CFTC Centro de Física Teórica e Computacional

Ciências ULisboa

Modelos de crescimento (fora de equilíbrio)

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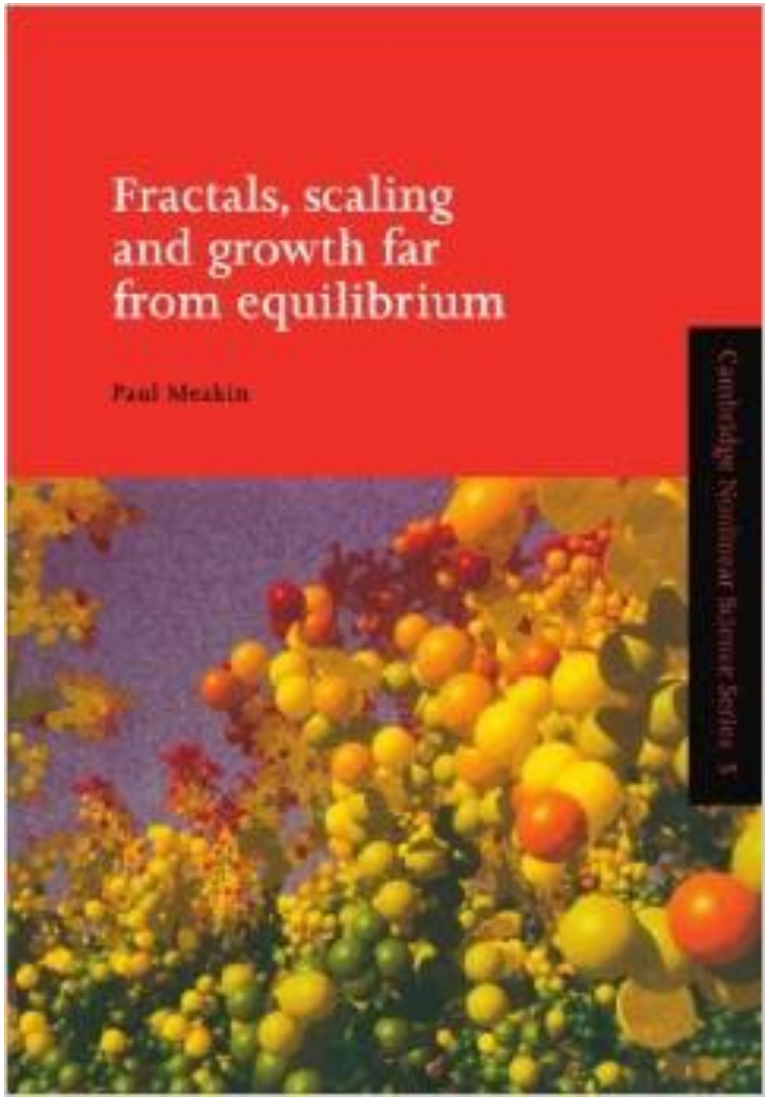
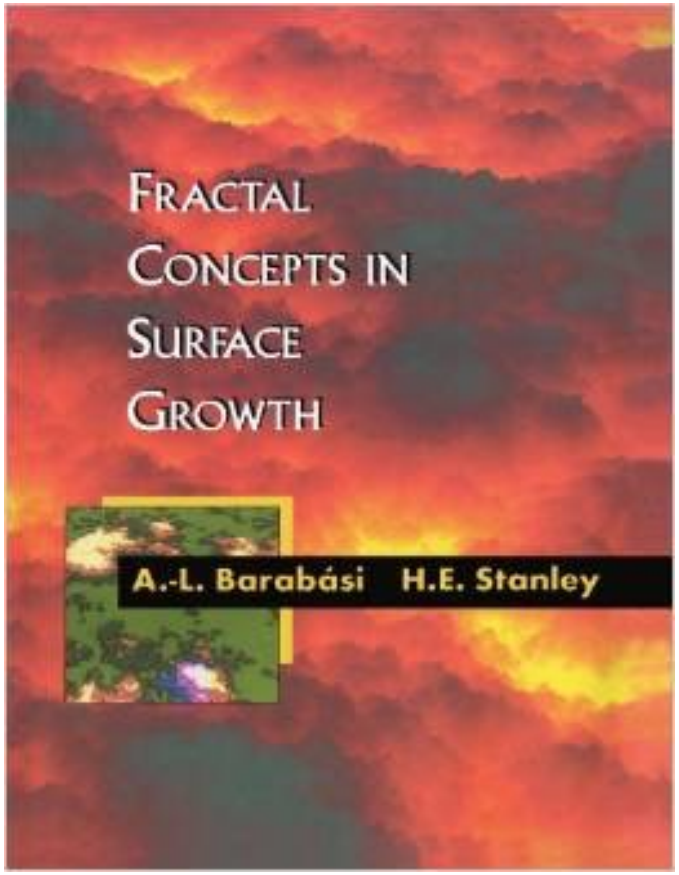
Bibliography

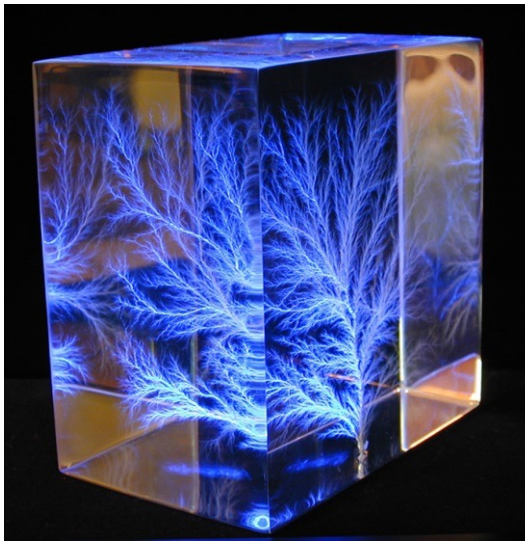
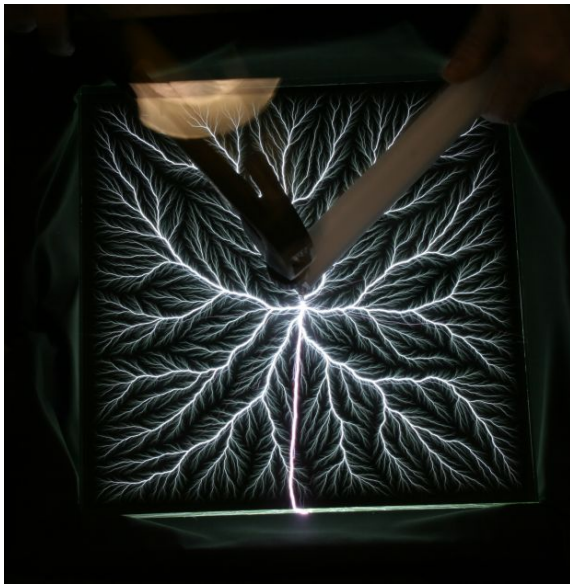
Fractals, scaling and growth far from equilibrium
P. Meakin
Cambridge University Press





