UNIVERSO PRIMITIVO

Mestrado em Física Astronomia 2024-2025

Exercise Sheet 3

1. As derived in class, the entropy of the primordial fluid can be defined as $S = V\,(\rho + P)/T$. Use the continuity equation to show that this definition leads to the conservation of entropy [**Hint**: prove that the time derivative of S is zero].

2. According to the standard model of particle physics, photons, electrons, neutrinos and their anti-particles are the main relativistic thermal species in the temperature plateau $1 \leq T/\text{MeV} \leq 30$.

   2.1. Compute the number and energy densities of primordial photons at $T = 1\text{MeV}$, in cgs units. Compare your findings with present CMB observations of these quantities: $n_{\gamma,0} \simeq 410\ \text{cm}^{-3}$; $\rho_{\gamma,0} \simeq 4 \times 10^{-34}\ \text{gcm}^{-3}$.

   2.2. Compute the effective number of relativistic species in energy and entropy (g_* and g_{*S}) during the temperature period from 1 to 30 MeV. What would be the value of g_* and g_{*S} in a particle physics model with 4 families of massless neutrinos, all with just one helicity state?

   2.3. Derive exact expressions for the plasma temperature, energy density and entropy density, $s = S/V$, as a function of redshift, z, assuming that $T = 1\ \text{MeV}$ at $z = 6 \times 10^9$.

3. Consider the Friedmann equation written as in exercise 3.2 of problems sheet 1.

   3.1. Explain why the energy density of relativistic particles (the radiation term in this equation) should be modified to:

   $$\Delta_R(a)\ \Omega_{r0}\left(\frac{a_0}{a}\right)^4, \qquad \text{where:} \qquad \Delta_R(a) = \frac{g_*(a)}{g_*(a_0)}\left(\frac{g_{*S}(a_0)}{g_{*S}(a)}\right)^{4/3}.$$

   3.2. Compute the age of the universe by the end of the Big Bang Nucleosynthesis, $T = 0.1$ MeV, assuming the following approximation for g_* and g_{*S}:

   $$g_* \simeq g_{*S} \simeq \begin{cases} 100 & T > 300\ \text{MeV} \\ 10 & 300\ \text{MeV} > T > 1\ \text{MeV} \\ 3 & T <\ \text{MeV} \end{cases}$$

   Use the present-day values $H_0 \simeq 1.44 \times 10^{-42}$ GeV, $\Omega_{r0} \simeq 9.2 \times 10^{-5}$. Consider that $T = 0.1\text{MeV}$ at $z = 4 \times 10^8$, for the normalisation of the temperature – z relation.

   3.3. Repeat the calculation now using the tabulated values of g_* and g_{*S} in Ref. astro-ph/1609.04979 (Table A1). Compare with your findings in 3.2.

4. Read sections 3.1 and 3.2 in Ref. astro-ph/1808.08968 where the authors discuss the effect of extra-degrees of freedom in the spectrum of gravitational waves (GW) from a network of cosmic strings. Explain by your own words their findings in Fig. 6 and say if the LISA space mission would be able to discriminate between the models they investigate.

5. Consider a particle species that remains relativistic after decoupling from the primordial fluid. Derive expressions for the scaling of that species' temperature and number density with redshift after decoupling. Consider now the case of a particle species that decouples from the fluid when it is non-relativistic. How the species' temperature and number density scale with redshift?
   [Hint: Assume that no particles are created or destroyed after decoupling. Study the scaling of momentum with the scale factor to prove that the shape of the distribution function of the species does not change after decoupling]