

Constante de Stefan-Boltzmann: $\sigma = 5.67 \times 10^{-8} \text{ W} \cdot \text{m}^{-2} \text{K}^{-4}$

HEAT TRANSFER

Thermal resistance

$$\text{Thermal resistance: } R = \frac{L}{kA}$$

$$\text{Resistances in serie: } R_{eq} = \sum_{i=1}^n R_n$$

$$\text{Resistances in parallel: } R_{eq} = \left(\sum_{i=1}^n R_n^{-1} \right)^{-1}$$

U-value and Psi-value

$$U = \frac{1}{R_{eq}A}$$

$$\Psi = \frac{q * / \Delta\theta - U_x A_x - U_y A_y}{B}$$

Heat conductance: $H=UA$

SURFACE-TO-AIR HEAT TRANSFER

Fundamentals

$$\text{Nusselt number: } Nu = \frac{hL}{k_f}$$

$$\text{Prandtl number: } Pr = \frac{\nu}{\alpha}$$

$$\text{Rayleigh number: } Ra_L = \frac{g\beta(T - T_\infty)L^3}{\nu\alpha}$$

$$\text{Grashof number: } Gr_L = \frac{g\beta(T - T_\infty)L^3}{\nu^2}$$

$$\text{Reynolds number: } Re_L = \frac{\rho U_\infty L}{\nu}$$

Vertical Walls

$$Nu_L = 0.59 Ra_L^{1/4} \quad 10^4 \leq Ra_L \leq 10^9 (\text{laminar})$$

$$Nu_L = 0.1 Ra_L^{1/3} \quad 10^9 \leq Ra_L \leq 10^{13} (\text{turbulent})$$

Horizontal Walls

Upwards heat flow:

$$Nu_L = 0.54 Ra_L^{1/4} \quad 10^4 \lesssim Ra_L \lesssim 10^7, Pr \gtrsim 0.7$$

$$Nu_L = 0.15 Ra_L^{1/3} \quad 10^7 \lesssim Ra_L \lesssim 10^{11}, \text{ toda a gama de } Pr$$

Downwards heat flow:

$$Nu_L = 0.27 Ra_L^{1/4} \quad 10^5 \lesssim Ra_L \lesssim 10^{10}$$

Surface film resistance [m²K/W]

$$R''_s = \frac{1}{h_c + h_r} = \frac{1}{h_s}$$

External environment: $R''_{se} = 0.04$

Internal, horizontal heat flow: $R''_{si} = 0.13$

Internal, vertical upwards heat flow: $R''_{si} = 0.10$

Internal, vertical downwards heat flow: $R''_{si} = 0.17$

SURFACE-TO-SURFACE HEAT TRANSFER

Air cavities

(print slides pp. 24 and 25 SEE-Radiation)

NETWORK ANALYSIS

Ventilation

Heat flow: $q = \dot{m} c \theta = \rho c \dot{v} \theta$

Conductance: $H_v = \rho c \dot{v} \simeq 1200 \dot{v}$

Simplification rules

Multiple conductances connected to different temperature

nodes: $H_{eq} = H_1 + H_2 + \dots + H_n$

$$\theta_{eq} = \frac{\theta_1 H_1 + \theta_2 H_2 + \dots + \theta_n H_n}{H_1 + H_2 + \dots + H_n}$$

Heat flow (q_0) with a conductance (H_0) connected to a temperature node (θ_0): $H_{eq} = H_0$

$$\theta_{eq} = \theta_0 + \frac{q_0}{H_0}$$

RADIATIVE HEAT TRANSFER

$$q_i = A_i \varepsilon^* h_r^* (T_i - T_j)$$

$$h_r^* = 4\sigma \bar{T}_{ij}^3$$

Enclosure with two surfaces

$$\varepsilon^* = \left(\frac{1 - \varepsilon_i}{\varepsilon_i} + \frac{1}{F_{ij}} + \frac{1 - \varepsilon_j}{\varepsilon_j} \frac{A_i}{A_j} \right)^{-1}$$

Enclosure with two parallel surfaces

$$\varepsilon^* = \left(\frac{1}{\varepsilon_i} + \frac{1}{\varepsilon_j} - 1 \right)^{-1}$$