Fractal concepts in surface growth
A.-L. Barabási and H. E. Stanley
Cambridge University Press

Motivation

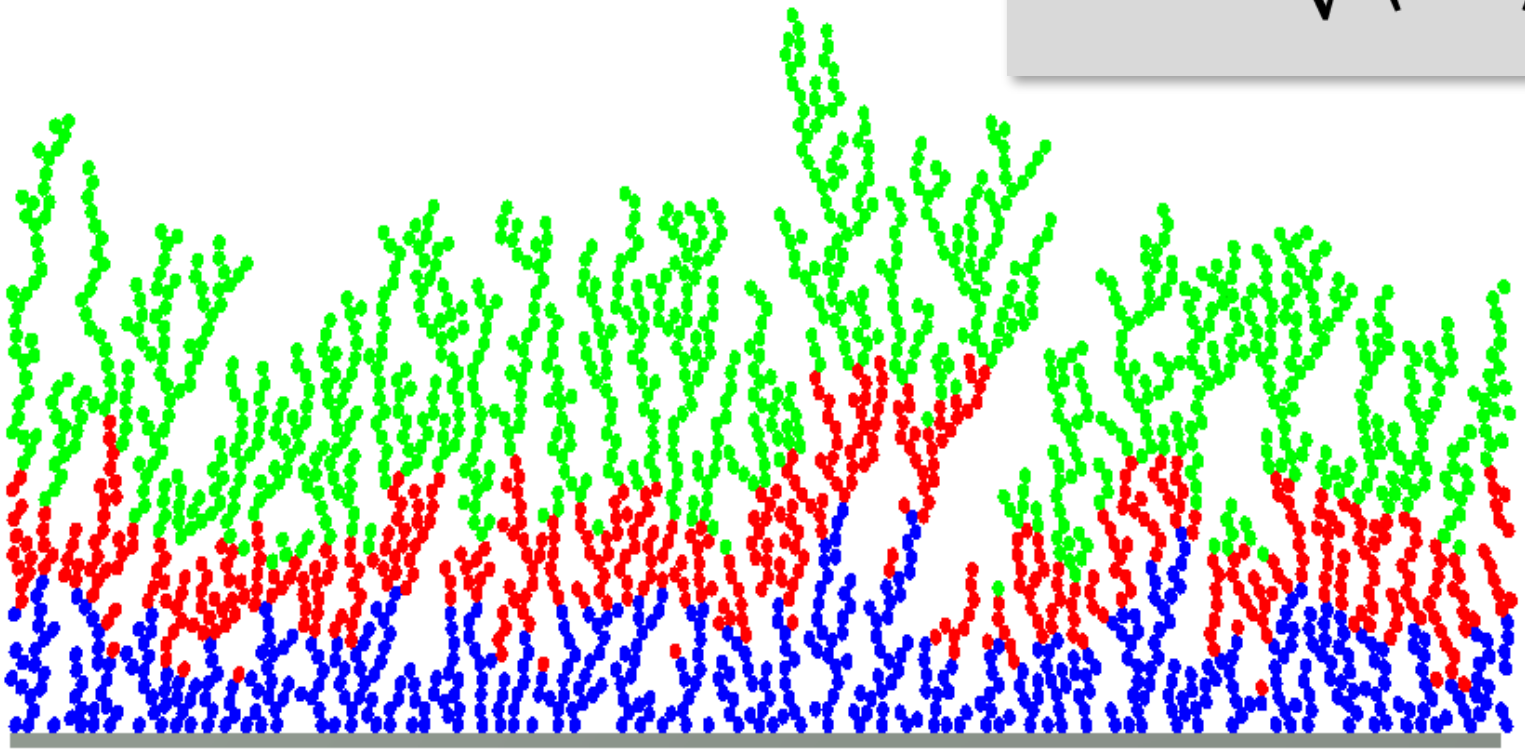



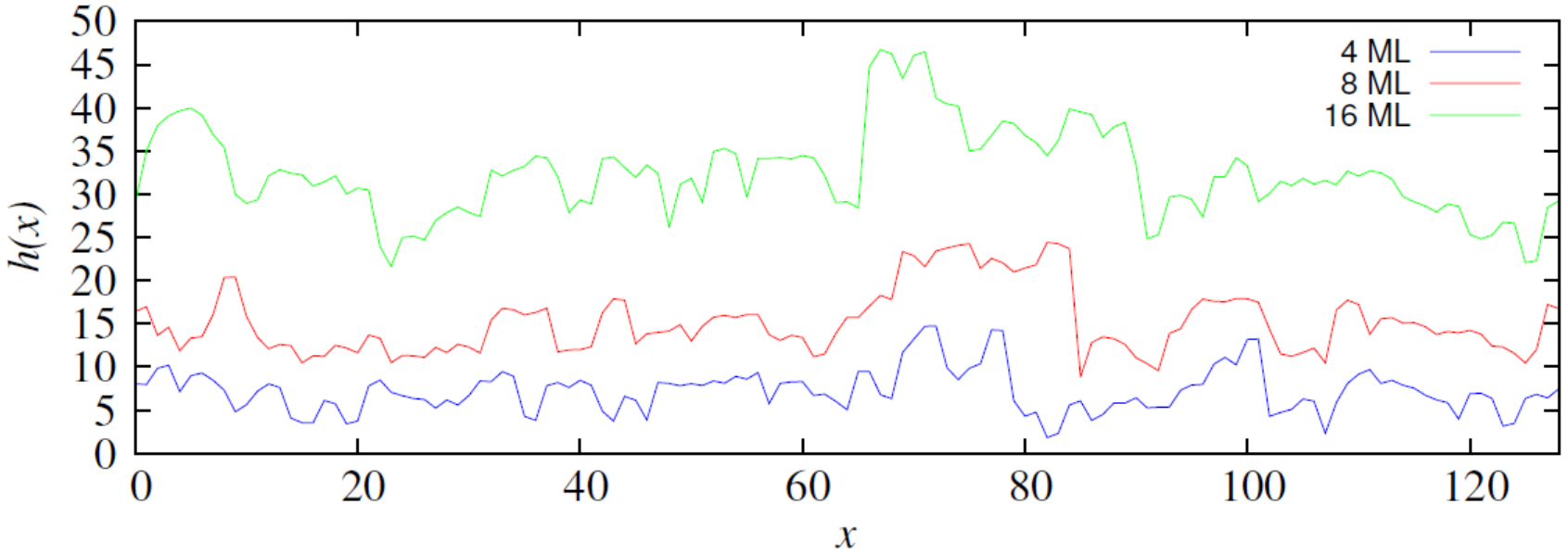


Sources: wikipedia, http://capturelightning.com/, http://thejefkins.wordpress.com/

