

Para a resistência

We begin with the resistor. If the current through a resistor R is $i = I_m \cos(\omega t + \phi)$, the voltage across it is given by Ohm's law as

$$v = iR = RI_m \cos(\omega t + \phi) \quad (9.29)$$

The phasor form of this voltage is

$$\mathbf{V} = RI_m \angle \phi \quad (9.30)$$

But the phasor representation of the current is $\mathbf{I} = I_m \angle \phi$. Hence,

$$\mathbf{V} = R\mathbf{I} \quad (9.31)$$

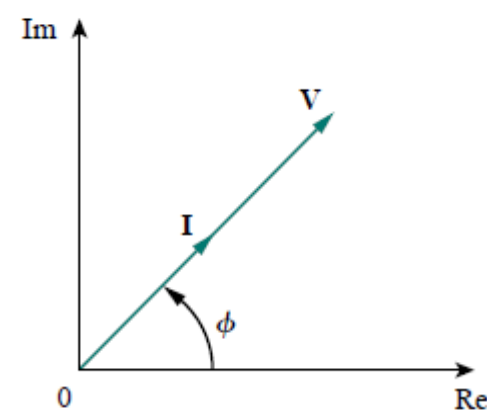
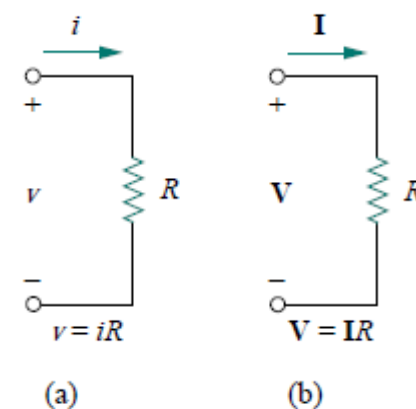


Figure 9.10 Phasor diagram for the resistor.

Para o indutor

For the inductor L , assume the current through it is $i = I_m \cos(\omega t + \phi)$. The voltage across the inductor is

$$v = L \frac{di}{dt} = -\omega L I_m \sin(\omega t + \phi) \quad (9.32)$$

Recall from Eq. (9.10) that $-\sin A = \cos(A + 90^\circ)$. We can write the voltage as

$$v = \omega L I_m \cos(\omega t + \phi + 90^\circ) \quad (9.33)$$

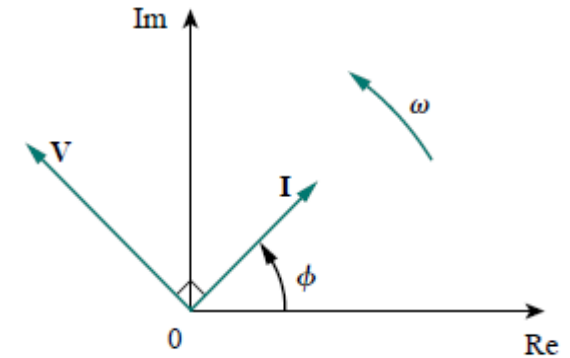
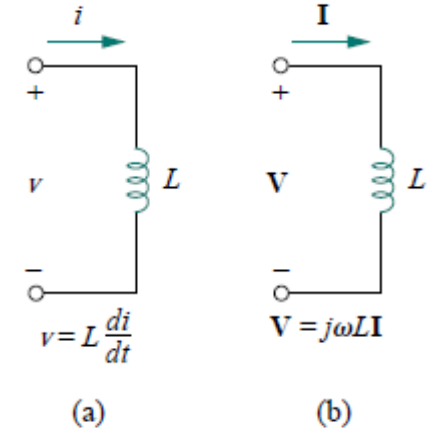
which transforms to the phasor

$$\mathbf{V} = \omega L I_m e^{j(\phi + 90^\circ)} = \omega L I_m e^{j\phi} e^{j90^\circ} = \omega L I_m \angle \phi e^{j90^\circ} \quad (9.34)$$

But $I_m \angle \phi = \mathbf{I}$, and from Eq. (9.19), $e^{j90^\circ} = j$. Thus,

$$\mathbf{V} = j\omega L \mathbf{I} \quad (9.35)$$

showing that the voltage has a magnitude of $\omega L I_m$ and a phase of $\phi + 90^\circ$.



