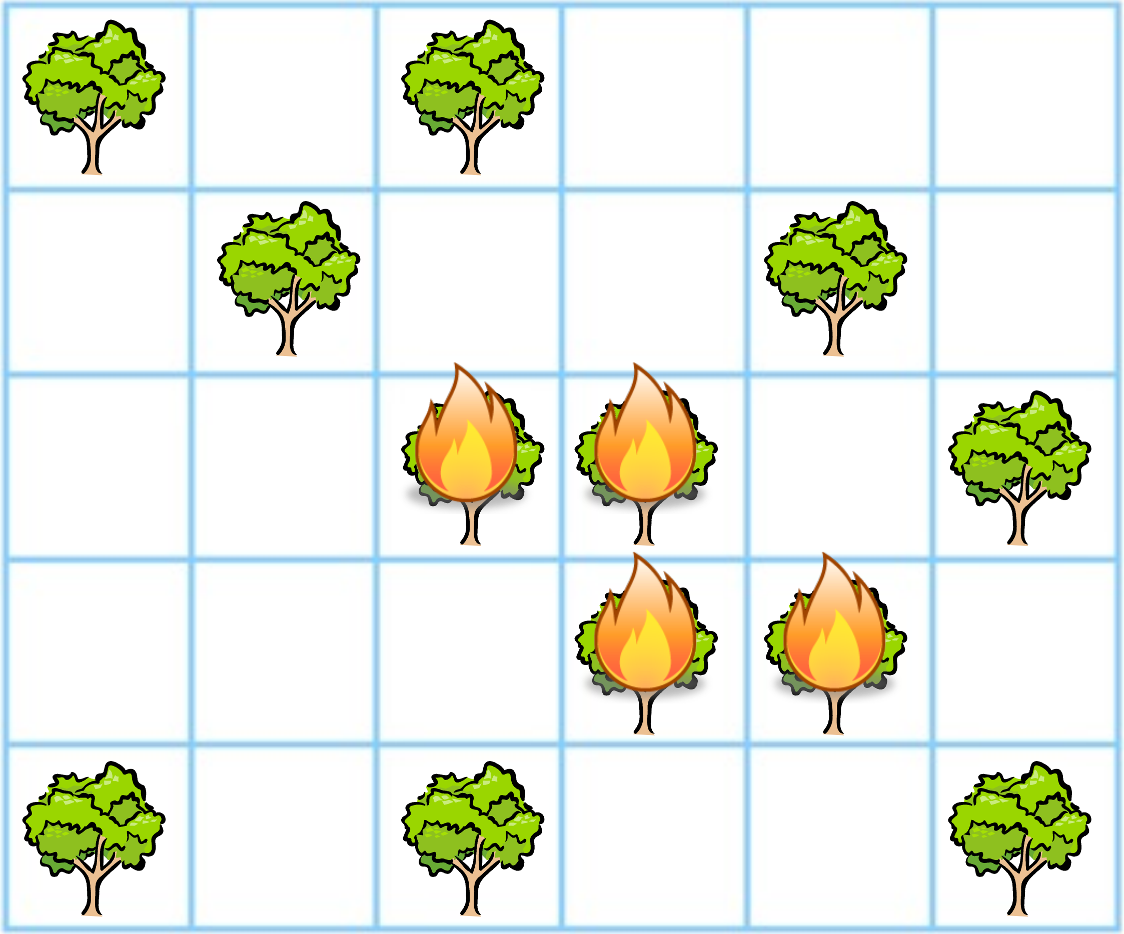
➢M. Sahimi, *Applications of percolation theory*. Taylor & Francis (1994).

➢K. Christensen and N. R. Moloney, *Complexity and criticality*. Imperial College Press (2005).

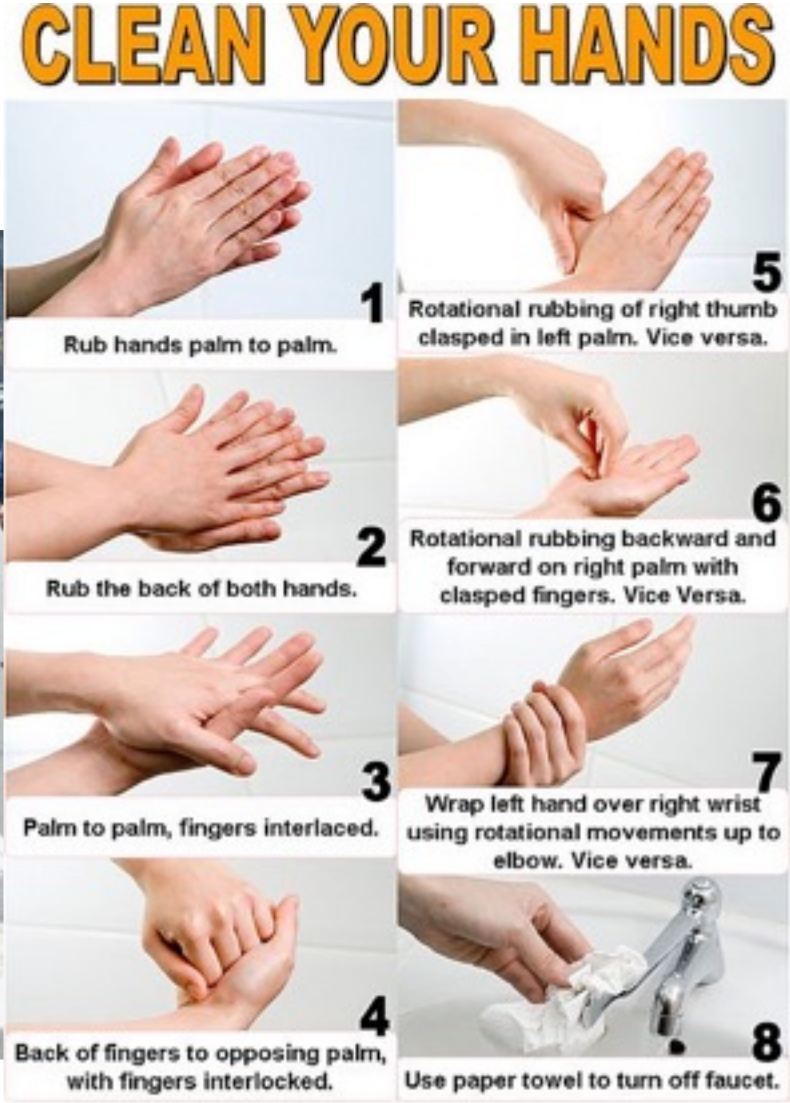
Forest fire

Photo - John McColgan BLM Alaska Fire Service

Forest fire



Spreading of epidemics



CLEAN YOUR HANDS

1 Rub hands palm to palm.

2 Rub the back of both hands.

3 Palm to palm, fingers interlaced.

4 Back of fingers to opposing palm, with fingers interlocked.

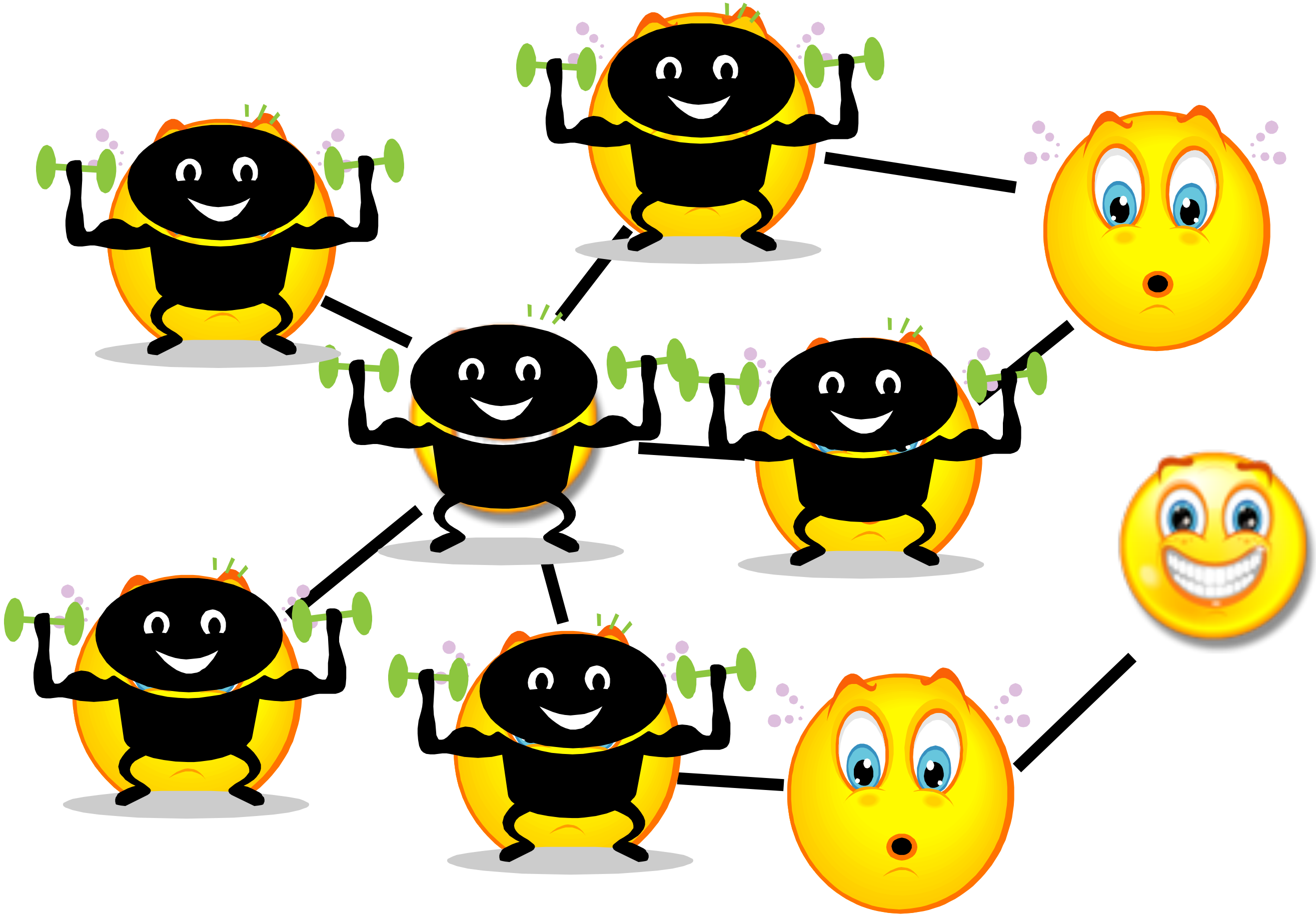
5 Rotational rubbing of right thumb clasped in left palm. Vice versa.

6 Rotational rubbing backward and forward on right palm with clasped fingers. Vice Versa.

7 Wrap left hand over right wrist using rotational movements up to elbow. Vice versa.

8 Use paper towel to turn off faucet.

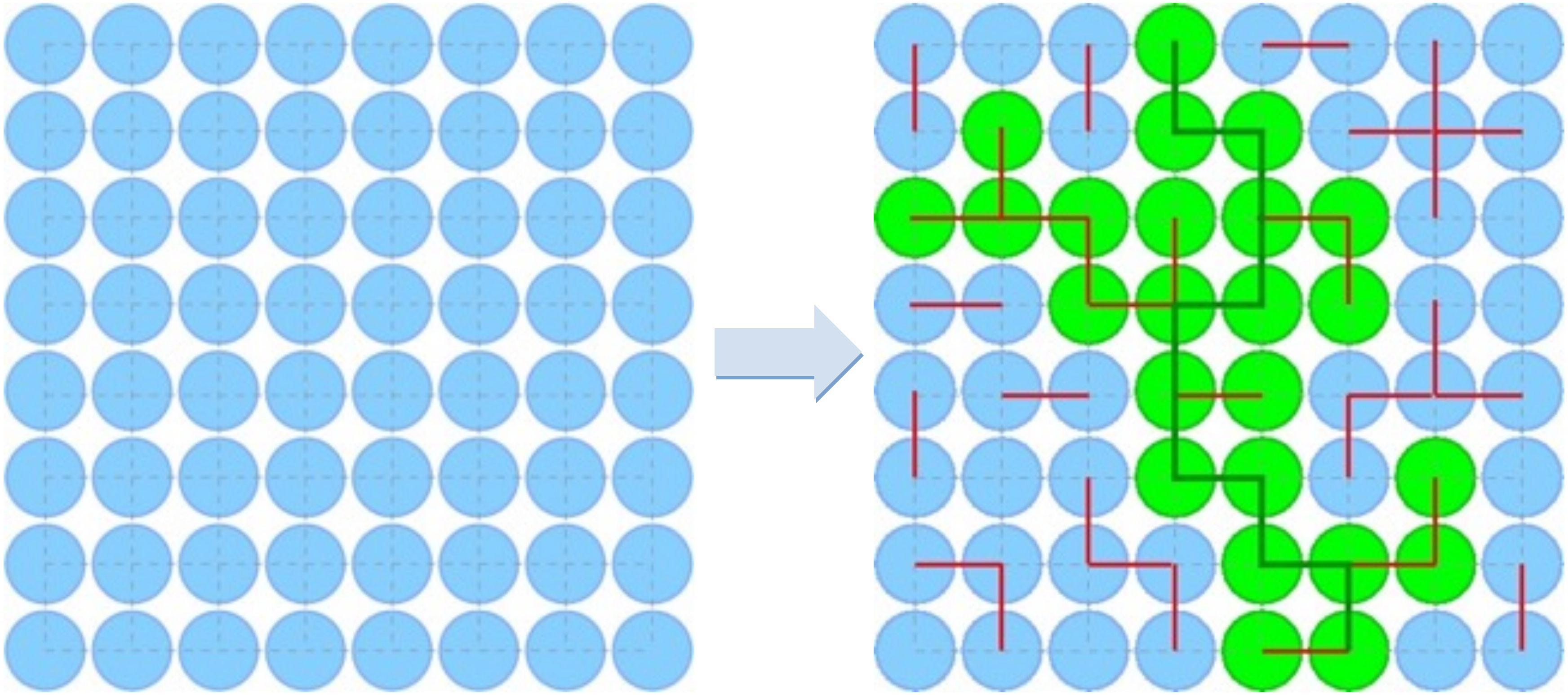
Spreading of epidemics



Oil fields

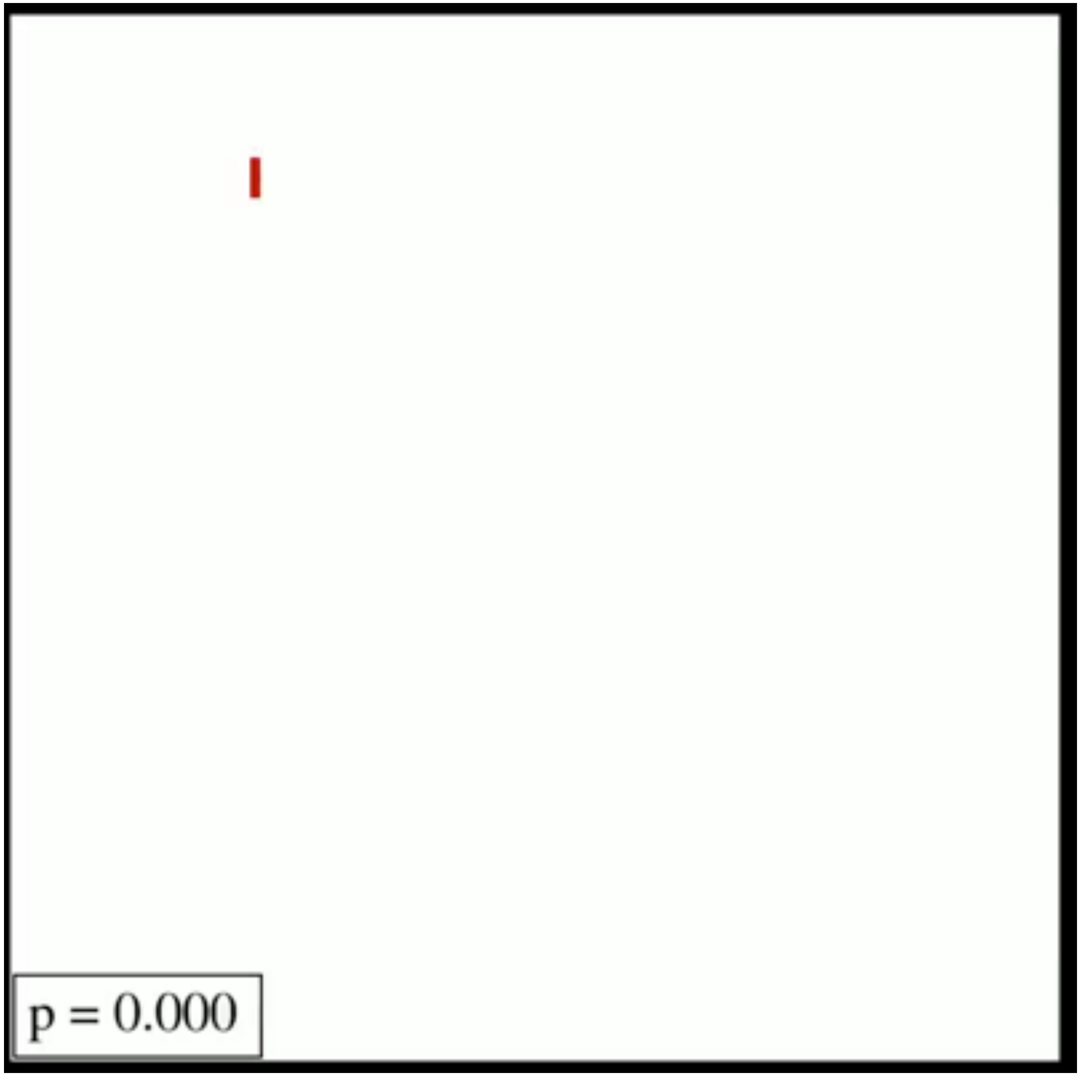
at *Barrancabermeja (Colombia)*, photo by *Melissa Jiménez*.