SOLAR GEOMETRY

$$\alpha = \arcsin(\cos \phi \cos \delta \cos \omega + \sin \phi \sin \delta)$$

$$\varphi = \arccos\left(\frac{\sin \alpha \sin \phi - \sin \delta}{\cos \alpha \cos \phi}\right)$$

$$\delta = 0.4093 \sin\left(2\pi \frac{284 + J}{365}\right) \text{ [rad]}$$

$$\omega = 0.2618(h - 12) \text{ [rad]}$$

$$\vartheta = \arccos(\cos \alpha \cos \gamma \sin \beta + \sin \alpha \cos \beta)$$

ϑ - incidence angle of solar radiation on a β sloped surface (angle between the sloped surface the horizontal plane)

α - solar altitude

γ - azimuth difference between the sun (φ) and the surface (ψ)

ϕ - latitude

δ - solar declination

ω - angular solar hour

J - Julian day

SOLAR GAINS

g-value

Glazing-to-internal node conductance: H_{ji}

Glazing-to-external node conductance: H_{je}

$$\text{Simple glazing: } g = \tau + \alpha \frac{H_{ji}}{H_{ji} + H_{je}}$$

$$\text{Angular correction: } g(\vartheta) = F_w g_{\perp}$$

$$\text{Frame correction or glazing factor: } F_g = \frac{A_g}{A_w}$$

Obstruction factors: F_o , F_f and F_h

Solar gains

Transparent (no shading elements)

$$A_{sol} = F_w g_{\perp} F_g A_w$$

Opaque

$$A_{sol} = \alpha R''_{se} U A_{op}$$

ENERGY BALANCE

$$q_{in} + q_{int} = q_{out} \text{ [W]}$$

Steady-state

Steady-state:

$$Q_{in} + Q_{int} = Q_{out} \text{ [J or Wh]}$$

Auxiliary systems (thermostatic):

$$Q_{aux} = Q_{out} - (Q_{in} + Q_{int}) \text{ [J or Wh]}$$

Quasi-steady-state

Heating needs:

$$Q_{nd,H} = Q_{ht} - \eta_g Q_{gn} \text{ [J or Wh]}$$

Cooling needs:

$$Q_{nd,C} = (1 - \eta_g) Q_{gn} \text{ [J or Wh]}$$

Utilisation gain factor:

$$\gamma = \frac{Q_{gn}}{Q_{ht}}$$

$$\eta_g = \frac{1 - \gamma^a}{1 - \gamma^{a+1}}$$

$$\text{For } \gamma = 0, \eta_g = \frac{a}{a+1}$$

$$\text{For } \gamma < 0, \eta_g = 1/\gamma$$

Heat transfer:

$$Q_{ht} = (H_{tr} + H_{ve})L(\theta_{set} - \bar{\theta}_{ext}) = (H_{tr} + H_{ve})GD_{\theta_{set}} \times 24$$

$$H_{tr} = H_d + H_g + H_u + H_a$$

$$H_x = b_{tr,x}(\sum A_i U_i + \sum B_i \Psi_i)$$

$$H_{ve} = \rho_a c_a \sum_k b_{ve,k} \phi_{ve,k}$$

$$\text{Adjust factor: } b_{tr} = \frac{\theta_i - \theta_u}{\theta_i - \theta_e}$$

Heat gains: $Q_{gn} = Q_{sol} + Q_{int} - \Delta Q_{sky}$

$$Q = \Phi L, \text{ with } \Phi \text{ the average power}$$

$$\Phi_{sol} = \sum_j \Phi_{sol,j}$$

$$\Phi_{sol,j} = G_j \sum_n F_{sh,n} A_{sol,n}$$

Effective collecting area for opaque elements

$$A_{sol,n} = \alpha_n R_{se} U_n A_n$$

Effective collecting area for transparent elements

$$A_{sol,n} = \bar{g}_{\theta,n} F_g A_{w,n}$$

$$\bar{g}_{\theta} = \bar{g}_{sh,\theta} F_{ms} + \bar{g}_{g,\theta} (1 - F_{ms})$$

$$\text{Shading elements: } \bar{g}_{sh,\theta} \simeq \bar{g}_{sh,\perp}$$

$$\text{Only glazing: } \bar{g}_{g,\theta} = F_w \bar{g}_{g,\perp}$$

$$\text{Alternative formulation for solar gains: } Q_{sol} = \sum_j E_j \left(\sum_n F_{sh,n} A_{sol,n} \right)_j$$

$$\text{Sky radiative exchanges: } \Delta Q_{sky} = h_r F_{sky} \Delta \theta_{sky} A_s$$