Interface roughness

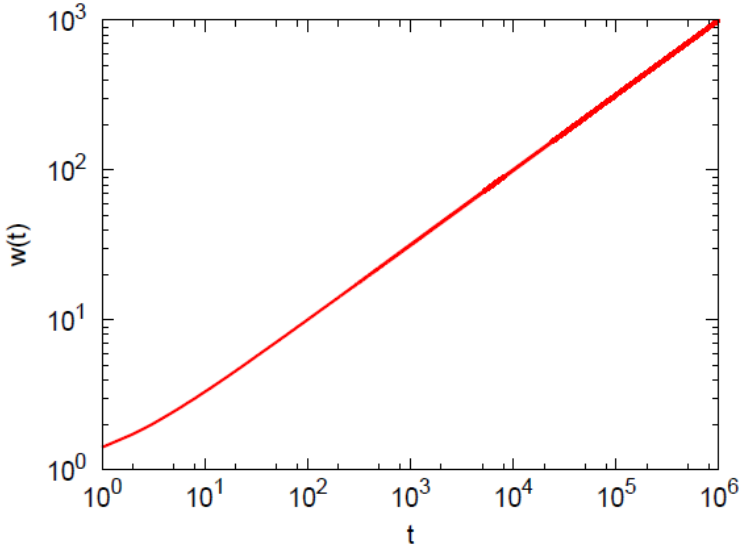
$$W = \sqrt{\langle h^2 \rangle - \langle h \rangle^2}$$