Percolation model



$$p^{O}(1-p)^{E}$$

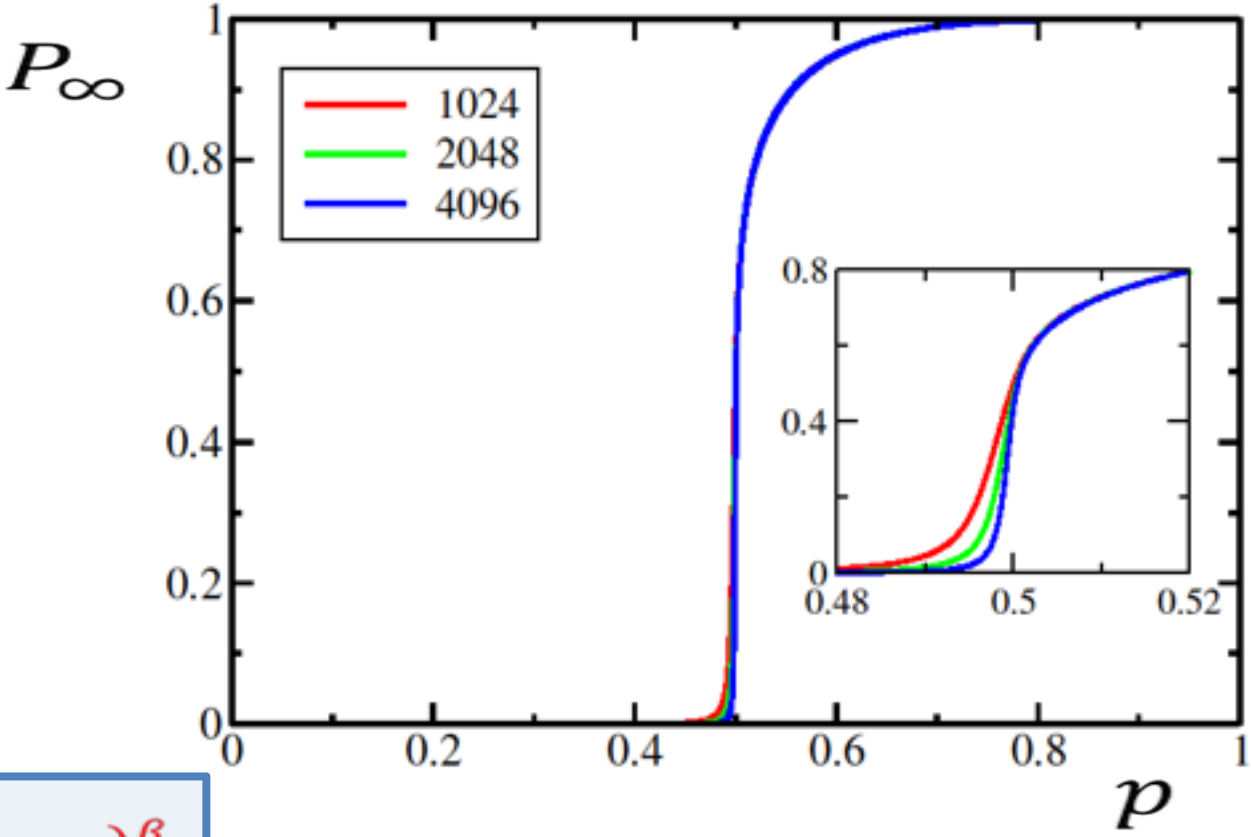
Percolation model



Percolation model

order parameter

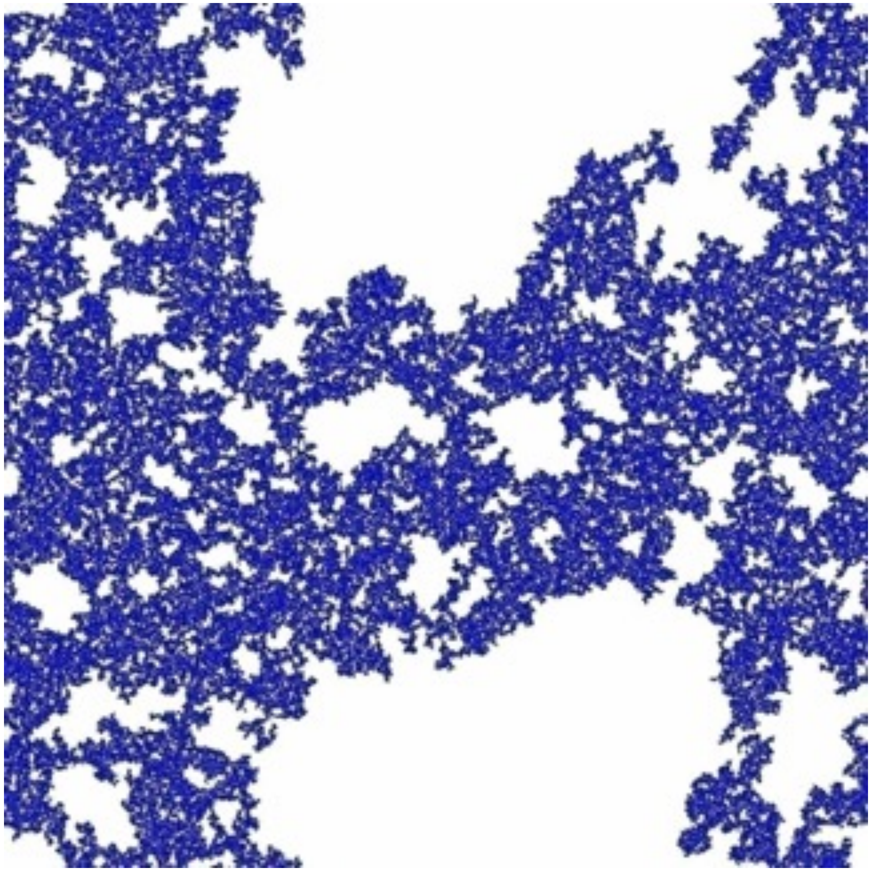
$$P_\infty = \frac{S_{max}}{N}$$



$$P_\infty \sim (p - p_c)^\beta$$

Percolation threshold

largest cluster: fractal dimension

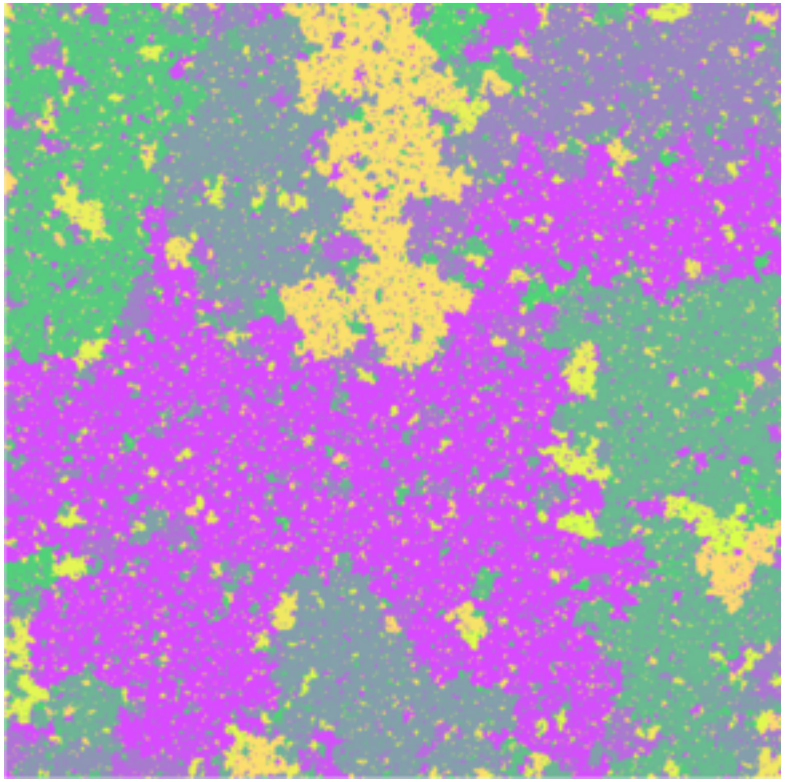


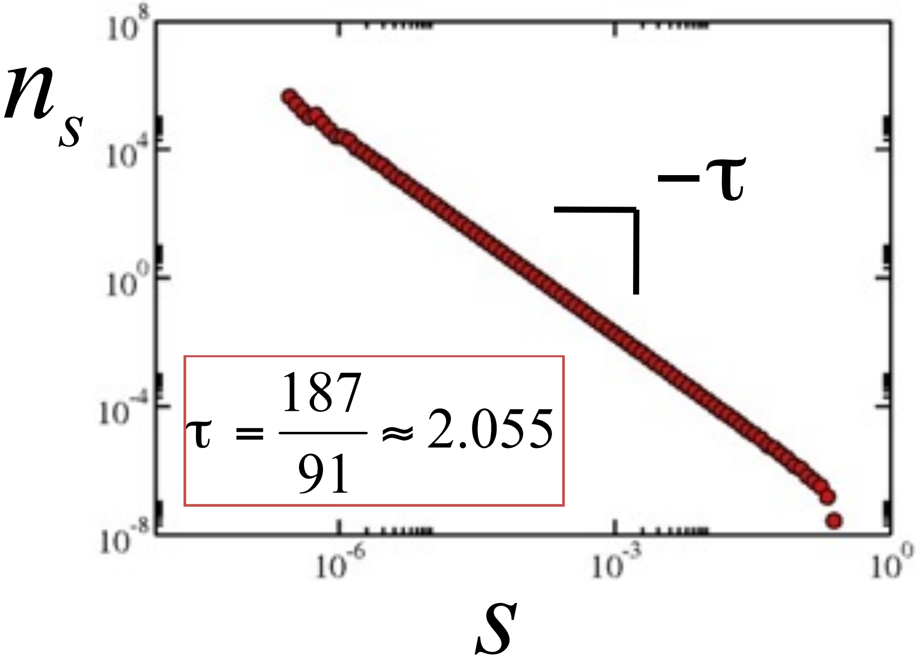
$$\boxed{d_f = \frac{91}{48} \approx 1.896}$$

Percolation threshold

cluster-size distribution

$$\boxed{n_s \sim s^{-\tau}}$$

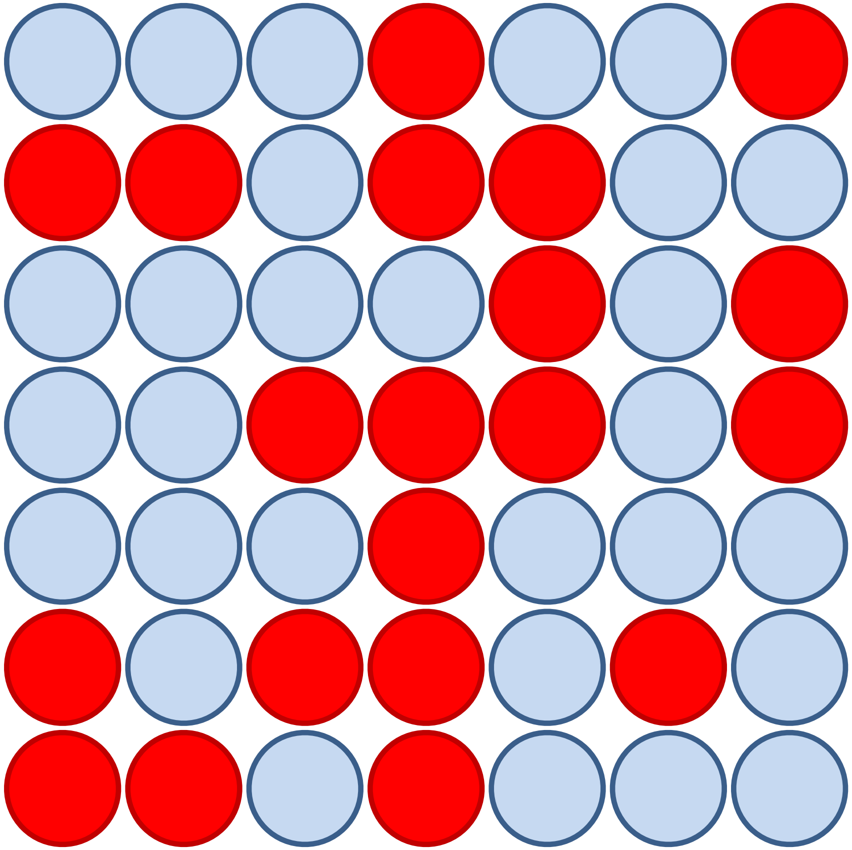




$$\boxed{\tau = \frac{187}{91} \approx 2.055}$$

Algorithms

generate canonical configurations



```
For each site i:
1. random number $\varepsilon$;
2. if
      $\varepsilon < p$: i is occupied;
      else: i is empty.
```

Algorithms

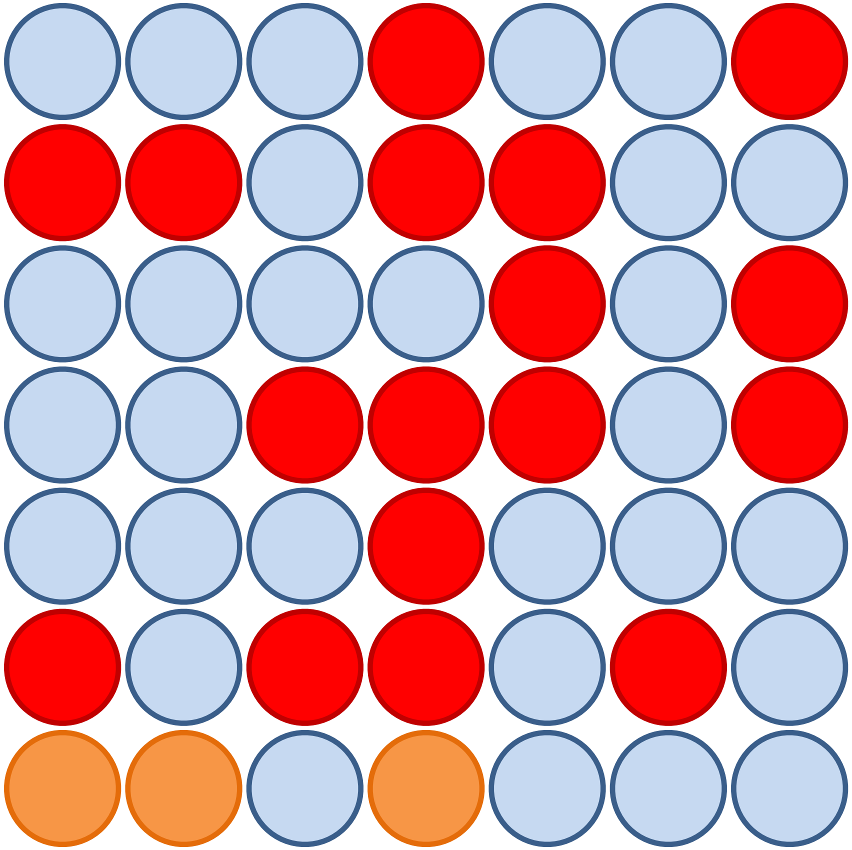
Burning method

🔵 empty

🔴 occupied

🟠 burning

⚫ burned



🔵	🔵	🔵	🔴	🔵	🔵	🔴
🔴	🔴	🔵	🔴	🔴	🔵	🔵
🔵	🔵	🔵	🔵	🔴	🔵	🔴
🔵	🔵	🔴	🔴	🔴	🔵	🔴
🔵	🔵	🔵	🔴	🔵	🔵	🔵
🔴	🔵	🔴	🔴	🔵	🔴	🔵
🟠	🟠	🔵	🟠	🔵	🔵	🔵

