CFTC Centro de Física Teórica e Computacional

Ciências ULisboa

Percolation theory

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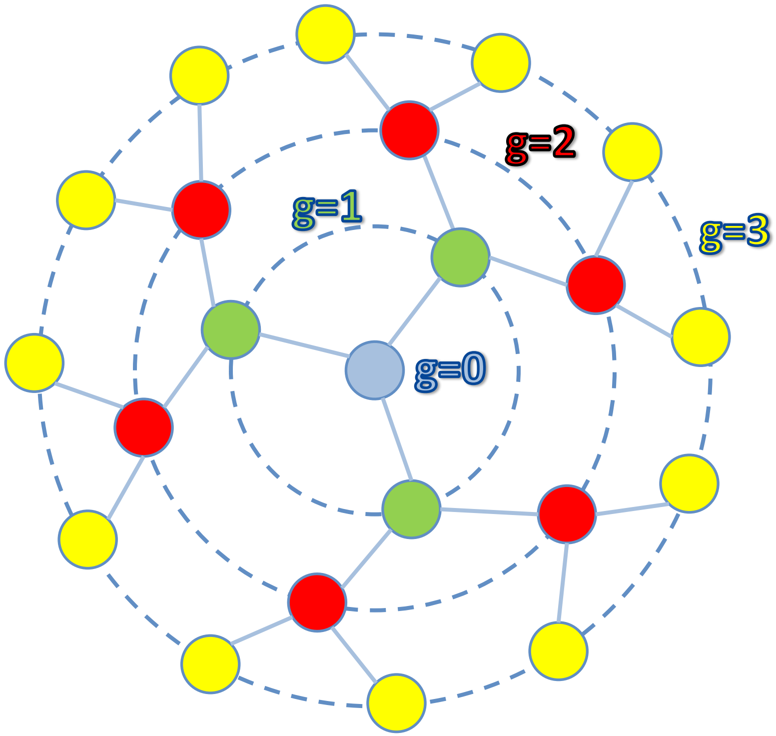
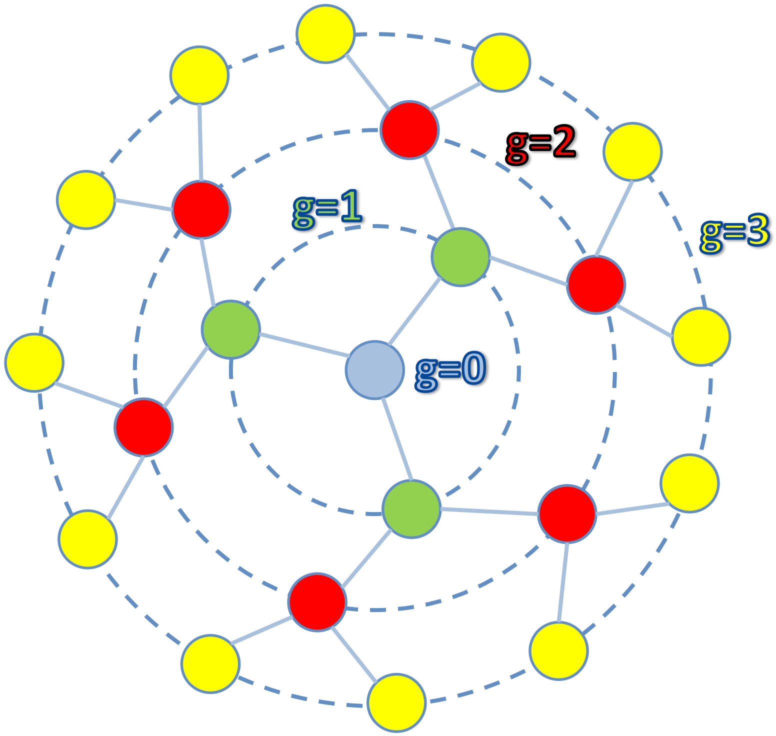
Site percolation on the Bethe lattice

mean-field (e.g. z=3)