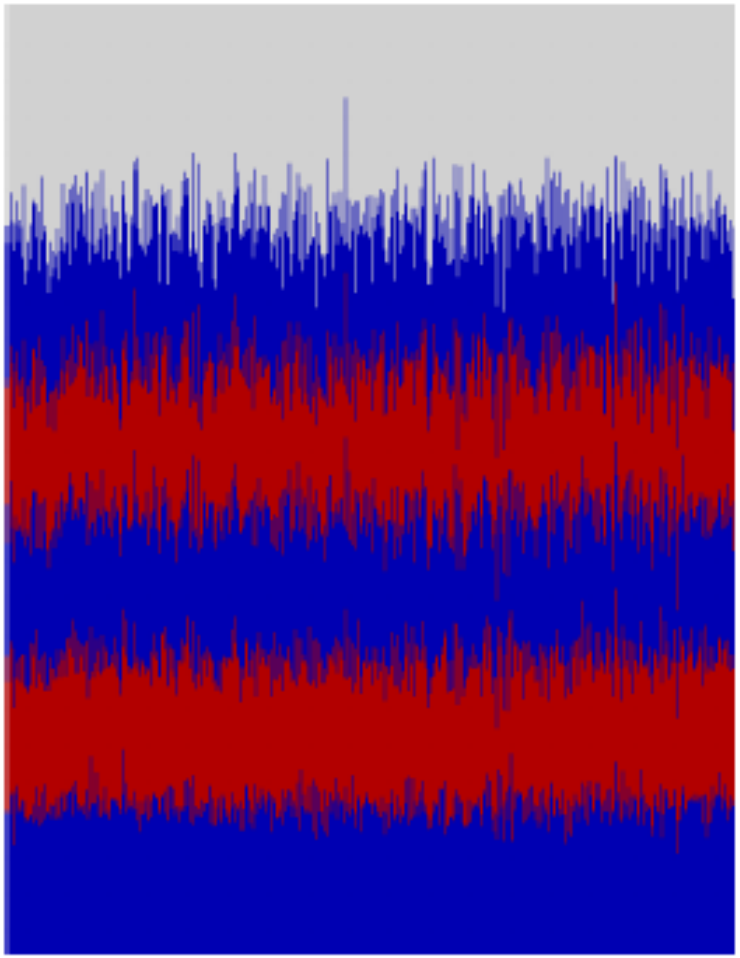


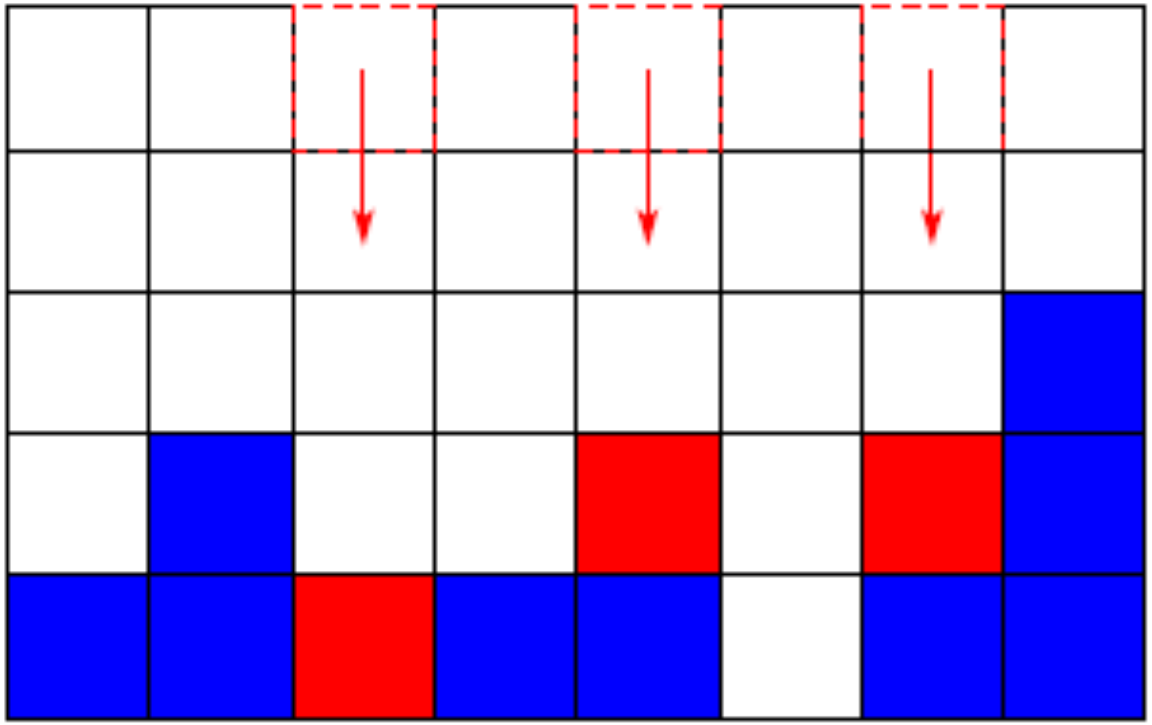


Random deposition

$$W \sim t^{\beta}$$







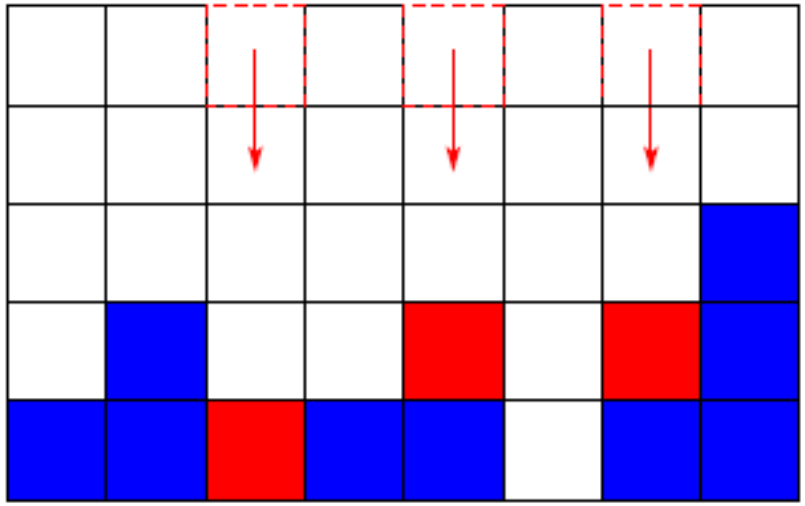
$$\beta = \frac{1}{2}$$

Random deposition analytic solution

$N \equiv$ number of deposited particles

$p = \frac{1}{L}$, L is the system size

$t = \frac{N}{L}$



$$P(h,N) = \binom{N}{h} p^h (1-p)^{N-h}$$

$$\langle h \rangle = \sum_{h=0}^{h=N} hP(h,N) = Np = t$$

$$\langle h^2 \rangle = \sum_{h=0}^{h=N} h^2 P(h,N) = Np(1-p) + N^2 p^2$$

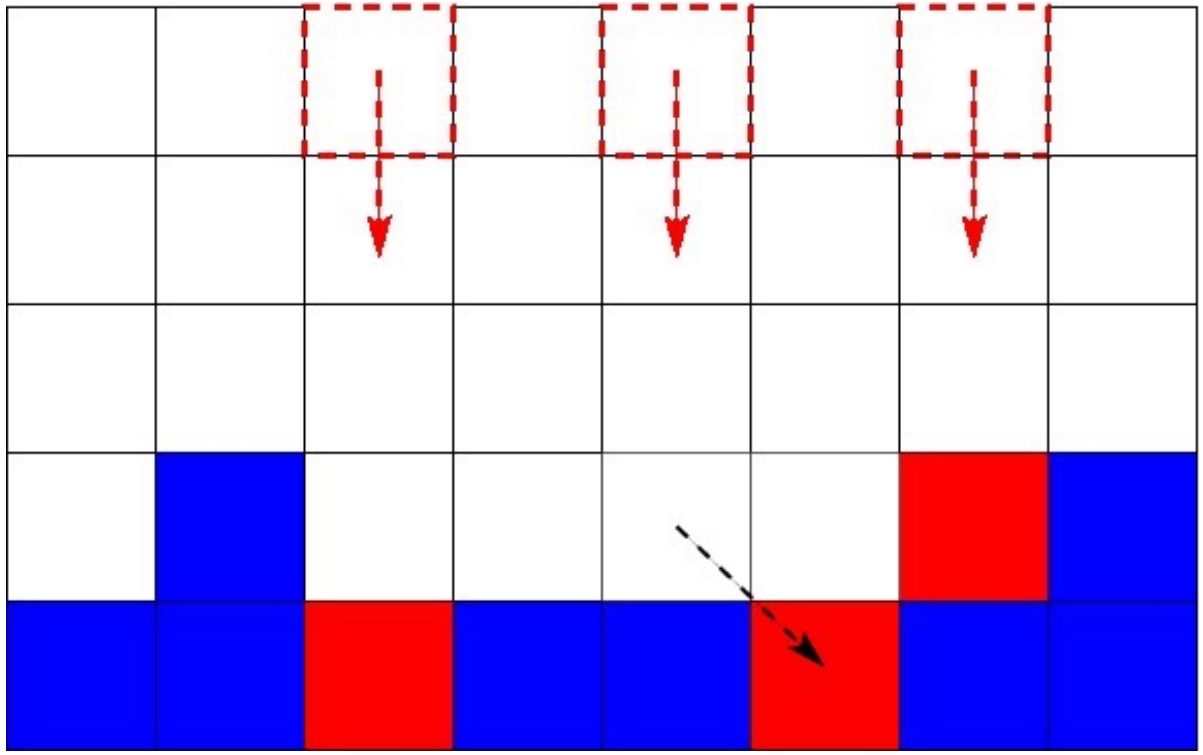
$$w^2 = \langle h^2 \rangle - \langle h \rangle^2 = Np(1-p) = \frac{N}{L}\left(1 - \frac{1}{L}\right)$$