1. **set first** row **burning**;

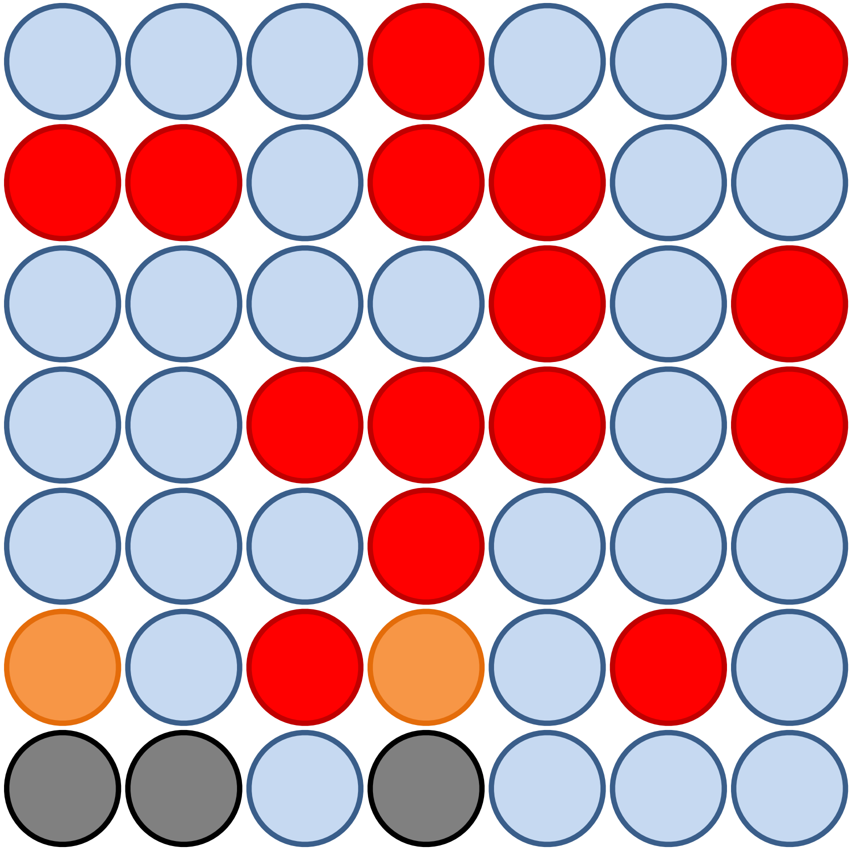
H. J. Herrmann, D. C. Hong, and H. E. Stanley. J. *Phys. A* **17**, L261 (1984)

Algorithms

Burning method

○ **empty** 🟠 **burning**

🔴 **occupied** ⚫ **burned**



1. **set first** row **burning**;

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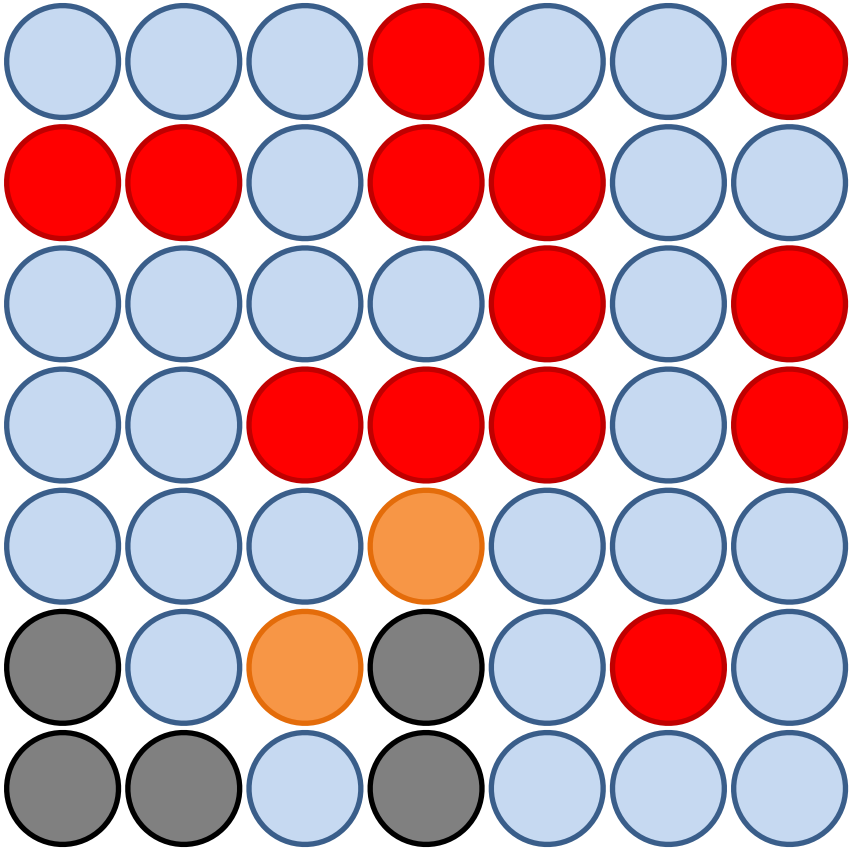
H. J. Herrmann, D. C. Hong, and H. E. Stanley. J. *Phys. A* **17**, L261 (1984)

Algorithms

Burning method

○ empty 🟠 burning

🔴 occupied ⚫ burned



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2. **set neighbors of burning** to **burning** and **burning** to **burned**;

3. **repeat** until everything is **burned**.

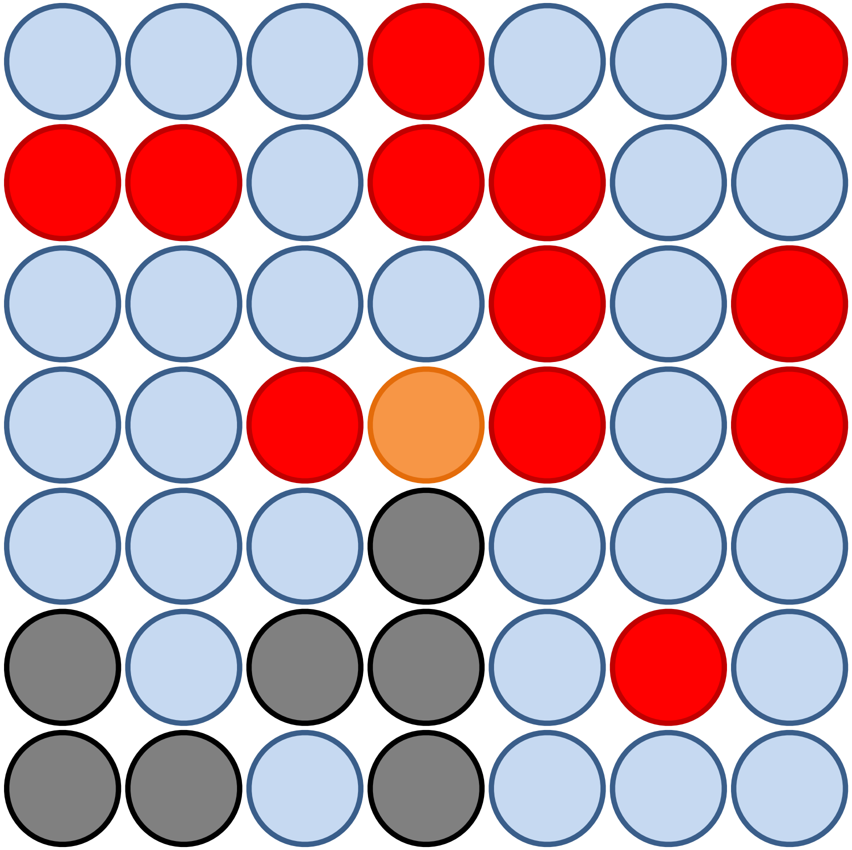
H. J. Herrmann, D. C. Hong, and H. E. Stanley. J. *Phys. A* **17**, L261 (1984)

Algorithms

Burning method

🔵 empty 🟠 burning

🔴 occupied ⚫ burned



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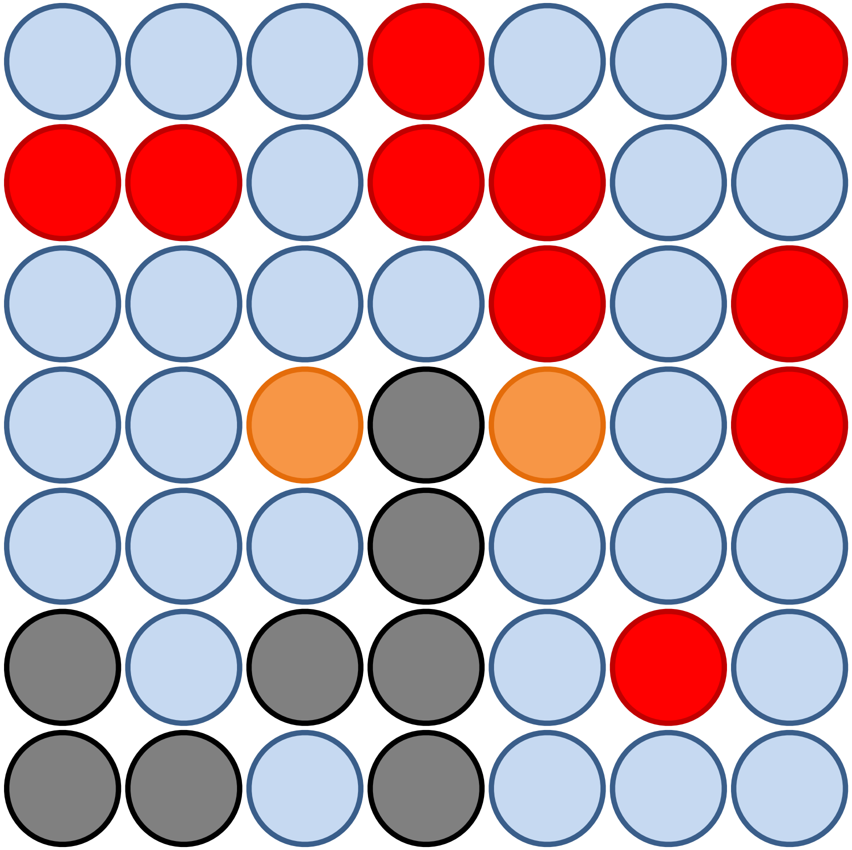
Algorithms

Burning method

○ empty 🟠 burning

🔴 occupied ⚫ burned



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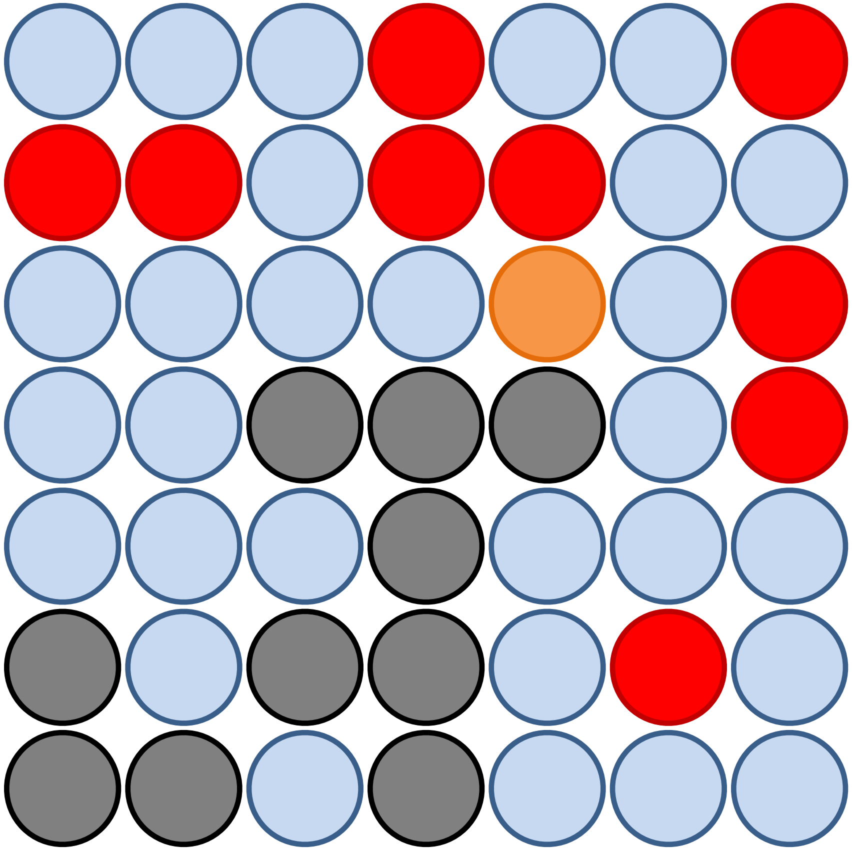
Algorithms

Burning method

○ **empty** 🟠 **burning**

🔴 **occupied** ⚫ **burned**



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Algorithms

Burning method

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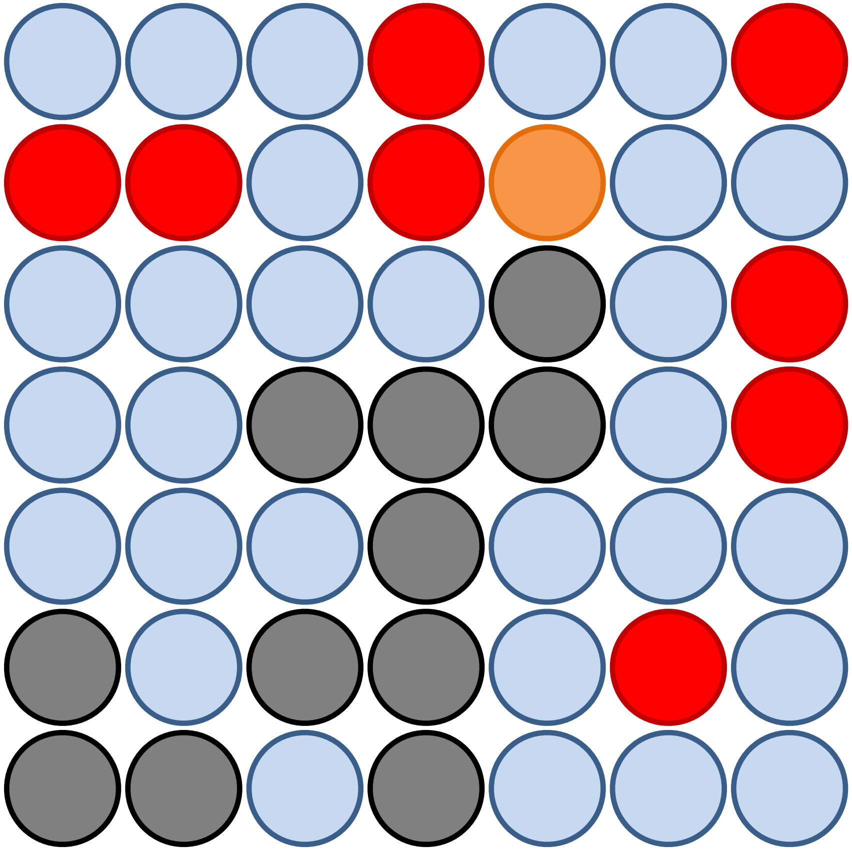
🟠 **burning**