Site percolation on the Bethe lattice

mean-field



$$p(z-1) \geq 1$$

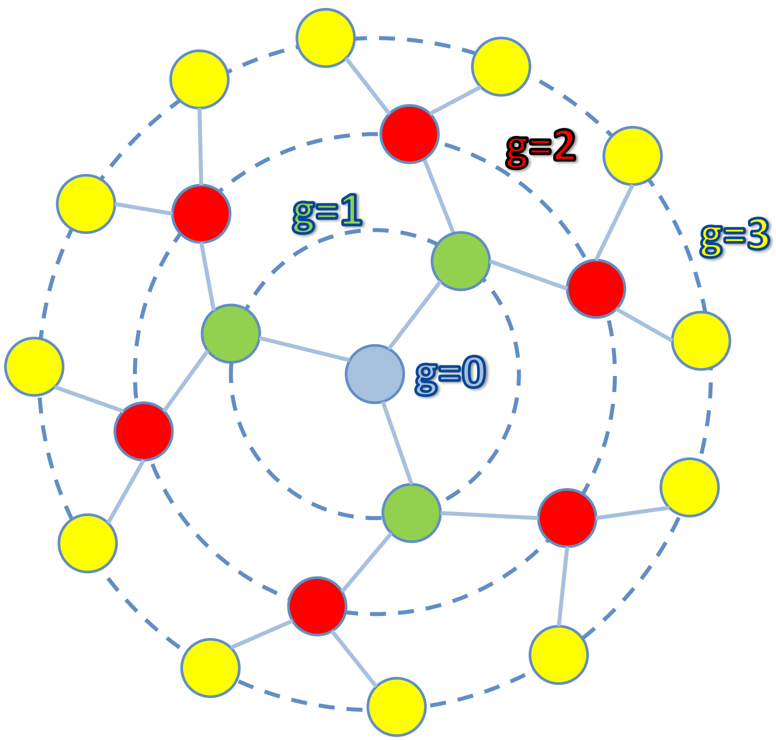
$$p_c = \frac{1}{z-1}$$

$z = 2$: $p_c = 1$ (1D)

$z > 2$: $p_c < 1$

Site percolation on the Bethe lattice

z=3



Q prob. site not connected to infinity through one branch.

$$Q = 1 - p + pQ^2$$

neighbor empty

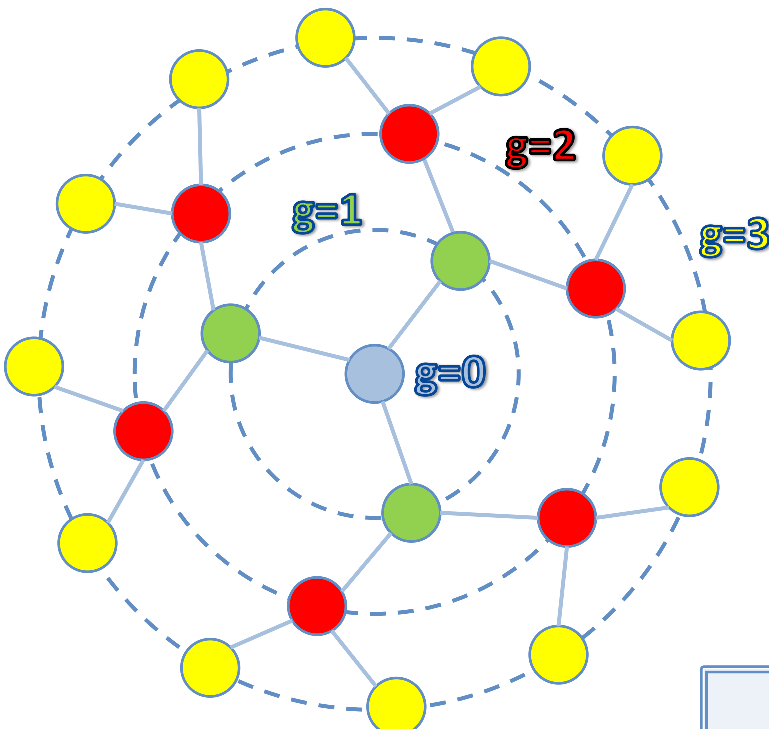
neighbor occupied

$$Q = 1$$

$$Q = \frac{1-p}{p}$$

Site percolation on the Bethe lattice

z=3 (order parameter)