$$w \sim t^{1/2}$$

Random deposition with surface diffusion

$$W \sim t^{\beta}$$

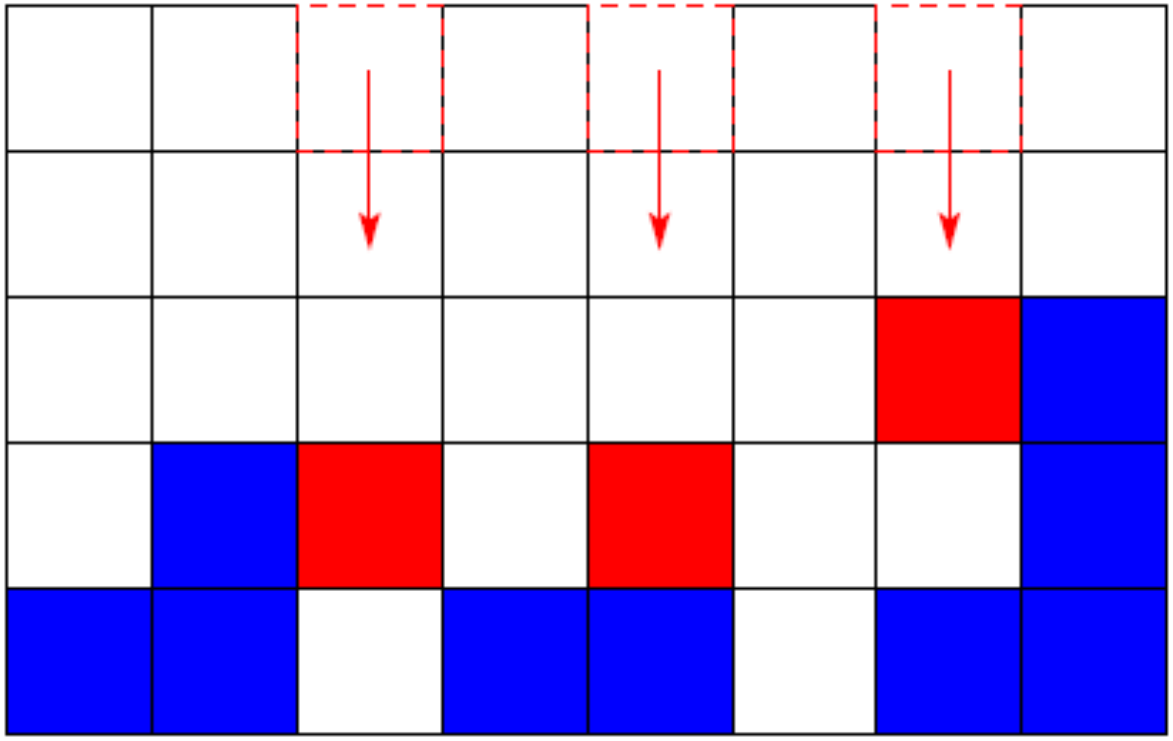
$$W_{sat} \sim L^{\alpha}$$

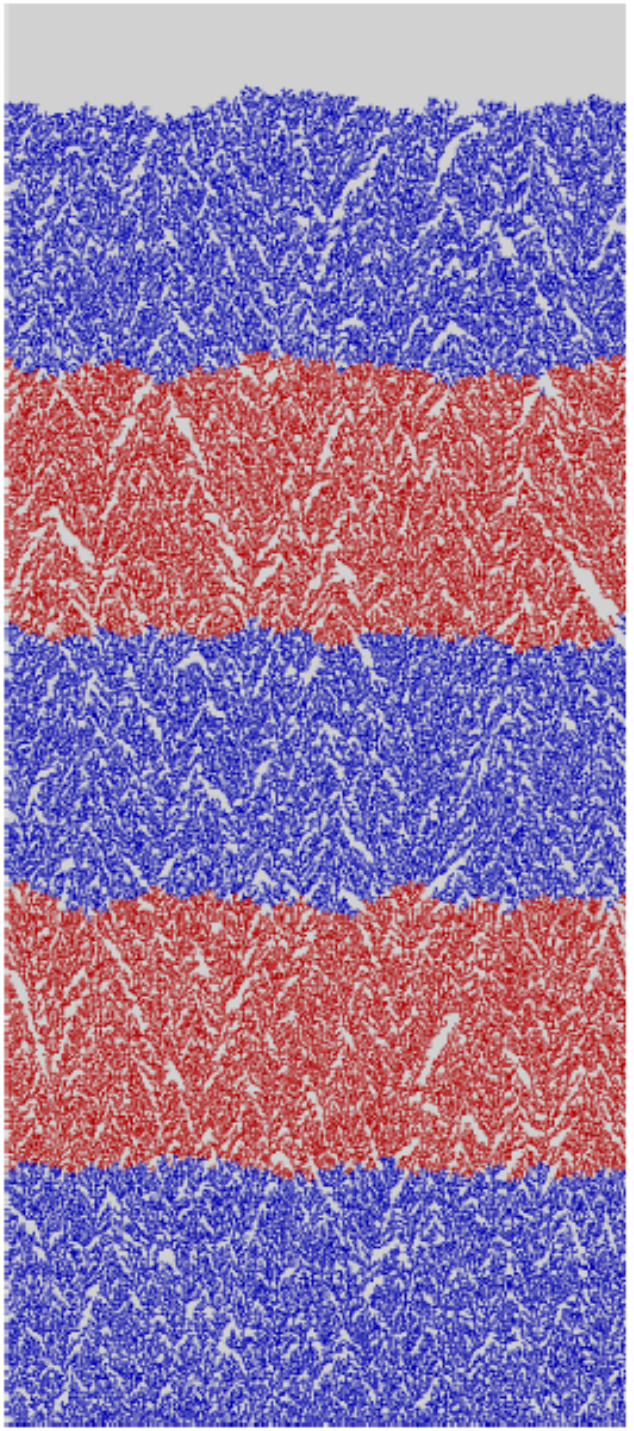


$$\beta = \frac{1}{4}$$

$$\alpha = \frac{1}{2}$$

Ballistic deposition





M. J. Vold, *J. Coll. Sci.* **14**, 168 (1959)

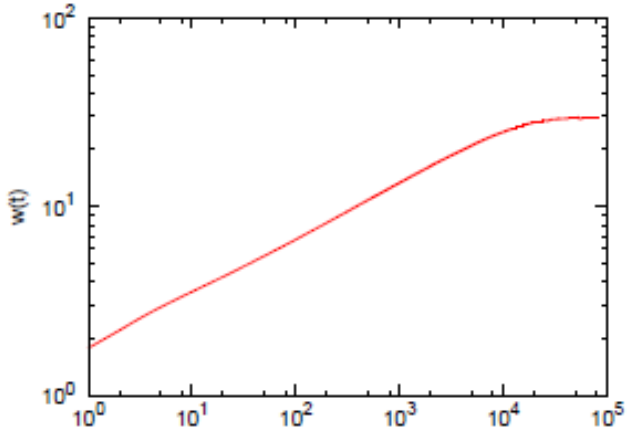
Ballistic deposition