🔴 **occupied**

⚫ **burned**



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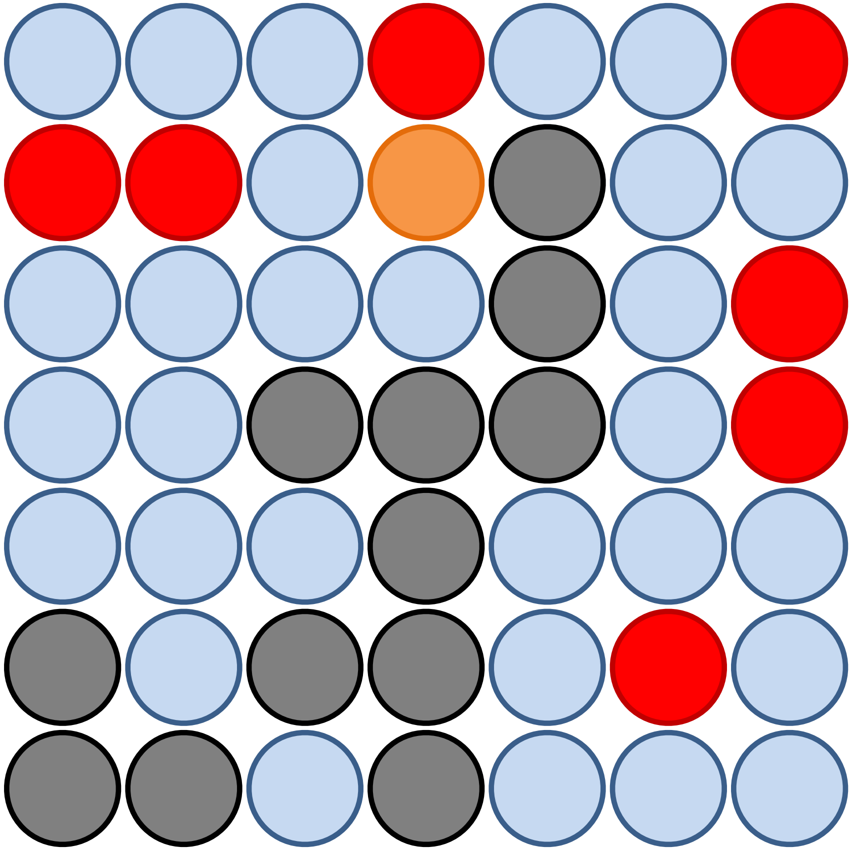
Algorithms

Burning method

○ empty 🟠 burning

🔴 occupied ⚫ burned



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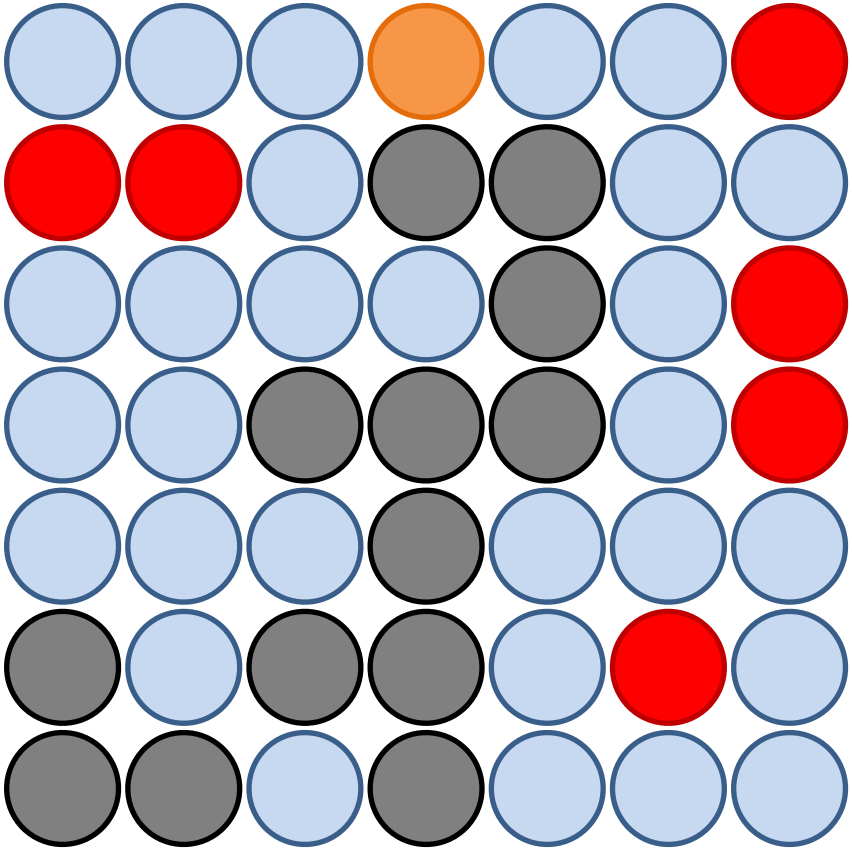
Algorithms

Burning method

🔵 **empty** 🟠 **burning**

🔴 **occupied** ⚫ **burned**



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Algorithms

Burning method

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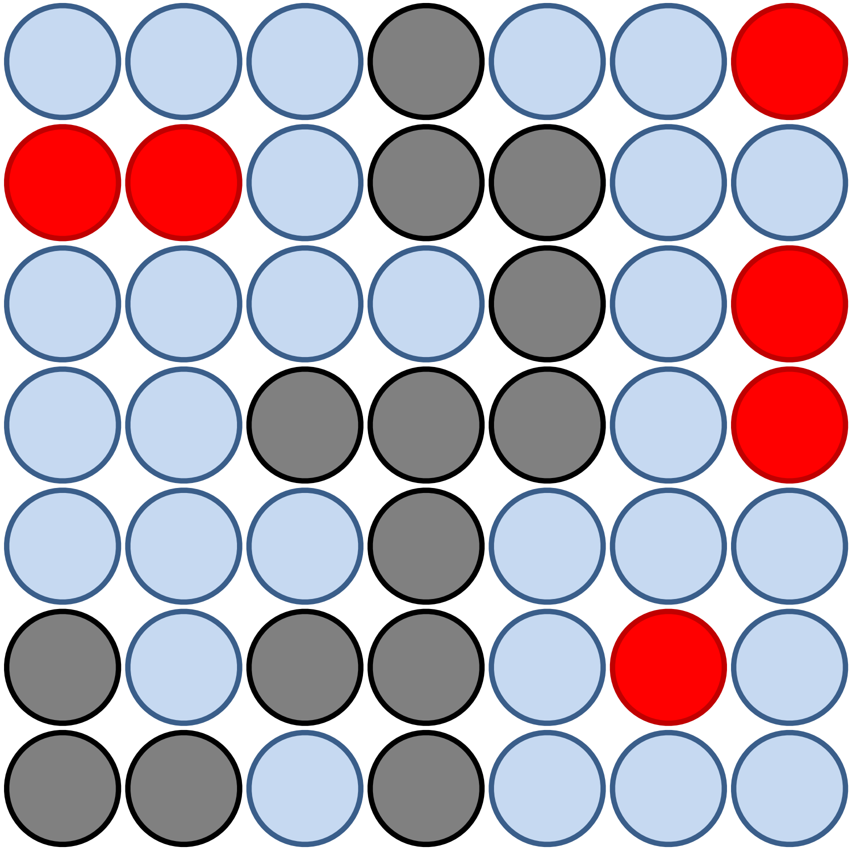
🔴 **occupied** ⚫ **burned**



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Algorithms

Burning method

○ empty 🟠 burning

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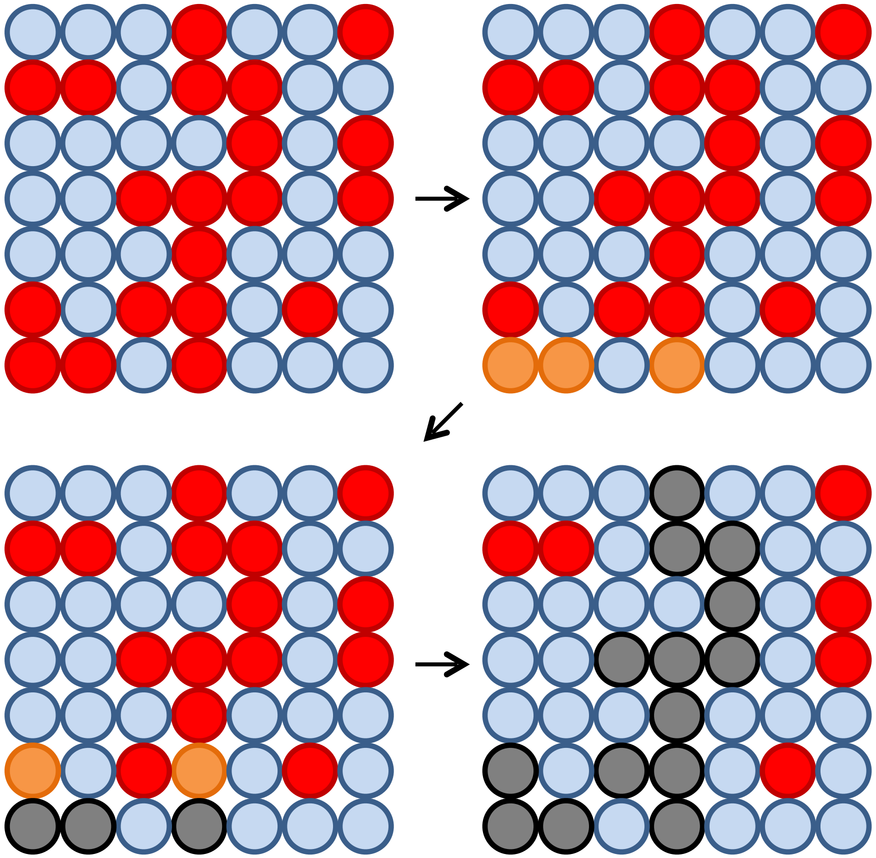


One can determine if the set of occupied sites percolates or not.

Number of clusters and cluster size distribution?

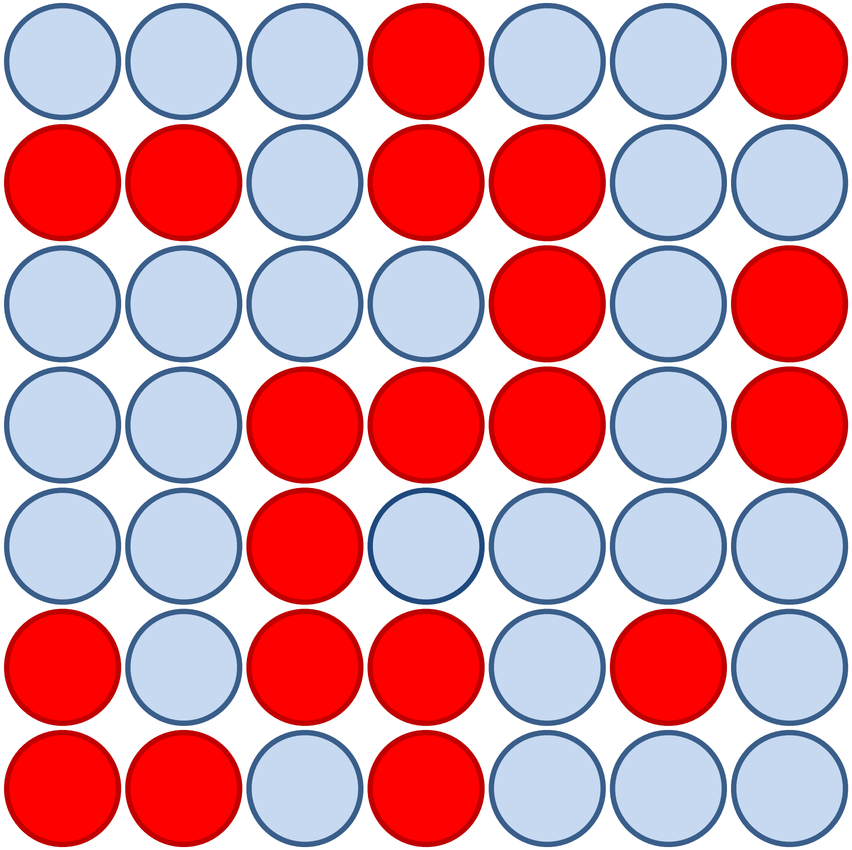
H. J. Herrmann, D. C. Hong, and H. E. Stanley. J. *Phys. A* **17**, L261 (1984)

Algorithms

Hoshen and Kopelman

k = 2

M(k) = 0



1. **start** from the site in the **left-bottom corner**;

2. **sweep** from **left to right bottom to top**;

3. **only** verify **left** and **bottom** neighbors.

J. Hoshen and R. Kopelman. *Phys. Rev. B* **14**, 3438 (1976)

Algorithms

Hoshen and Kopelman

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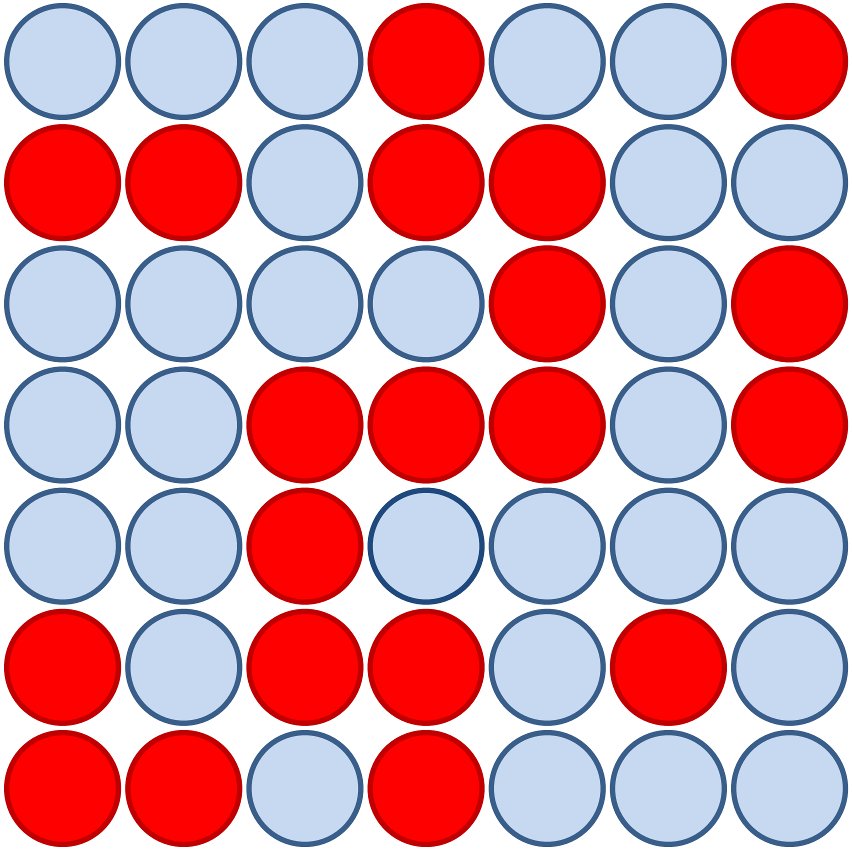