P_∞ prob. site connected to infinity

$$p - P_\infty = pQ^3$$

Occupied but not connected to infinity

$$P_\infty = p(1 - Q^3)$$

$p < p_c$

$$P_\infty = 0$$

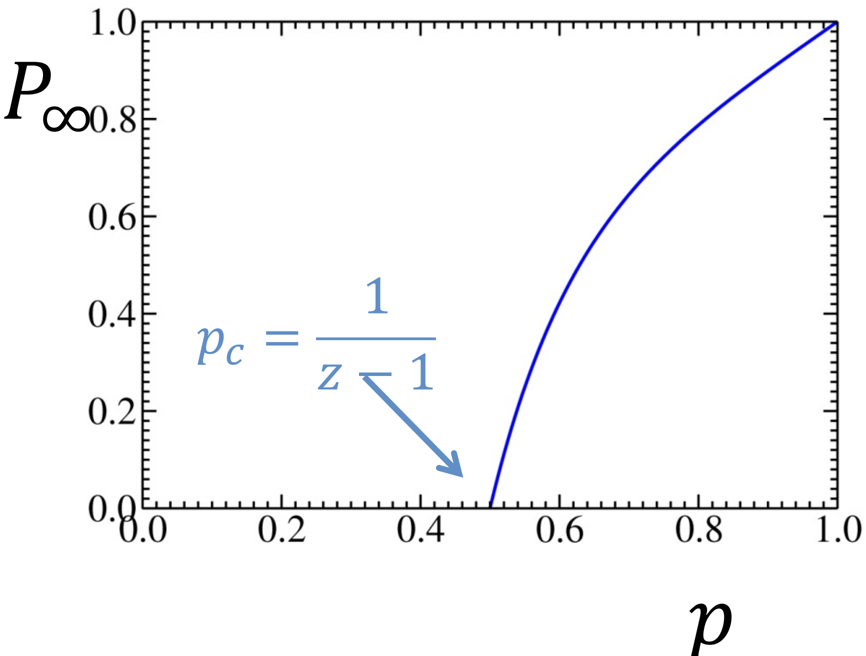
$p > p_c$

$$P_\infty = p\left[1 - \left(\frac{1-p}{p}\right)^3\right]$$

Site percolation on the Bethe lattice

z=3 (order parameter)

P_∞ prob. site connected to infinity



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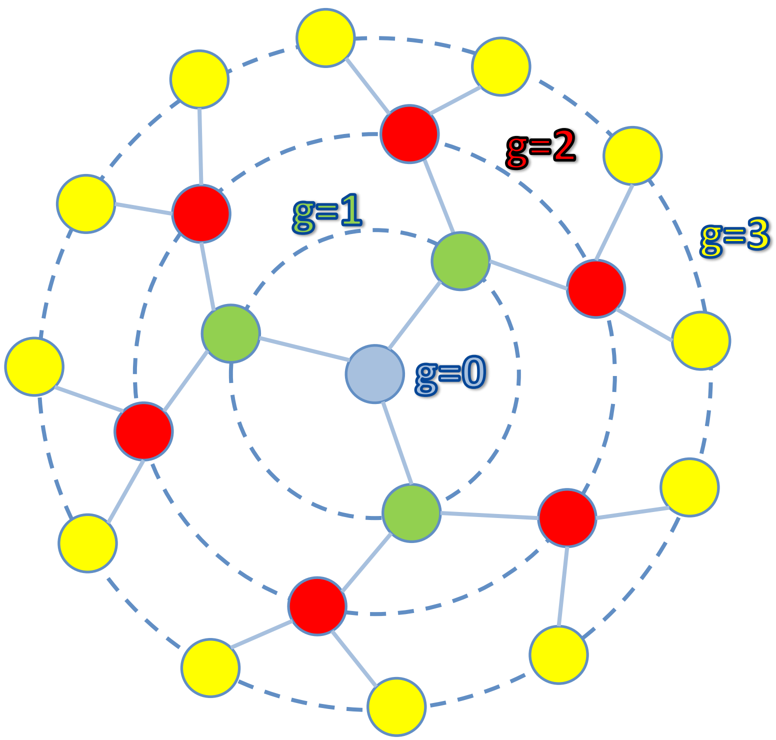
$$P_\infty \sim (p - p_c)^1$$

Site percolation on the Bethe lattice

z=3 (fluctuations)

T contribution to the mean cluster size for one branch.



