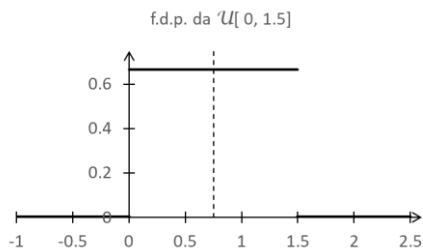


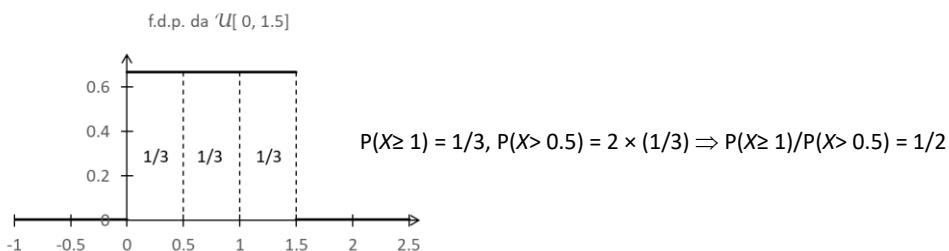
Exercícios 28 a 31 – Resoluções

28. X – v.a. que representa o tempo de espera (em horas) de um utente do consultório, escolhido ao acaso. Sabe-se que $X \sim \mathcal{U}[0, 1.5]$. A f.d.p. de X é, então: $f(x) = 1/(1.5 - 0) = 2/3, 0 \leq x \leq 1.5$

a) $E(X) = (0 + 1.5)/2 = 0.75$ horas (a f.d.p. de X é simétrica em torno de $x = 0.75$)

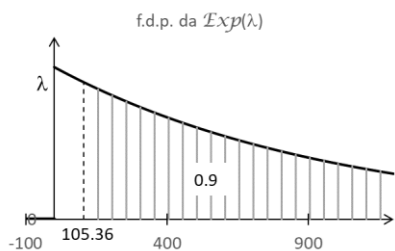


b) $P(X \geq 1 | X > 0.5) = P(X \geq 1 \wedge X > 0.5) / P(X > 0.5) = P(X \geq 1) / P(X > 0.5) = (0.5 \times 2/3) / (1 \times 2/3) = 0.5$

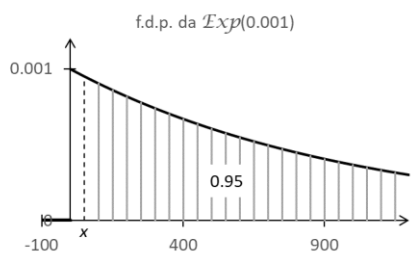


29. $X \sim \text{Exp}(\lambda) \Leftrightarrow F(x) = P(X \leq x) = 1 - e^{-\lambda x}, x \geq 0 \Leftrightarrow S(x) = P(X > x) = e^{-\lambda x}, x \geq 0$

a)



$$S(105.36) = 0.9 \Leftrightarrow e^{-105.36\lambda} = 0.9 \Leftrightarrow \lambda = -(\ln 0.9)/105.36 = 0.001$$



$$x: S(x) = 0.95 \Leftrightarrow e^{-\lambda x} = 0.95 \Leftrightarrow x = -(\ln 0.95)/\lambda = -1000 \ln 0.95 \approx 51.29 \text{ horas}$$

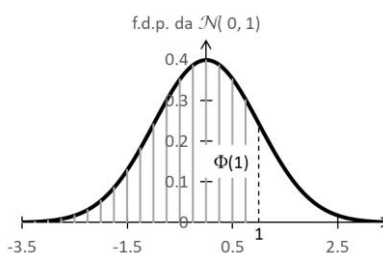
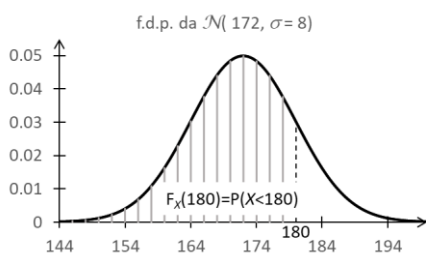
b) $P(X \geq 20 | X > 5) = P(X \geq 20 \wedge X > 5) / P(X > 5) = P(X \geq 20) / P(X > 5) = S(20) / S(5) = e^{-20\lambda} / e^{-5\lambda} = e^{-15\lambda} = S(15) = P(X > 15)$

A distribuição exponencial não tem “memória”:

$$X \sim \text{Exp}(\lambda) \Rightarrow P(X \geq t | X > s) = P(X \geq t - s)$$

30. Seja X – v.a. que representa a altura, em cm, de um homem escolhido ao acaso na região em questão; $X \sim \mathcal{N}(172, \sigma = 8)$

a) $P(X < 180) = \Phi((180 - 172)/8) = \Phi(1) = 0.84134$



b) $P(170 < X < 180) = \Phi((180 - 172)/8) - \Phi((170 - 172)/8) = \Phi(1) - \Phi(-0.25) = \Phi(1) + \Phi(0.25) - 1 =$
 $= 0.84134 + 0.59871 - 1 \approx 0.44 \Rightarrow 44\% \text{ dos homens}$

c) $x: P(X > x) = 0.99 \Leftrightarrow P(X \leq x) = 0.01 \Leftrightarrow x = \chi_{0.01} = 172 + 8 \times z_{0.01} = 172 - 8 \times z_{0.99} = 172 - 8 \times 2.326 =$
 153.4 cm

31. Seja T – v.a. que representa o tempo, em min, que o técnico do CI gasta na resolução de um problema;
 $T \sim \mathcal{N}(16, \sigma = 4)$

a) $P(T < 8) = \Phi((8 - 16)/4) = \Phi(-2) = 1 - \Phi(2) = 1 - 0.97725 = 0.02275$

b) $P(T \leq 20 | T > 12) = P(12 < T \leq 20) / P(T > 12) = [\Phi((20 - 16)/4) - \Phi((12 - 16)/4)] / [1 - \Phi((12 - 16)/4)] =$
 $= [\Phi(1) - \Phi(-1)] / [1 - \Phi(-1)] = (2\Phi(1) - 1) / \Phi(1) = 2 - 1/\Phi(1) =_{(\Phi(1)=0.84134)} 0.81142$

c) $t: P(T \leq t) = 0.95 \Leftrightarrow t = \chi_{0.95} = 16 + 4 \times z_{0.95} = 16 + 4 \times 1.645 = 22.58 \text{ min}$