k	M(k)
2	0

J. Hoshen and R. Kopelman. *Phys. Rev. B* **14**, 3438 (1976)

Algorithms

Hoshen and Kopelman

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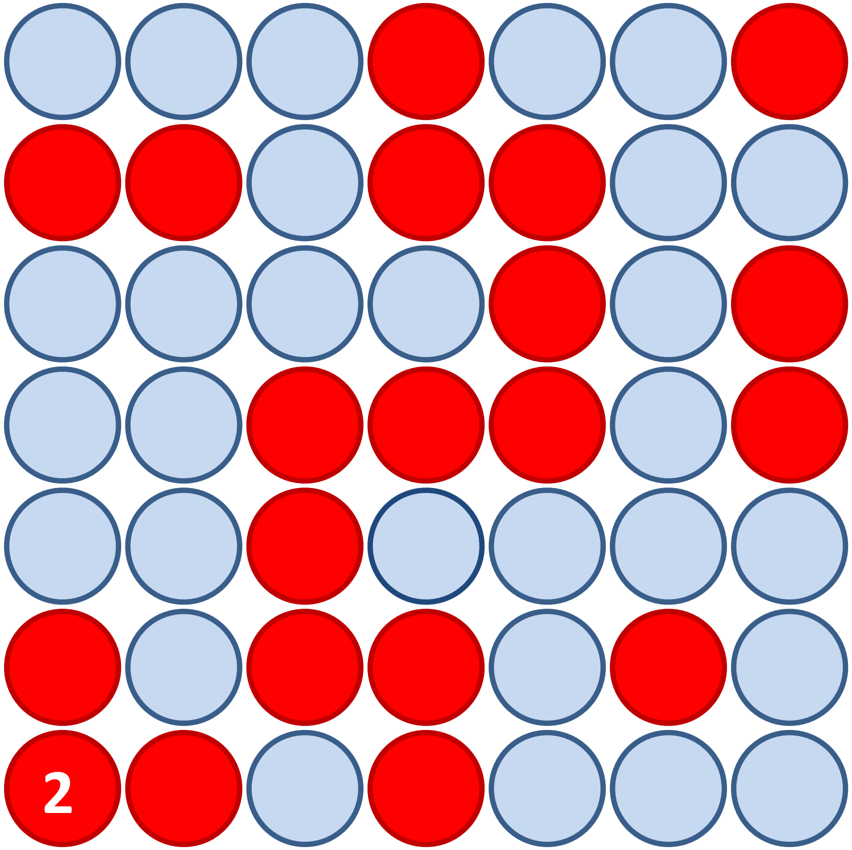

k	M(k)
2	1

J. Hoshen and R. Kopelman. *Phys. Rev. B* **14**, 3438 (1976)

Algorithms

Hoshen and Kopelman

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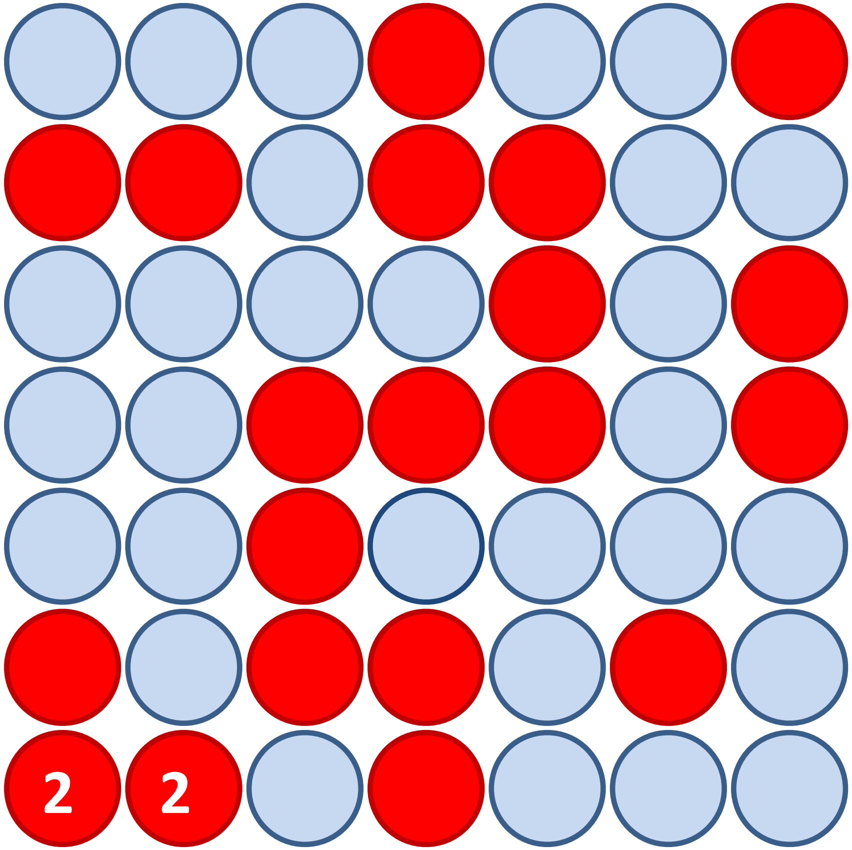
3. **only** verify **left** and **bottom** neighbors.

k	M(k)
2	2



J. Hoshen and R. Kopelman. *Phys. Rev. B* **14**, 3438 (1976)

Algorithms

Hoshen and Kopelman

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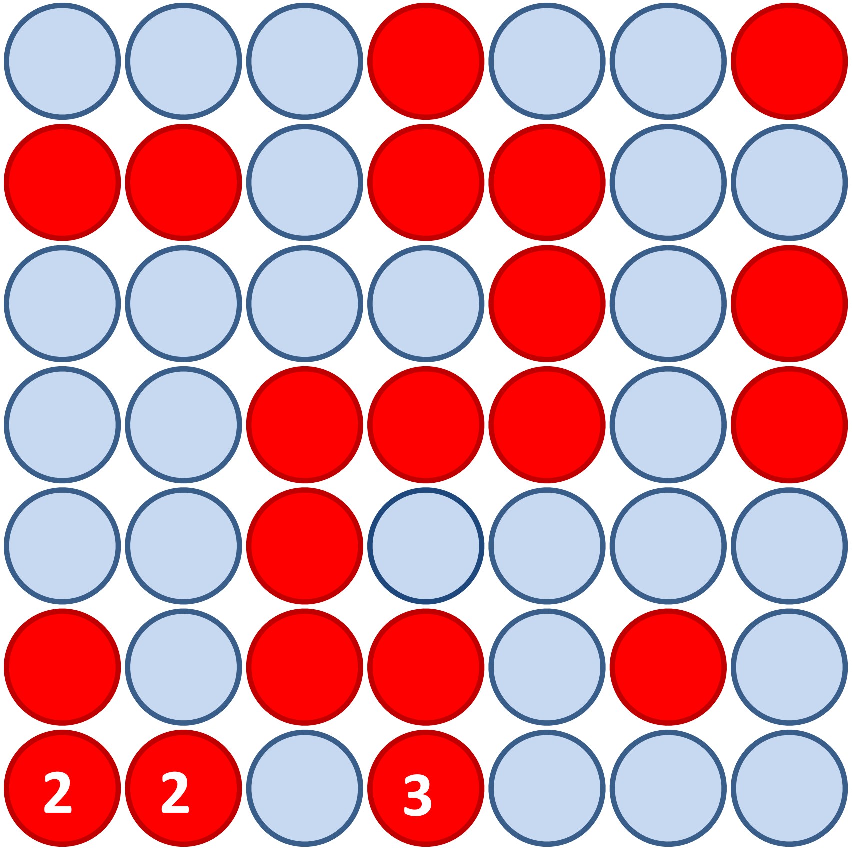
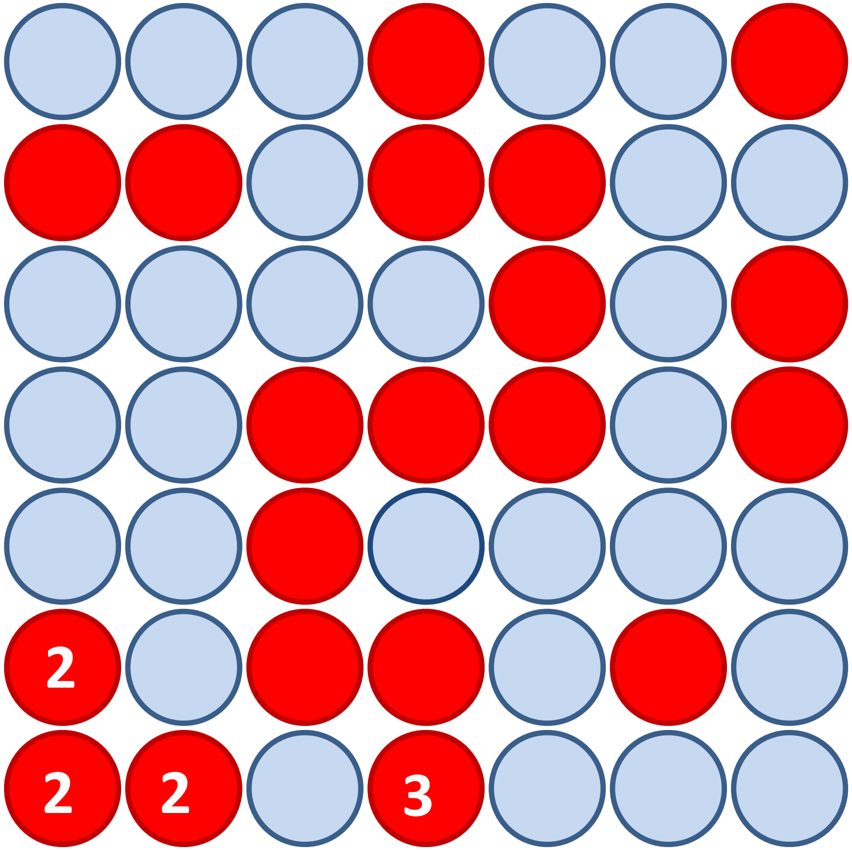


k	M(k)
2	2
3	1

J. Hoshen and R. Kopelman. *Phys. Rev. B* **14**, 3438 (1976)

Algorithms

Hoshen and Kopelman



1. **start** from the site in the **left-bottom corner**;

2. **sweep** from **left to right bottom to top**;

3. **only** verify **left** and **bottom** neighbors.

k	M(k)
2	3
3	1

J. Hoshen and R. Kopelman. *Phys. Rev. B* **14**, 3438 (1976)

Algorithms

Hoshen and Kopelman

1. **start** from the site in the **left-bottom corner**;

2. **sweep** from **left to right bottom to top**;

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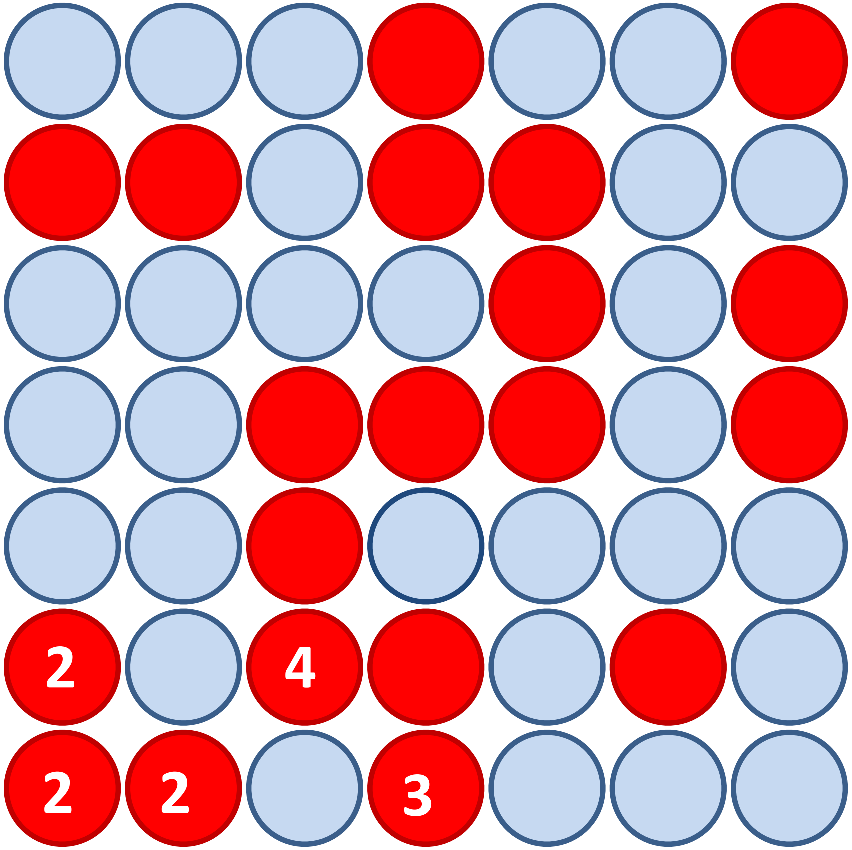

k	M(k)
2	3
3	1
4	1

J. Hoshen and R. Kopelman. *Phys. Rev. B* **14**, 3438 (1976)

Algorithms

Hoshen and Kopelman

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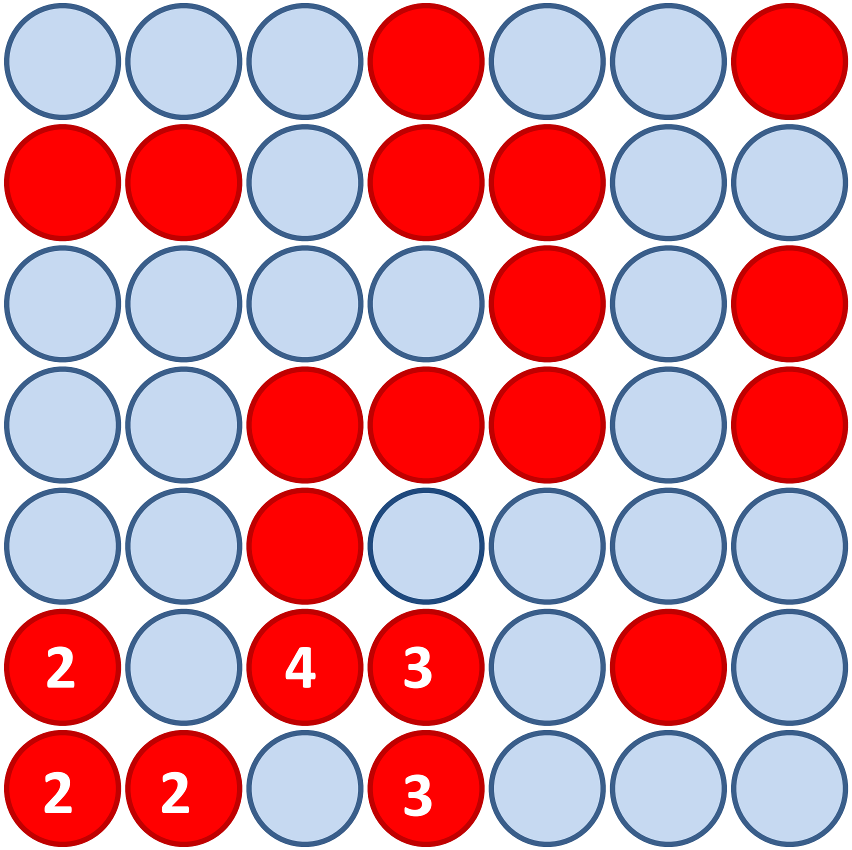