$$T = (1-p)0 + p(1+2T)$$

neighbor empty

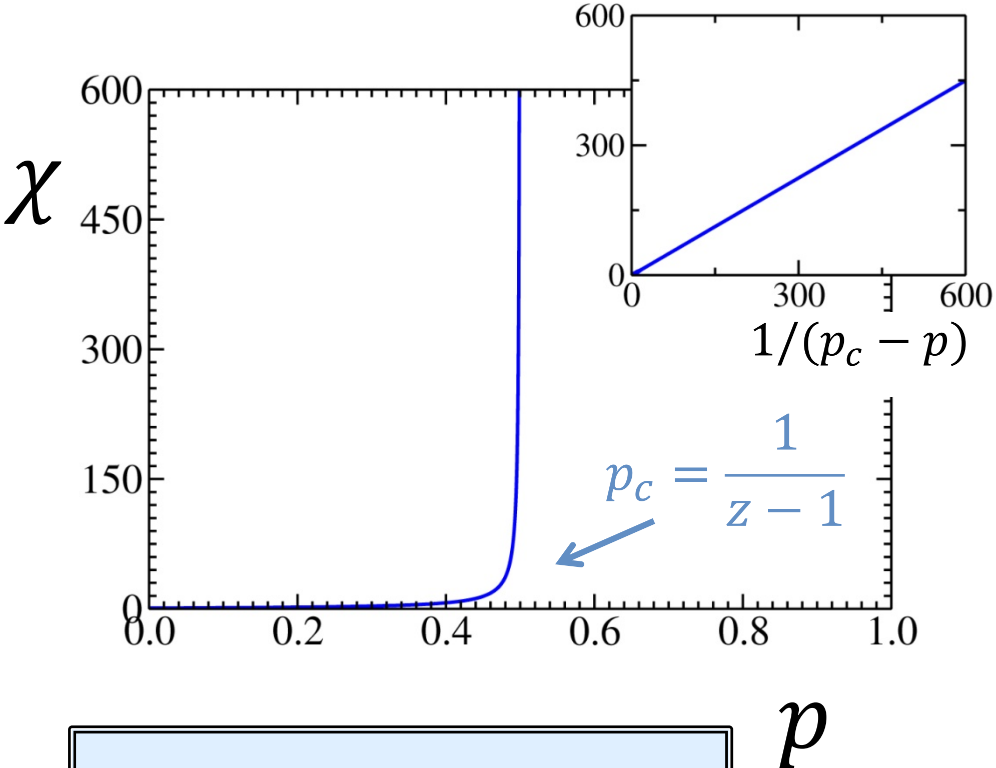
neighbor occupied

$$T = \frac{p}{1-2p}$$

Diverges for $p = p_c = \frac{1}{2}$

Site percolation on the Bethe lattice

z=3 (fluctuations)



$$\chi \sim (p_c - p)^{-1}$$

χ mean cluster size (fluctuations)

Mean cluster size for the occupied origin

$$\chi = (1 + 3T)$$

$$T = \frac{p}{1-2p}$$

$p < p_c$

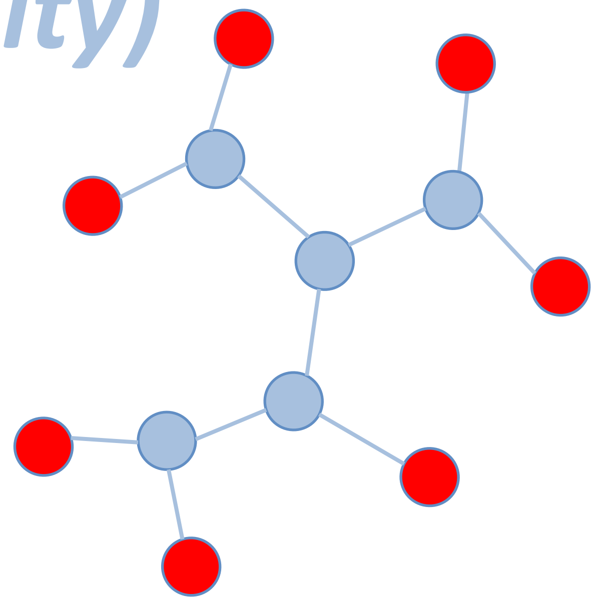
$$\chi = \frac{2(1+p)}{\frac{1}{2} - p}$$

Site percolation on the Bethe lattice

z=3 (cluster number density)

$$n(s,p) = \sum_{t=1}^{\infty} g(s,t)(1-p)^t p^s$$

degeneracy factor



size s, perimeter t

$$t = 2 + s(z-2)$$

$$n(s,p) = g[s, 2+s(z-2)](1-p)^{2+s(z-2)}p^s$$

$$n(s,p) = g(s, 2+s)(1-p)^{2+s}p^s$$

$$z = 3$$

Site percolation on the Bethe lattice

z=3 (characteristic cluster size)

$$\frac{n(s,p)}{n(s,p_c)} = \left[\frac{1-p}{1-p_c}\right]^2 \left[\frac{(1-p)p}{(1-p_c)p_c}\right]^s$$

$$= \left[\frac{1-p}{1-p_c}\right]^2 \exp\left(s\, ln\left[\frac{(1-p)p}{(1-p_c)p_c}\right] \right)$$

$$= \left[\frac{1-p}{1-p_c}\right]^2 \exp(-s/s_\xi)$$

$$s_\xi = -\frac{1}{\ln\left[\frac{(1-p)p}{(1-p_c)p_c}\right]}$$

$z = 3$

$$s_\xi \sim (p-p_c)^{-2}$$

Site percolation on the Bethe lattice

mean-field exponents

$$P_\infty \sim (p - p_c)^1$$

$$P_\infty \sim (p - p_c)^\beta$$

$$\beta = 1$$

$$\chi \sim (p_c - p)^{-1}$$

$$\chi \sim (p_c - p)^{-\gamma}$$

$$\gamma = 1$$

$$s_\xi \sim (p - p_c)^{-2}$$

$$s_\xi \sim (p - p_c)^{-\frac{1}{\sigma}}$$

$$\sigma = \frac{1}{2}$$