Cosmologia Física

Homework 1

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Exercise 1: Cosmological Principle

1.1)
a) What is the cosmological principle?
b) Which observations support this principle?

1.2)
a) What are comoving coordinates?
b) In a general 3-d space, we can consider that the scale factor is a 3x3 matrix A(t) that is applied to a comoving position vector $\vec{\chi}$ to obtain the physical position vector $\vec{r}$: $\vec{r}(t) = A(t)\vec{\chi}$. Say (in words, no calculations are needed) what properties the matrix A(t) can have for the Universe to be isotropic.
Hint: examples of matrix properties would be: symmetric, anti-symmetric, traceless, diagonal, etc.

Exercise 2: Expansion

2.1)
a) Does the Robertson-Walker metric impose an expanding Universe?
b) Is the Milky Way a perturbation to the RW metric?
c) Can the Milky Way be described by a perturbed RW metric?
d) Does the Milky Way expand? Why (or why not)?
e) How does the comoving size of the Milky Way evolve as the Universe expands?

2.2) The detected frequency of an electromagnetic wave emitted by a cosmological source is different from the emitted frequency.
a) Derive the relation between this so-called cosmological redshift and the scale factor of the Universe at emission time.

Exercise 3: Olbers' Paradox

3.1) Consider a sample of N galaxies with identical luminosity and constant comoving number density n_0, contained in a comoving volume V_C, where the comoving volume element is given by $dV_C = D_A^2(1+z)^2/H(z)\, d\Omega dz$.

a) Show that the number of galaxies per redshift and solid angle is given by,

$$\frac{dN}{d\Omega dz} = n_0 D_A^2 (1+z)^2 \frac{c}{H(z)} \,.$$

b) Inserting the luminosity distance, show that the flux function is given by

$$\frac{dN}{d\Omega dF} = -\frac{n_0}{2}\left(\frac{L}{4\pi}\right)^{3/2} F^{-5/2} \frac{c}{H(z)} \frac{1}{(1+z)^2} \frac{dz}{dD_L} \,,$$

where F is the flux.

c) Show that in the local universe ($z \ll 1$) this expression reduces to

$$\frac{dN}{d\Omega dF} = -\frac{n_0}{2}\left(\frac{L}{4\pi}\right)^{3/2} F^{-5/2} \,.$$

Hint: Consider the local Hubble function and a flat model.

Note that this expression is called the Euclidean limit of the flux function, while the remaining factor is a cosmological correction:

$$\frac{c}{H(z)} \frac{1}{(1+z)^2} \frac{dz}{dD_L} \,.$$

d) Using the Euclidean flux function compute the total flux in the volume, including sources with fluxes down to a minimum F_0, and find Olbers' paradox.

e) Consider now the Milne Universe, which is an open (i.e. with negative curvature) cosmological model where the Hubble function is $H(z) = H_0\,(1+z)$. Show that in this case the flux cosmological correction is given by $(1+z)^{-4}$.

Hint: Remember to use $D_L = (1+z)\,D_M$ in Milne's Universe.

f) Compute the total flux in Milne's Universe and solve Olbers' paradox!

Hint: It will be useful to write the cosmological correction as function of flux.

Exercise 4: Distances and Volume

4.1) Due to the existence of curvature and expansion in a cosmological spacetime metric, it is possible to consider different definitions of distances. Which of the two factors, curvature or expansion, is responsible for the distinction between:

a) Comoving distance and angular diameter comoving distance?

b) Angular diameter distance and angular diameter comoving distance?

c) Comoving distance and luminosity distance?

4.2)

a) Explain (in words) that the comoving volume element may be written as

$$dV_C = D_A^2 (1+z)^2 \, d\Omega \, dD_C \,,$$

where $d\Omega$ is the solid angle.

b) Derive similar expressions for

$$dV_C/dt \,, dV_C/da \,, dV_C/dz$$