k	M(k)
2	3
3	3
4	-3

J. Hoshen and R. Kopelman. *Phys. Rev. B* **14**, 3438 (1976)

Algorithms

Hoshen and Kopelman

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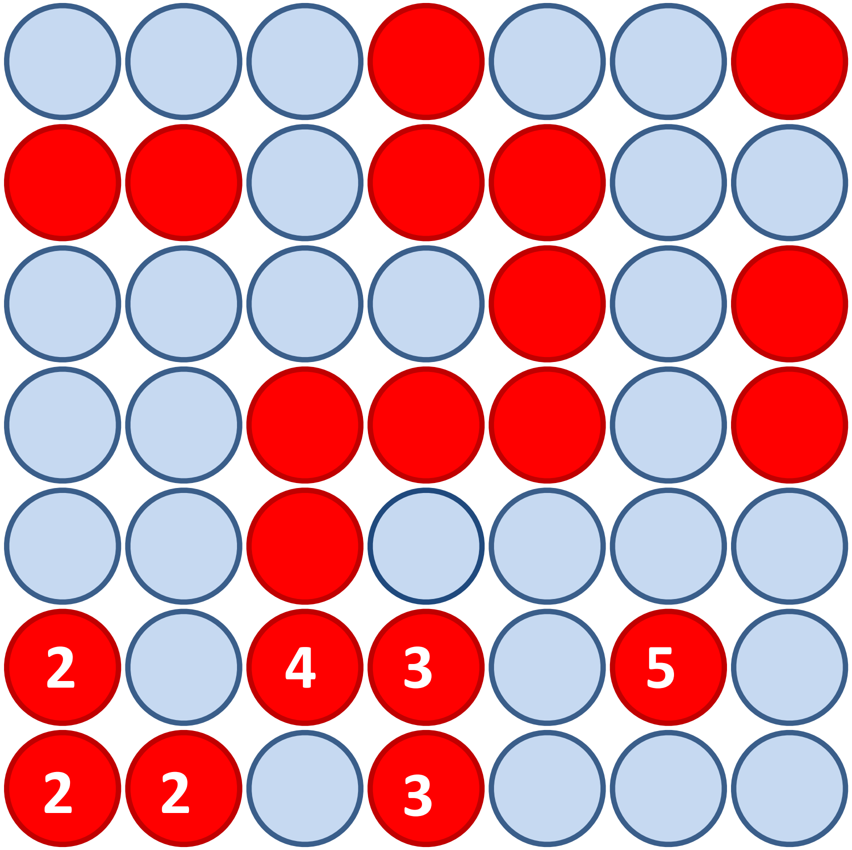

k	M(k)
2	3
3	3
4	-3
5	1

J. Hoshen and R. Kopelman. *Phys. Rev. B* **14**, 3438 (1976)

Algorithms

Hoshen and Kopelman

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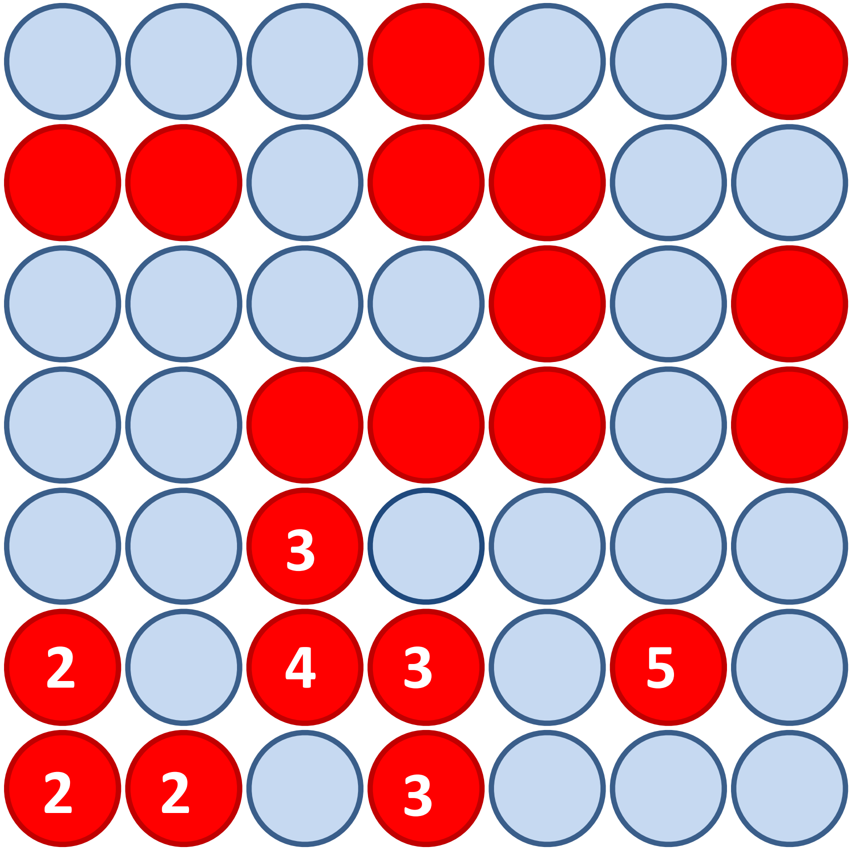

k	M(k)
2	3
3	4
4	-3
5	1

J. Hoshen and R. Kopelman. *Phys. Rev. B* **14**, 3438 (1976)

Algorithms

Hoshen and Kopelman

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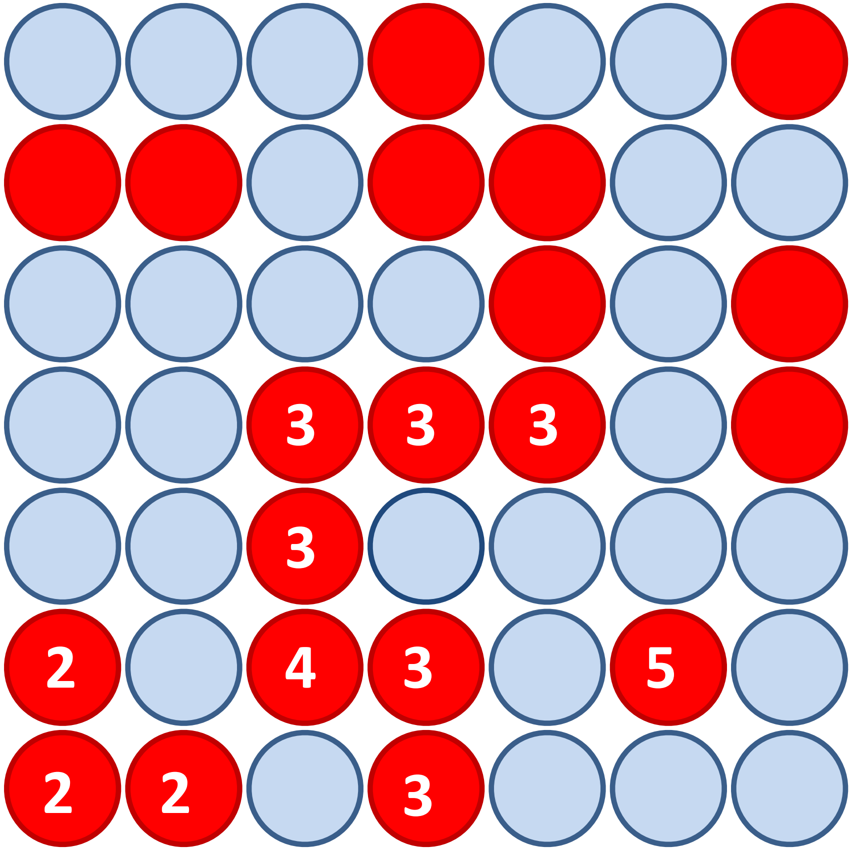

k	M(k)
2	3
3	7
4	-3
5	1

J. Hoshen and R. Kopelman. *Phys. Rev. B* **14**, 3438 (1976)

Algorithms

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1. **start** from the site in the **left-bottom corner**;

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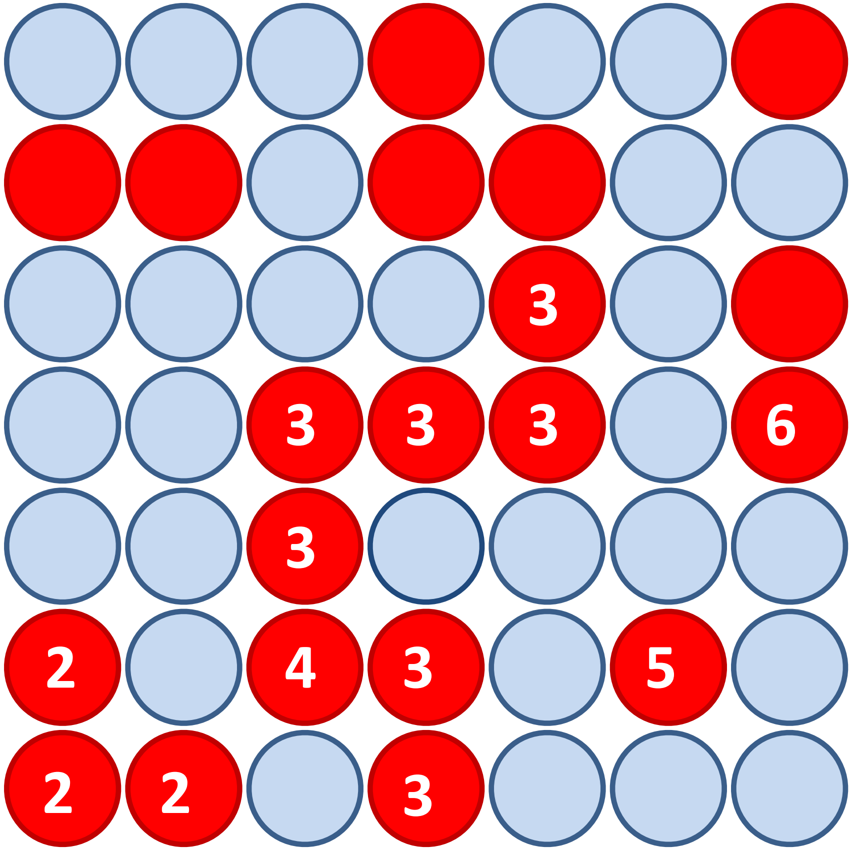

k	M(k)
2	3
3	8
4	-3
5	1
6	1

J. Hoshen and R. Kopelman. *Phys. Rev. B* **14**, 3438 (1976)

Algorithms

Hoshen and Kopelman

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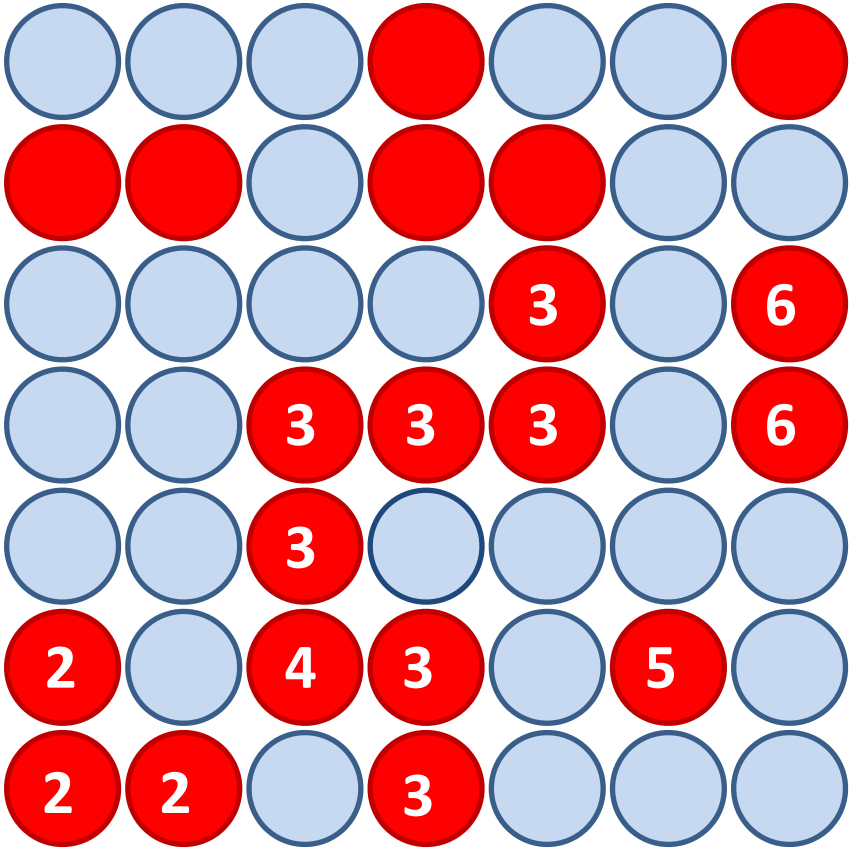

k	M(k)
2	3
3	8
4	-3
5	1
6	2

J. Hoshen and R. Kopelman. *Phys. Rev. B* **14**, 3438 (1976)

Algorithms

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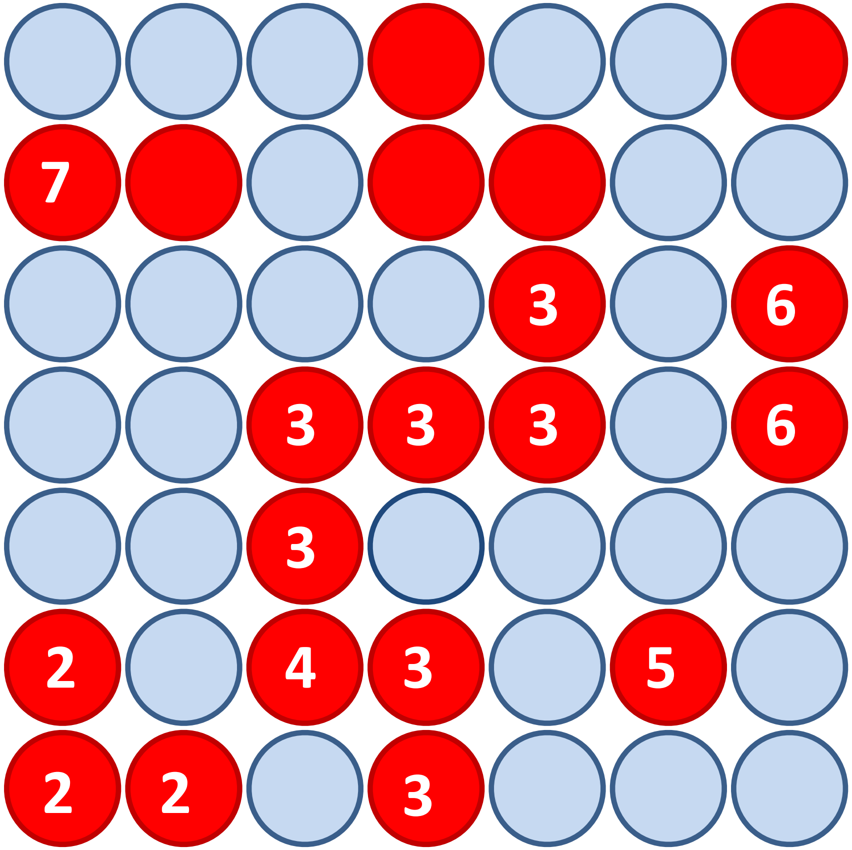

k	M(k)
2	3
3	8
4	-3
5	1
6	2
7	1

J. Hoshen and R. Kopelman. *Phys. Rev. B* **14**, 3438 (1976)

Algorithms

Hoshen and Kopelman

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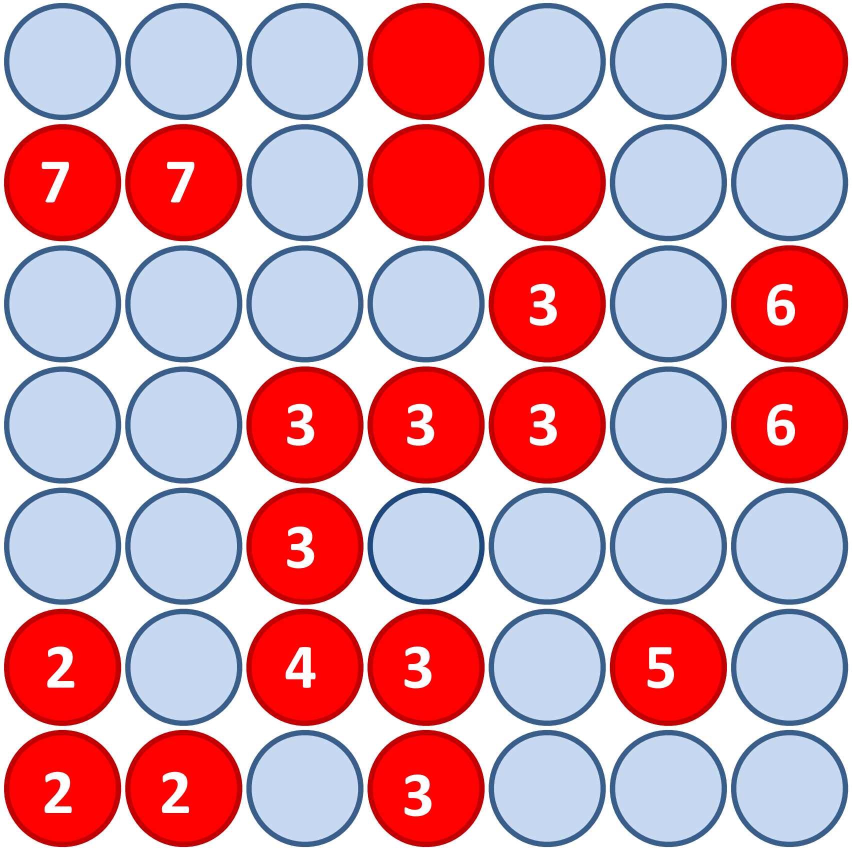