PSICROMETRY

$$\text{Humidity ratio: } W = \frac{m_v}{m_a}$$

$$\text{Relative humidity: } \phi = \frac{x_v}{x_s}$$

$$\text{Degree of saturation: } \mu = \frac{W}{W_s}$$

Enthalpy of moist air: $h = h_a + W h_v$ [J/kg_a]

$$h = c_{pa}\theta + W(h_{fg} + c_{pv}\theta) \simeq 1.0\theta + W(2500 + 1.86\theta) \text{ [kJ/kg}_a\text{]}$$

Pressure: $P = p_a + p_v$

$$\phi = \frac{p_v}{p_s} = \frac{\rho_v}{\rho_s}$$

$$\phi = \frac{W}{0.62198} \frac{p_a}{p_s}$$

$$W = 0.62198 \frac{p_v}{1 - p_v}$$

Heating moist air or cooling with no condensation

Energy balance: $\dot{m}_a h_1 + q = \dot{m}_a h_2$

Water mass flow balance: $\dot{m}_a W_1 = \dot{m}_a W_2$

(Note: for cooling q is negative.)

Cooling and dehumidifying of moist air

Energy balance: $\dot{m}_a h_1 + q = \dot{m}_a h_2 + \dot{m}_w h_w$

Water mass flow balance: $\dot{m}_a W_1 = \dot{m}_a W_2 + \dot{m}_w$

(Note: for cooling q is negative.)

Heating and humidifying moist air

Energy balance: $\dot{m}_a h_1 + q + \dot{m}_w h_w = \dot{m}_a h_2$

Water mass flow balance: $\dot{m}_a W_1 + \dot{m}_w = \dot{m}_a W_2$

Adiabatic humidification of moist air

Energy balance: $\dot{m}_a h_1 + \dot{m}_w h_w = \dot{m}_a h_2$

Water mass flow balance: $\dot{m}_a W_1 + \dot{m}_w = \dot{m}_a W_2$

Adiabatic mixing of two streams of moist air

Energy balance: $\dot{m}_{a1} h_1 + \dot{m}_{a2} h_2 = \dot{m}_{a3} h_3$

Water mass flow balance: $\dot{m}_{a1} W_1 + \dot{m}_{a2} W_2 = \dot{m}_{a3} W_3$

Interstitial condensation

Mass flow rate of vapour: $\dot{m}_v'' = \frac{p_{v1} - p_{v2}}{R_v} = \frac{p_{v1} - p_{v2}}{r_v L}$

Transient

$$V \frac{dC_i}{dt} = C_o \dot{V} + P - C_i(t) \dot{V}$$

$$C_i(t) = C_e + \frac{P}{\dot{V}} \left[1 - \exp\left(-\frac{\dot{V}}{V} t\right) \right]$$

LOAD CALCULATION

Winter conditions

$T_o < T_i$

$$\rho_a c_{pa} V_{room} \frac{dT_i}{dt} = q_{heat,s} - q_{vent} - \sum_n A_n U_n (T_i - T_o)$$

$$q_{vent} = \rho_a c_{pa} \dot{V} (T_i - T_o)$$

Summer conditions

$T_o > T_i$

Sensible load

$$\rho_a c_{pa} V_{room} \frac{dT_i}{dt} = q_{cool,s} + q_{int} + q_{sol} + q_{vent} + \sum_n A_n U_n (T_o - T_i)$$

$$q_{vent} = \rho_a c_{pa} \dot{V} (T_o - T_i)$$

Latent load

$$\dot{m}_a W_i = \dot{m}_a W_e + \dot{m}_w$$

If $W_i > W_{max}$, $q_{cool,l} = \dot{m}_a (W_i - W_{max}) h_{fg}$

THERMAL COMFORT

Operative temperature: $T_{op} = a T_a + (1 - a) T_r$

$$a = 0.5 + 0.25 v_a$$

VENTILATION

Steady-state

$$\dot{V} = \frac{P}{C_i - C_o}$$

(Note that C_i and C_e are expressed in ppm $\times 10^{-6}$)

$$\dot{V} = \frac{P}{W_i - W_o}$$