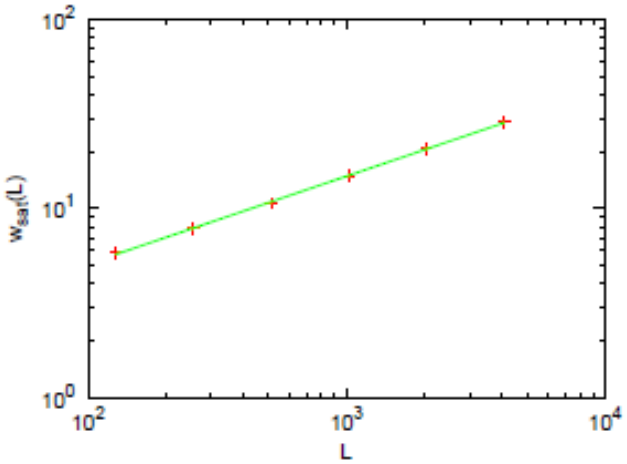
$$W \sim t^{\beta}$$

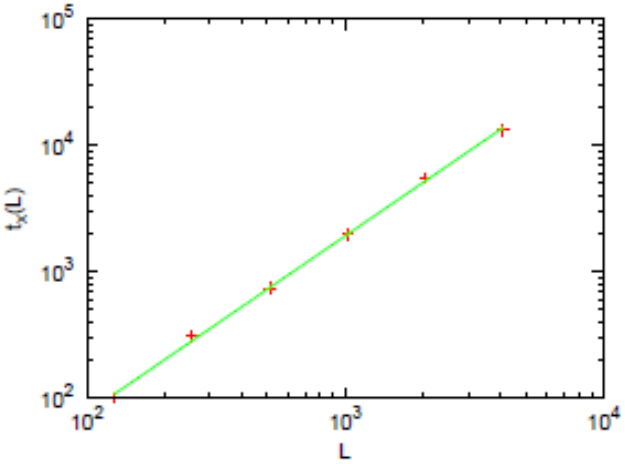
$$W_{sat} \sim L^{\alpha}$$

$$\beta = \frac{1}{3}$$

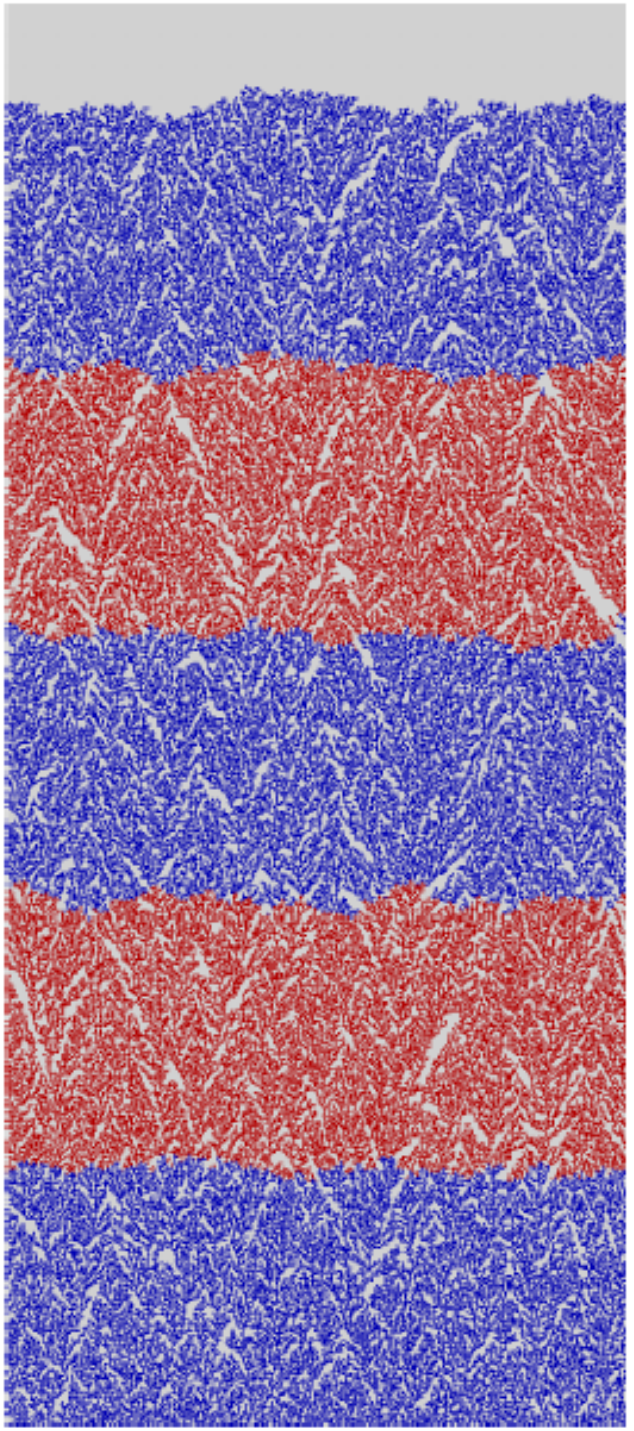
$$\alpha = \frac{1}{2}$$







$$t_{\times} \sim L^{z}$$

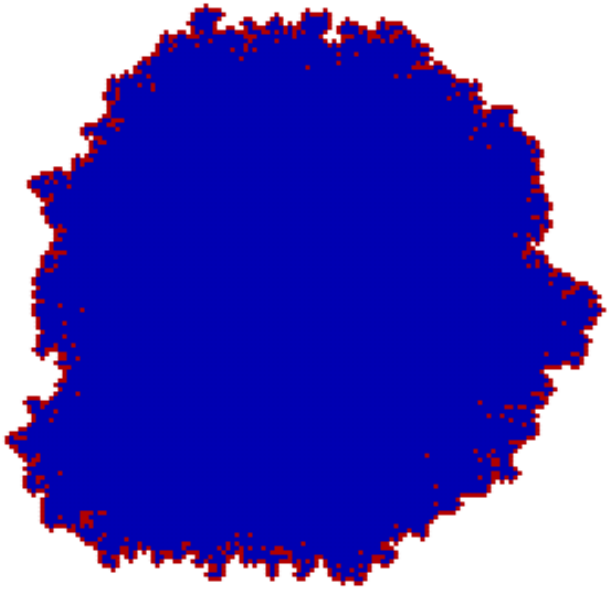


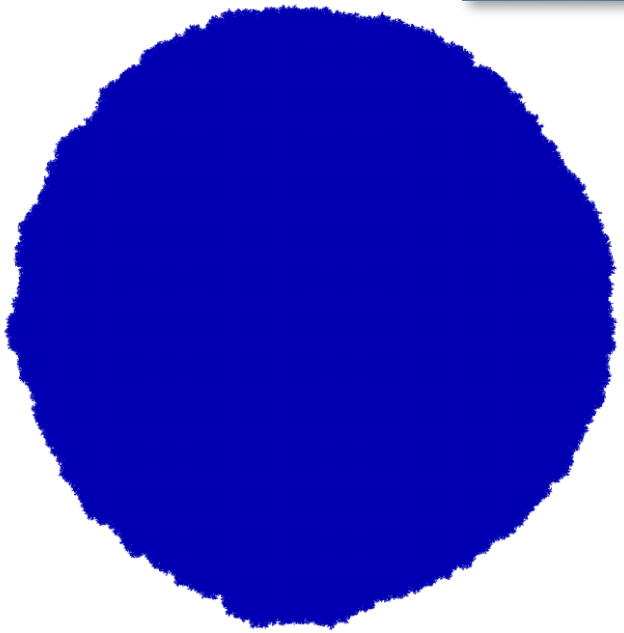
M. J. Vold, *J. Coll. Sci.* **14**, 168 (1959)