k	M(k)
2	3
3	8
4	-3
5	1
6	2
7	2

J. Hoshen and R. Kopelman. *Phys. Rev. B* **14**, 3438 (1976)

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Hoshen and Kopelman

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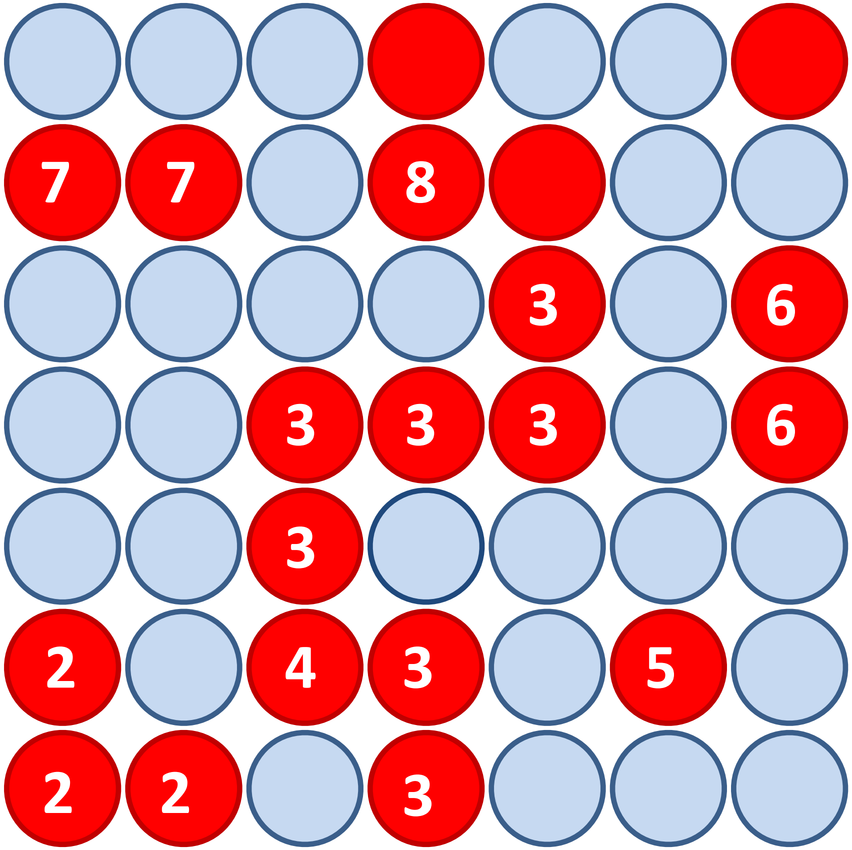

k	M(k)
2	3
3	8
4	-3
5	1
6	2
7	2
8	1

J. Hoshen and R. Kopelman. *Phys. Rev. B* **14**, 3438 (1976)

Algorithms

Hoshen and Kopelman

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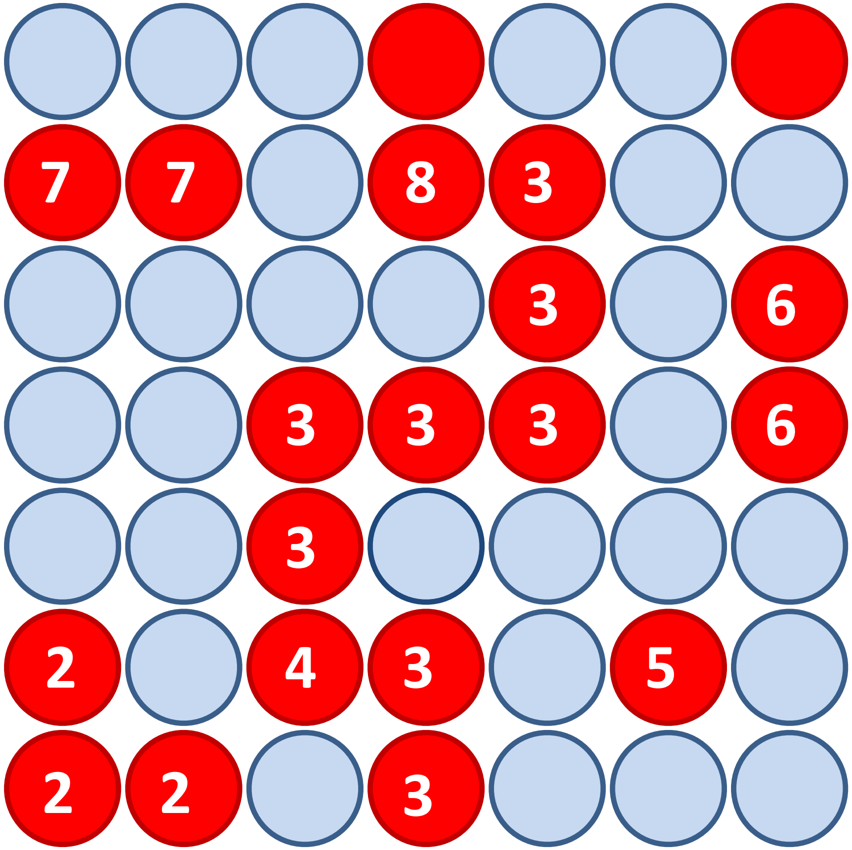

k	M(k)
2	3
3	10
4	-3
5	1
6	2
7	2
8	-3

J. Hoshen and R. Kopelman. *Phys. Rev. B* **14**, 3438 (1976)

Algorithms

Hoshen and Kopelman

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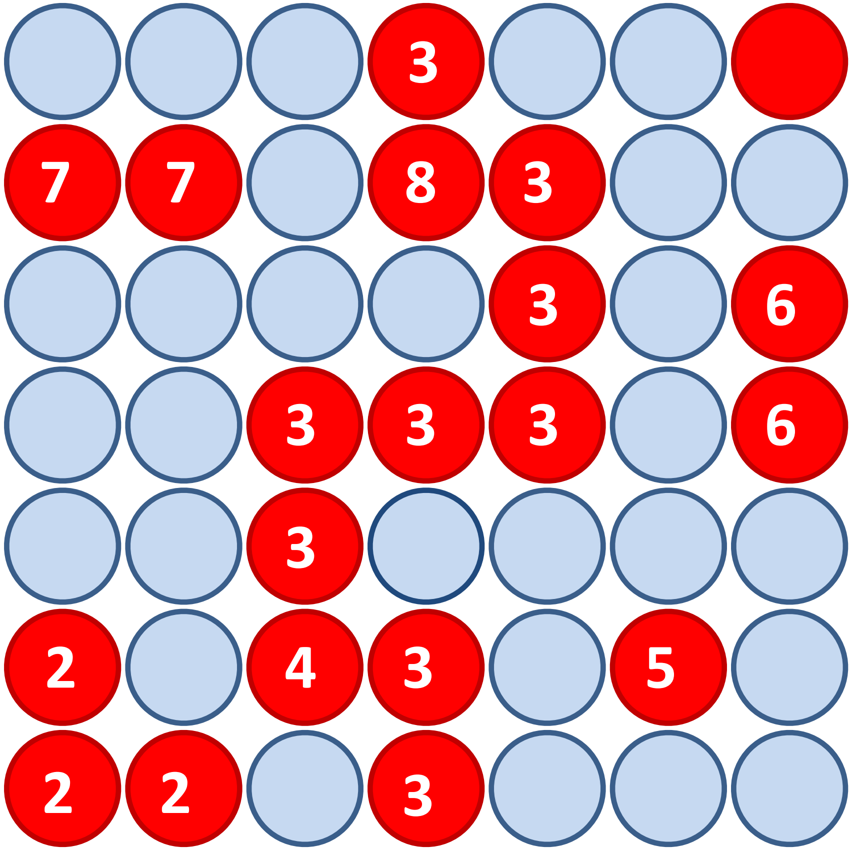
2. **sweep** from **left to right bottom to top**;

3. **only** verify **left** and **bottom** neighbors.



k	M(k)
2	3
3	11
4	-3
5	1
6	2
7	2
8	-3

J. Hoshen and R. Kopelman. *Phys. Rev. B* **14**, 3438 (1976)

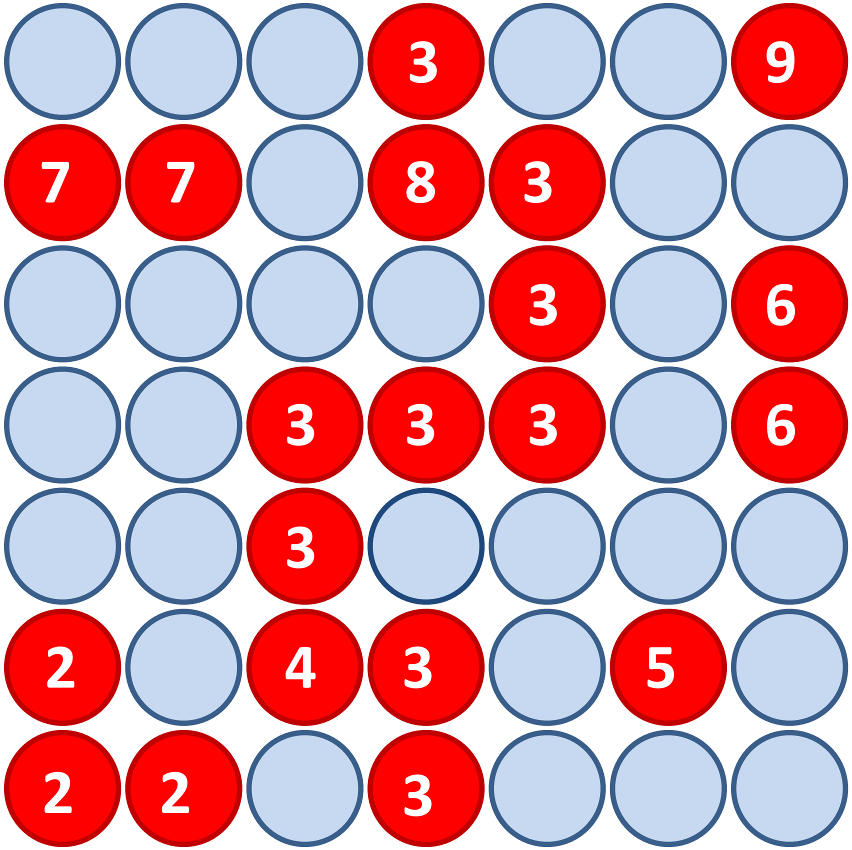
Algorithms

Hoshen and Kopelman

1. **start** from the site in the **left-bottom corner**;

2. **sweep** from **left to right bottom to top**;

3. **only** verify **left** and **bottom** neighbors.

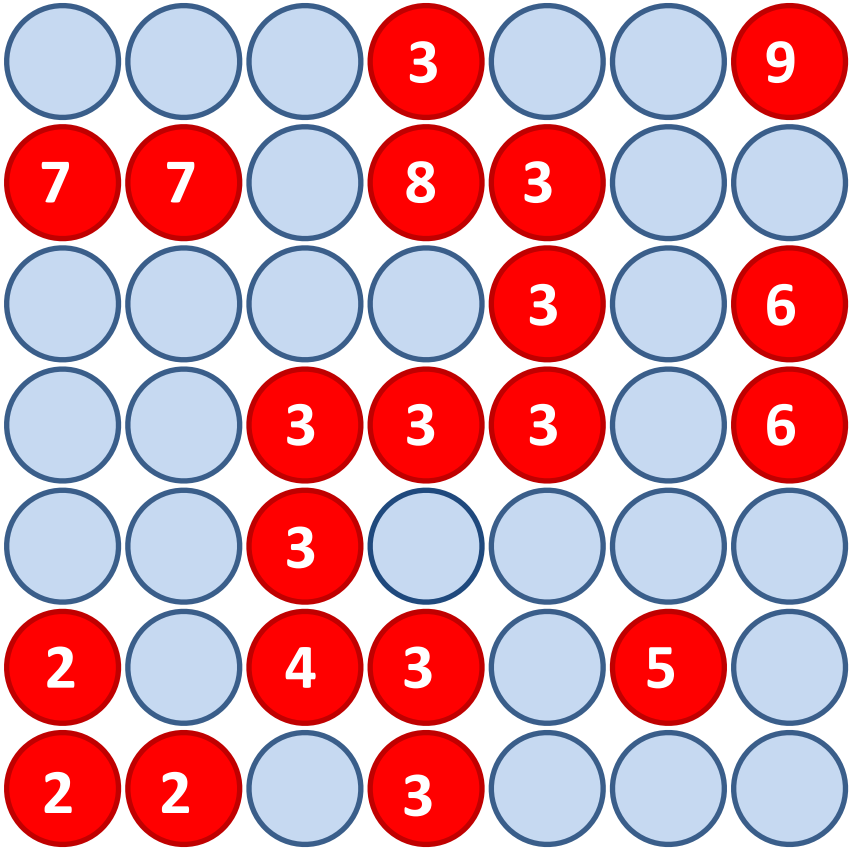


k	M(k)
2	3
3	11
4	-3
5	1
6	2
7	2
8	-3
9	1

J. Hoshen ar 38 (1976)

Algorithms

Hoshen and Kopelman



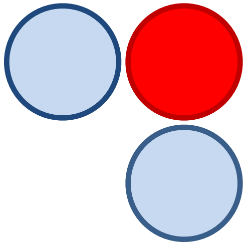
			3			9
7	7		8	3		
				3		6
		3	3	3		6
		3				
2		4	3		5	
2	2		3			

k	M(k)
2	3
3	11
4	-3
5	1
6	2
7	2
8	-3
9	1

J. Hoshen and R. Kopelman. *Phys. Rev. B* **14**, 3438 (1976)

Algorithms

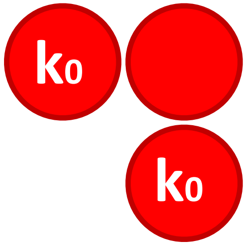
Hoshen and Kopelman



Isolated

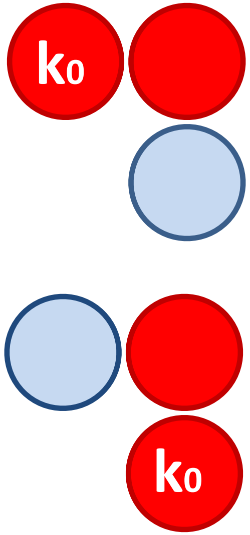
$k = k + 1$

$M(k) = 1$



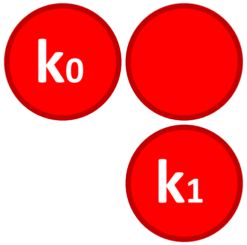
Two neighbor k_0:

$M(\underline{k_0}) = M(\underline{k_0}) + 1$



One neighbor k_0:

$M(\underline{k_0}) = M(\underline{k_0}) + 1$



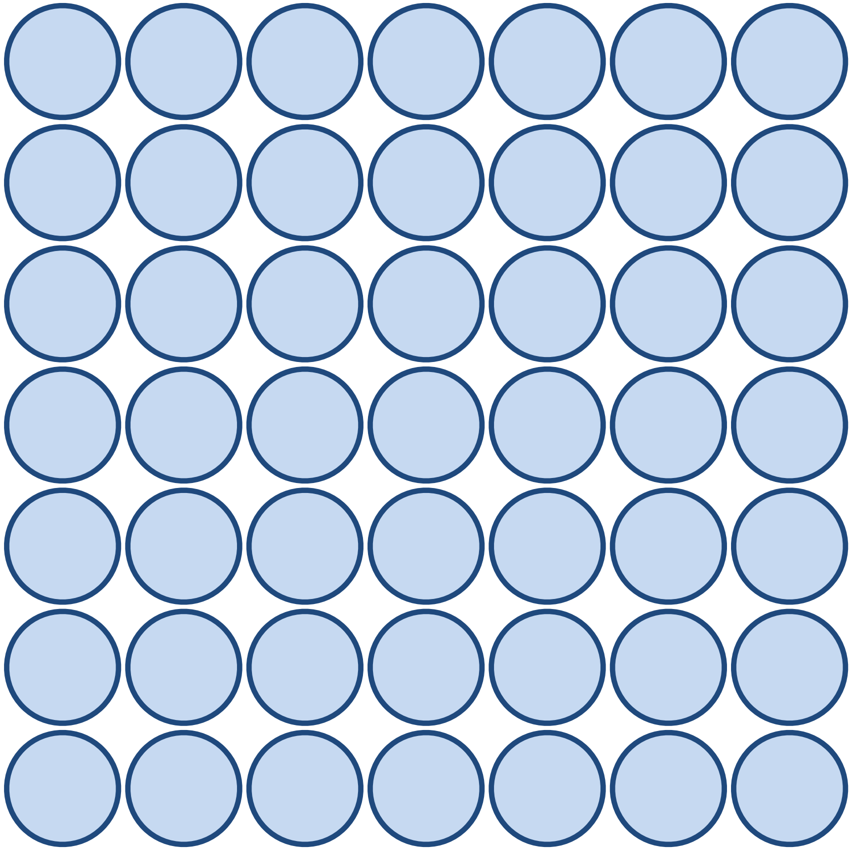
One neighbor k_0 and one neighbor k_1 :

$M(\underline{k_0}) = M(\underline{k_0}) + M(\underline{k_1}) + 1$

J. Hoshen and R. Kopelman. *Phys. Rev. B* **14**, 3438 (1976)

Algorithms

Newman and Ziff (microcanonical)

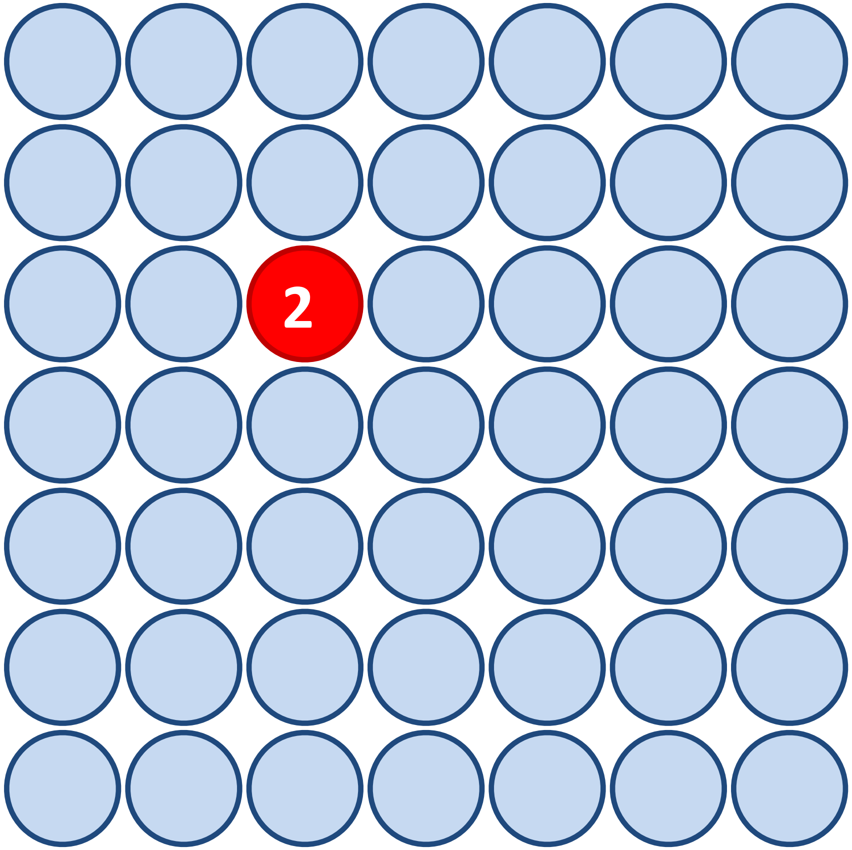


k	M(k)
2	0

M. E. J. Newman and R. M. Ziff. *Phys. Rev. Lett.* **85**, 4104 (2000)
M. E. J. Newman and R. M. Ziff. *Phys. Rev. E* **64**, 016706 (2001)

Algorithms

Newman and Ziff (microcanonical)

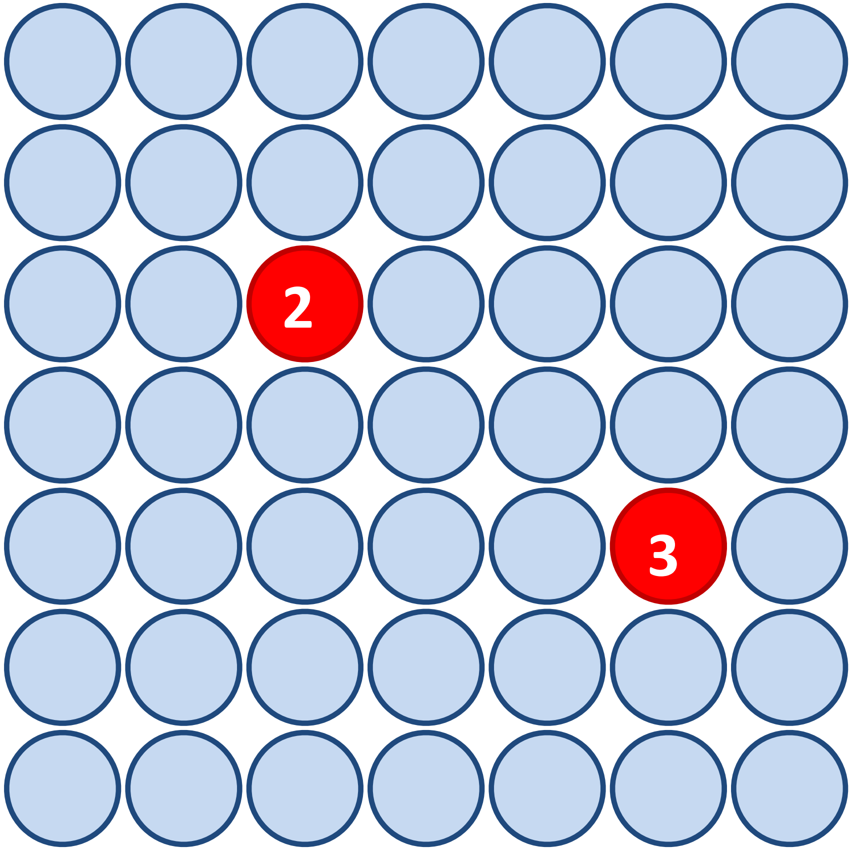


k	M(k)
2	1

M. E. J. Newman and R. M. Ziff. *Phys. Rev. Lett.* **85**, 4104 (2000)
M. E. J. Newman and R. M. Ziff. *Phys. Rev. E* **64**, 016706 (2001)

Algorithms

Newman and Ziff (microcanonical)

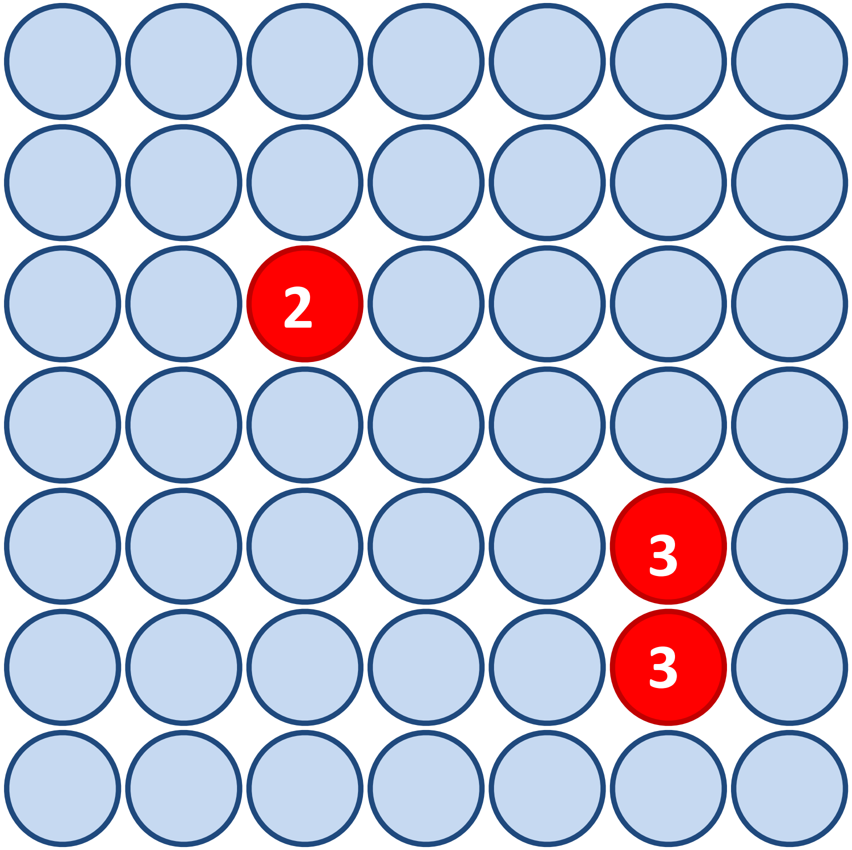


k	M(k)
2	1
3	1

M. E. J. Newman and R. M. Ziff. *Phys. Rev. Lett.* **85**, 4104 (2000)
M. E. J. Newman and R. M. Ziff. *Phys. Rev. E* **64**, 016706 (2001)

Algorithms

Newman and Ziff (microcanonical)

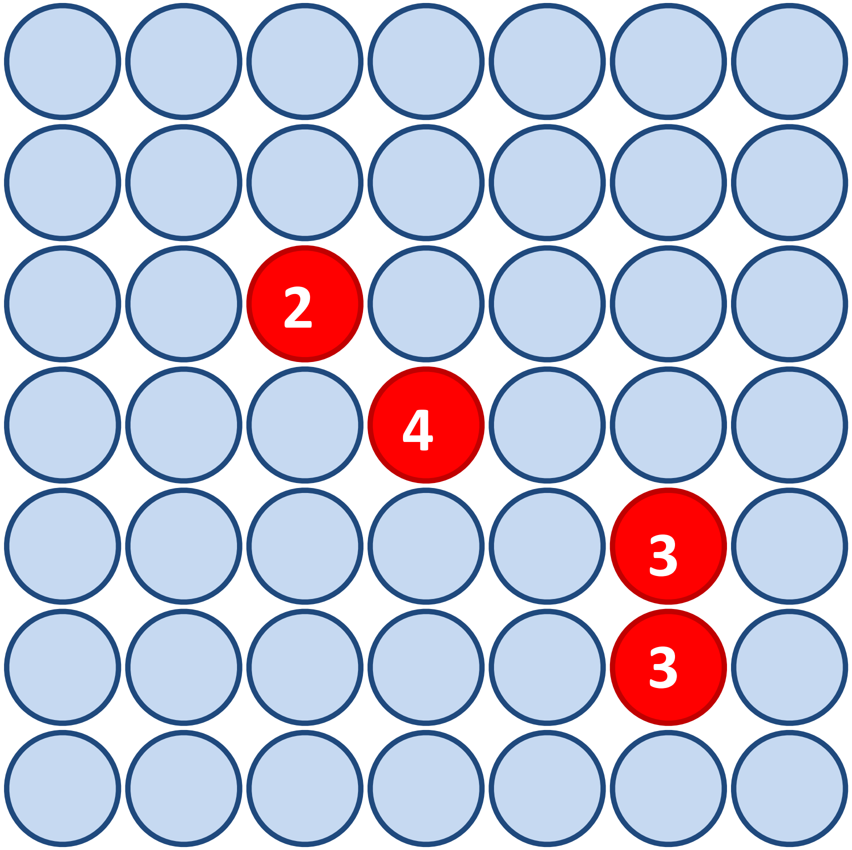


k	M(k)
2	1
3	2

M. E. J. Newman and R. M. Ziff. *Phys. Rev. Lett.* **85**, 4104 (2000)
M. E. J. Newman and R. M. Ziff. *Phys. Rev. E* **64**, 016706 (2001)

Algorithms

Newman and Ziff (microcanonical)

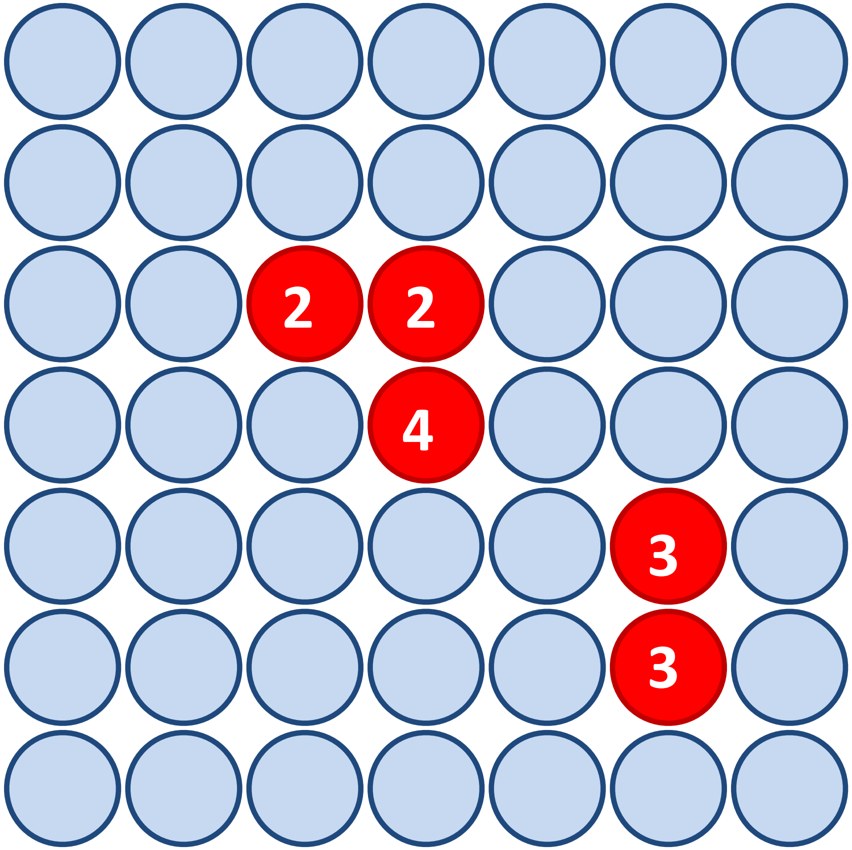


k	M(k)
2	1
3	2
4	1

M. E. J. Newman and R. M. Ziff. *Phys. Rev. Lett.* **85**, 4104 (2000)
M. E. J. Newman and R. M. Ziff. *Phys. Rev. E* **64**, 016706 (2001)

Algorithms

Newman and Ziff (microcanonical)



k	M(k)
2	3
3	2
4	-2

M. E. J. Newman and R. M. Ziff. *Phys. Rev. Lett.* **85**, 4104 (2000)
M. E. J. Newman and R. M. Ziff. *Phys. Rev. E* **64**, 016706 (2001)

Algorithms

Microcanonical vs canonical

Fixed number of occupied sites (*n*)

Fixed probability that a site is occupied (*p*)

$$\boldsymbol{B(N,n,p)} = \binom{N}{n} p^n (1-p)^{N-n}$$

$\boldsymbol{B(N,n,p)}$: probability that exactly n sites are occupied in a *canonical* configuration

$$\boxed{Q(p) = \sum_{n=0}^{N} \boldsymbol{B(N,n,p)} Q_n = \sum_{n=0}^{N} \binom{N}{n} p^n (1-p)^{N-n} Q_n}$$

M. E. J. Newman and R. M. Ziff. *Phys. Rev. Lett.* **85**, 4104 (2000)
M. E. J. Newman and R. M. Ziff. *Phys. Rev. E* **64**, 016706 (2001)