Cosmologia Física

Homework 2

Ismael Tereno (tereno@fc.ul.pt)

Exercise 1: Friedmann equations

1.1) Consider the Friedmann equation,

$$H^2(a) = H_0^2 \left(\Omega_r (1+z)^4 + \Omega_\mathrm{m} (1+z)^3 + \Omega_\mathrm{K} (1+z)^2 + \Omega_\Lambda \right).$$

This relation implies that the sum of all the densities today is 1 (which sometimes, when the density values are represented in a pie chart, it is also written as 100%).

a) Does this mean that Ω_m must be smaller than 1?

b) If not, what type of curvature (positive, negative or flat) a universe with $(\Omega_\mathrm{m} > 1, \Omega_\Lambda = 0)$ must have?

c) Do all universes with $\Omega_\mathrm{m} > 1$ necessarily have that same type of curvature found in b)?

1.2) Consider a 2-fluid flat universe with dark matter and cosmological constant.

a) Compute the redshift of the transition to the dark energy epoch.

b) Derive the expression of the line of no acceleration in a $(\Omega_m, \Omega_\Lambda)$ plane.

Exercise 2: Universe with 5 dimensions

2.1) Consider a 5d spacetime with one time coordinate and four spatial coordinates.

a) Write the continuity equation for this spacetime.

Hint: Use only simple arguments to write this equation, no formal derivation from the metric is needed.

b) Argue what must be the radiation's density evolution $\rho_r(a)$ and equation-of-state in this spacetime.

c) Consider a cosmological fluid with a constant equation-of-state of w=0.3. Is this a tachyonic or a non-tachyonic fluid in this spacetime? And in the standard 4d spacetime?

Exercise 3: Surprising results

3.1) Consider two identical galaxy clusters (meaning they have similar intrinsic sizes and luminosities) in the Einstein-de Sitter universe. The clusters are at different redshifts, z_1 and z_2, with $z_1 < z_2$

a) Show that the comoving distance $D_c(z)$ in this universe is

$$D_c = 2\frac{c}{H_0}\left[1 - (1+z)^{-1/2}\right].$$

b) Compute which of the 2 clusters appears to be the brightest to the observer.

c) Compute which of the 2 clusters appears to be the largest to the observer.

d) Which one of distances could be used as a time indicator in the evolution of the universe?

3.2) Consider the concordance universe of the ΛCDM 3-fluid model, i.e. a flat cosmology with $h = 0.7$, $\Omega_m = 0.3$, $\Omega_\Lambda = 0.7$ and $\Omega_r = 8 \times 10^{-5}$.

a) Even though the redshift range extends from 0 to ∞, the redshift at half of the age of the universe is a surprisingly low value. Compute it for the concordance universe
Hint: solve the integral numerically, for example using Wolfram Alpha online.

3.3)

a) In terms of the dynamics of the expansion, think about what is the physical parameter that defines the moment when a universe reaches $a = 1$. Is it a fixed moment in time? Is it a fixed size of the universe? Is it some other physical parameter (and in this case say what parameter is it)?

b) Show that a decelerating universe is younger than the Hubble time.

c) The result in b) may seem unexpected because it leads to the conclusion that a slower universe (the decelerating one) is younger than a faster growing accelerated universe. So the fact that the slower expansion is able to reach the current state $a = 1$ sooner than the faster universe may seem a paradox. Explain that this result is not a paradox.
Hint: Consider two universes with two different power law expansions $a \propto t^n$ and compare the evolution of their Hubble functions, H(a). From this comparison and taken into account the answer to a) it should become clear that there is no paradox.

3.4) The redshift of a source may vary with time as the universe expands (a galaxy with redshift z observed today will have a different z tomorrow). Indeed, the redshift observed today (at $t = t_0$) is

$$z_s(t_0) = a(t_0)/a(t_s) - 1,$$

while the redshift of the same source observed in a future time $t_0 + \Delta t_0$ (corresponding to an emission made at $t_s + \Delta t_s$) is

$$z_s(t_0 + \Delta t_0) = a(t_0 + \Delta t_0)/a(t_s + \Delta t_s) - 1.$$

The difference between the two is known as the *redshift drift* $\Delta z_s = z_s(t_0 + \Delta t_0) - z_s(t_0)$.

a) There is a certain expansion $a(t)$ that does not create a redshift drift. Which type of expansion is this?

b) Taylor expanding the scale factor to first order in time, both around t_0 and around t_s, show that the redshift drift can be computed from the Hubble function as

$$\frac{\Delta z_s}{1 + z_s} = H_0 \Delta t_0 \left[1 - \frac{H(z_s)}{(1 + z_s) H_0} \right].$$

c) The redshift drift is a very slow effect. Consider a quasar at $z = 6$ in the concordance model. If we observe the quasar 14 years from now, what will its redshift be?

d) The redshift drift of the quasar in c) is negative. However, for lower redshift sources in the concordance model, the redshift increases with time. Compute the source redshift threshold below which the redshift drift is positive.