Eden model

$$W \sim t^{\beta}$$

$$W_{sat} \sim L^{\alpha}$$





$$\beta = \frac{1}{3}$$

$$\alpha = \frac{1}{2}$$

12 000 particles ***1 200 000 particles***

M. Eden, in *Proceedings of the Fourth Berkeley Symposium on Mathematical Statistics and Probability* **IV**, 223 (1961)

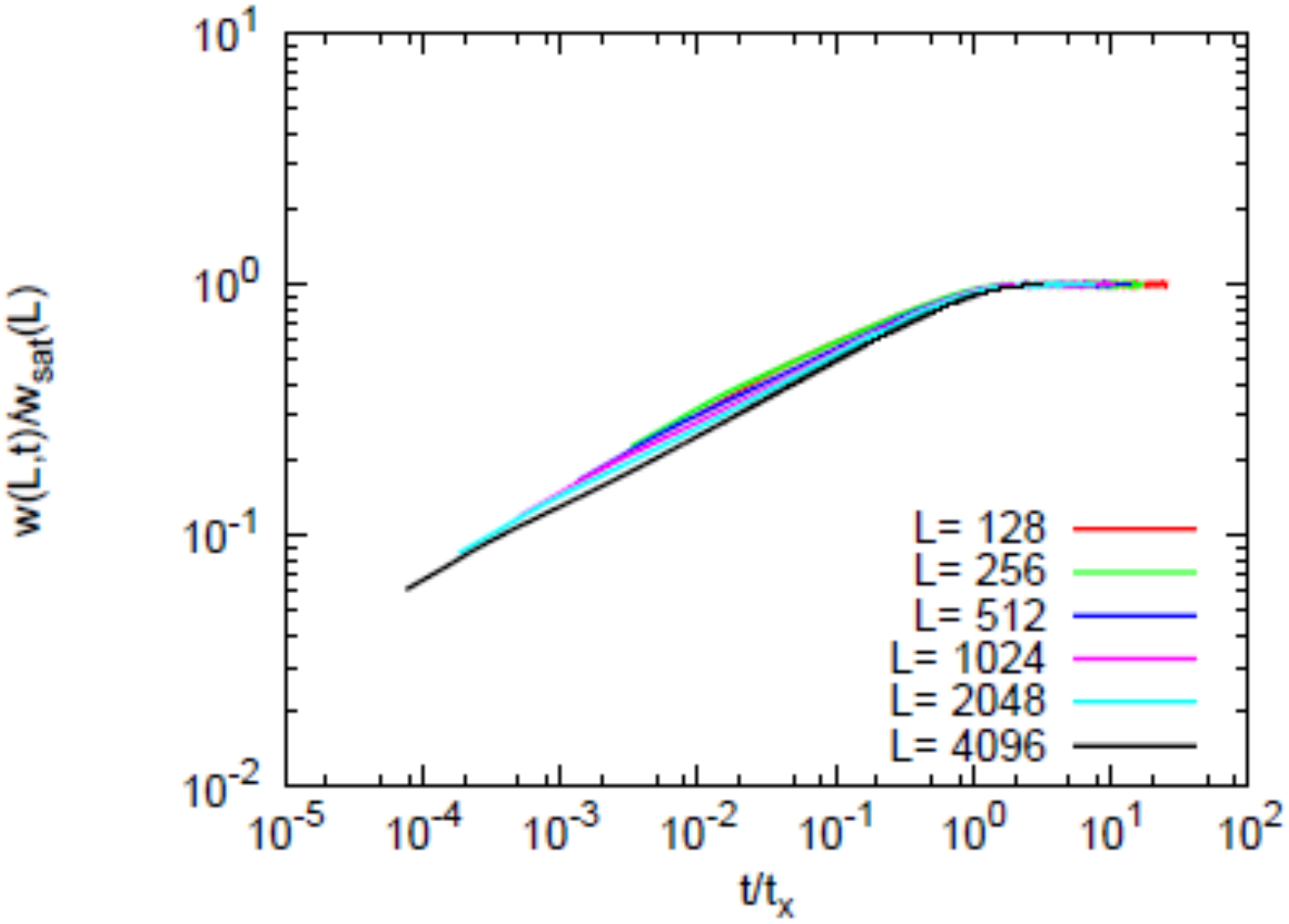
Family-Vicsek scaling

$$W(L,t) = L^{\alpha}\mathcal{F}\left[\frac{t}{L^{z}}\right]$$

$$W \sim t^{\beta}$$

$$W_{sat} \sim L^{\alpha}$$

$$t_{\times} \sim L^{z}$$



F. Family and T. Vicsek, *J.Phys. A* **18**, L75 (1985)

Family-Vicsek scaling

$$W(L,t) = L^{\alpha}\mathcal{F}\left[\frac{t}{L^{z}}\right]$$

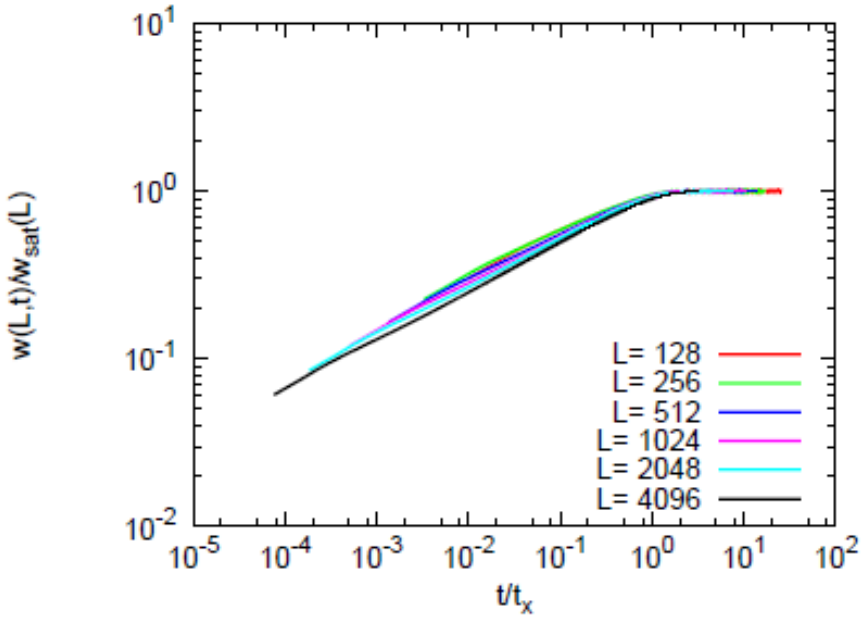
$$t \to \infty: W \sim L^{\alpha} \Rightarrow \mathcal{F}\left[\frac{t}{L^{z}} \to \infty\right] = const$$

$$L \to \infty: W \sim t^{\beta} \Rightarrow \mathcal{F}\left[\frac{t}{L^{z}} \to 0\right] \sim \left(\frac{t}{L^{z}}\right)^{\beta}$$

$$W \sim L^{\alpha}\left(\frac{t}{L^{z}}\right)^{\beta} = L^{\alpha-\beta z}t^{\beta}$$

$$\alpha - \beta z = 0$$

$$\beta = \frac{\alpha}{z}$$

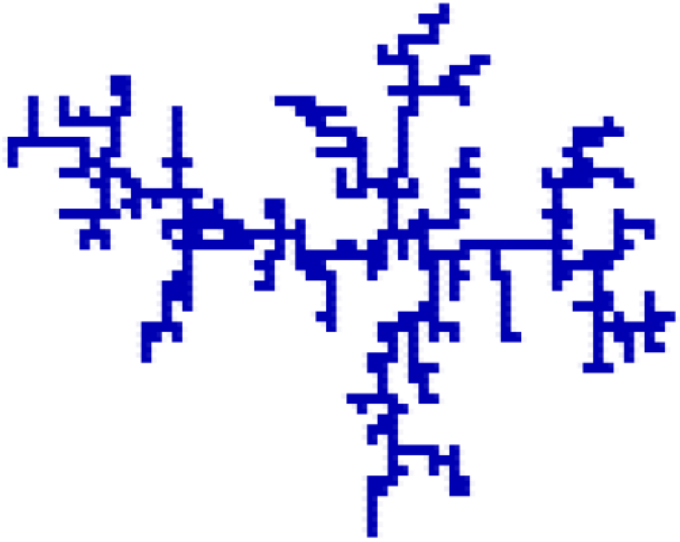


F. Family and T. Vicsek, *J.Phys. A* **18**, L75 (1985)

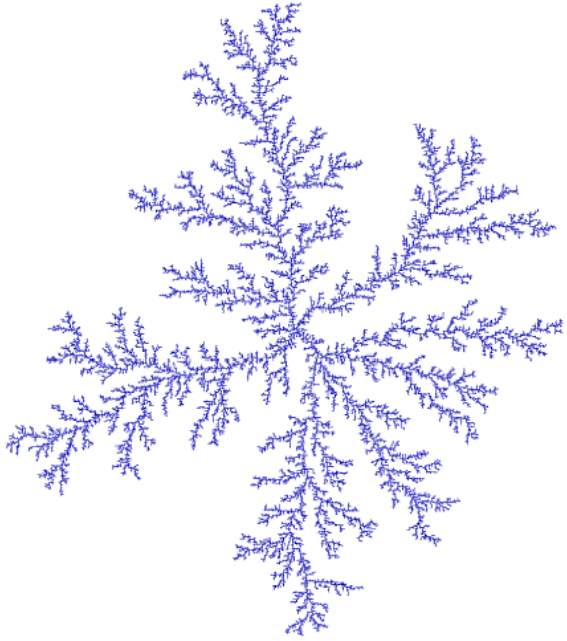
Diffusion limited aggregation

Self-similar (fractal)

$d_f \approx 1.7$ (2D)



500 particles

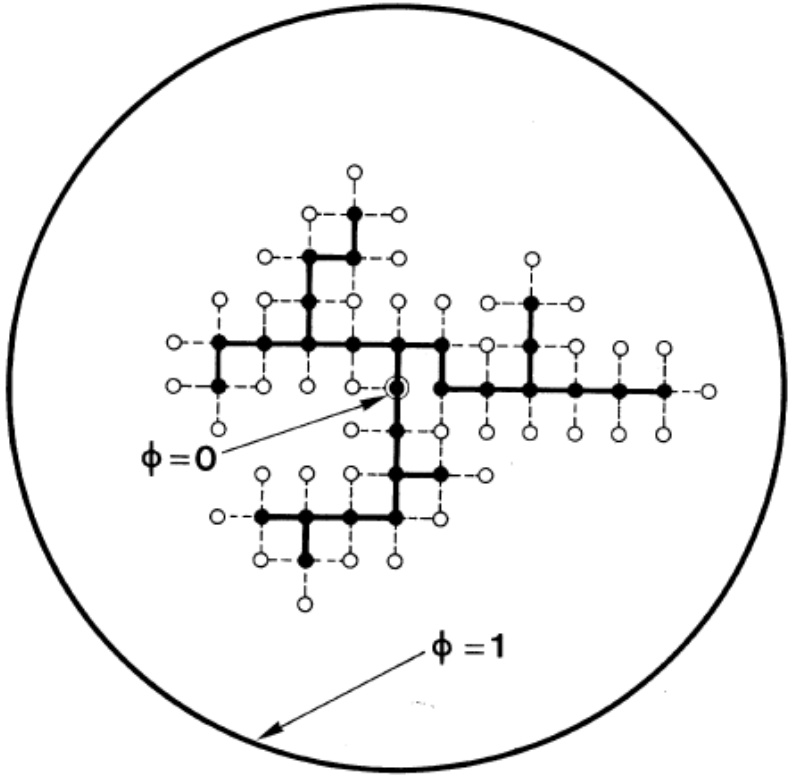


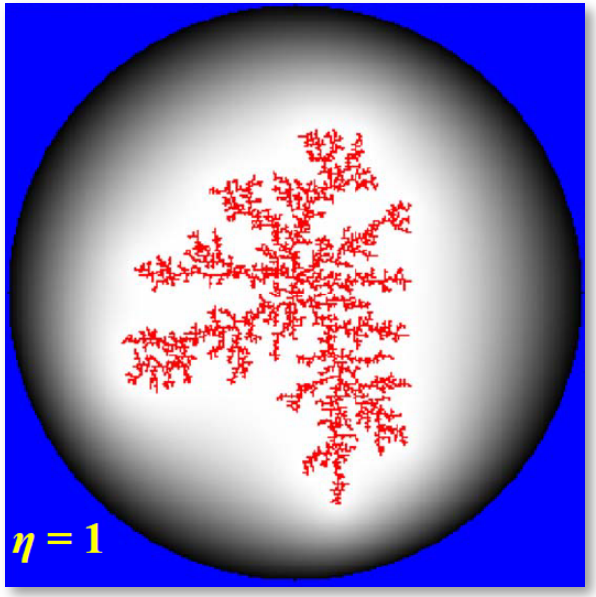
50 000 particles

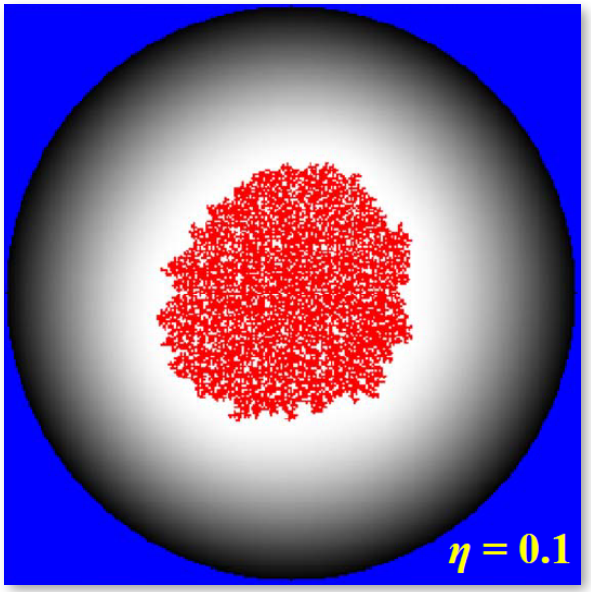
$$\nabla^2 \emptyset = 0$$

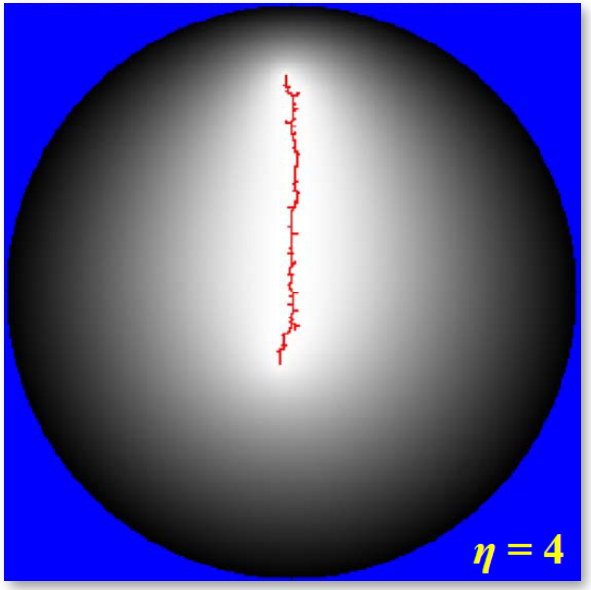
T. A. Witten and L. M. Sander, *Phys. Rev. Lett.* **47**, 1400 (1981)

Dielectric breakdown









1) $\nabla^2 \emptyset = 0$

2) $p \propto (\nabla \emptyset)^{\eta}$

L. Niemeyer, L. Pietronero, and H. J. Wiesmann, *Phys. Rev. Lett.* **52**, 1033 (1984)