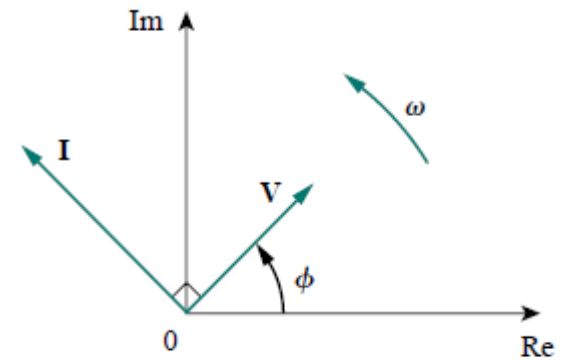
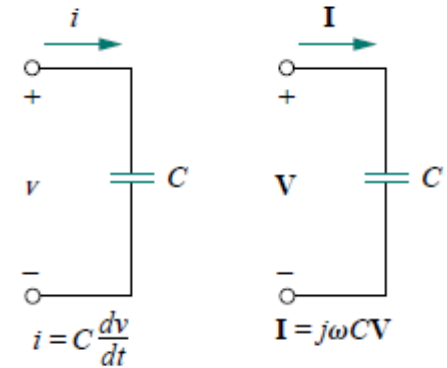
For the capacitor C , assume the voltage across it is $v = V_m \cos(\omega t + \phi)$. The current through the capacitor is

$$i = C \frac{dv}{dt} \quad (9.36)$$

By following the same steps as we took for the inductor or by applying Eq. (9.27) on Eq. (9.36), we obtain

$$\mathbf{I} = j\omega C \mathbf{V} \quad \Rightarrow \quad \mathbf{V} = \frac{\mathbf{I}}{j\omega C} \quad (9.37)$$

showing that the current and voltage are 90° out of phase. To be specific,



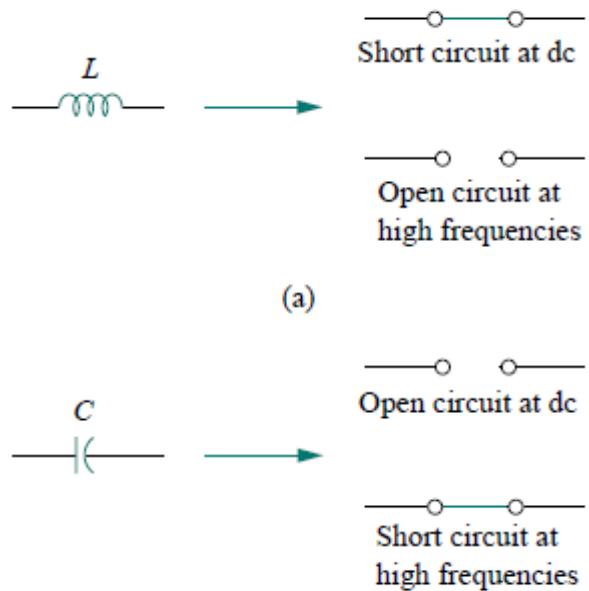
Element	Time domain	Frequency domain
R	$v = Ri$	$\mathbf{V} = R\mathbf{I}$
L	$v = L\frac{di}{dt}$	$\mathbf{V} = j\omega L\mathbf{I}$
C	$i = C\frac{dv}{dt}$	$\mathbf{V} = \frac{\mathbf{I}}{j\omega C}$

Noção de impedância

$$\mathbf{V} = R\mathbf{I}, \quad \mathbf{V} = j\omega L\mathbf{I}, \quad \mathbf{V} = \frac{\mathbf{I}}{j\omega C}$$

$$\frac{\mathbf{V}}{\mathbf{I}} = R, \quad \frac{\mathbf{V}}{\mathbf{I}} = j\omega L, \quad \frac{\mathbf{V}}{\mathbf{I}} = \frac{1}{j\omega C}$$

$$\mathbf{Z} = \frac{\mathbf{V}}{\mathbf{I}} \quad \text{or} \quad \mathbf{V} = \mathbf{Z}\mathbf{I}$$



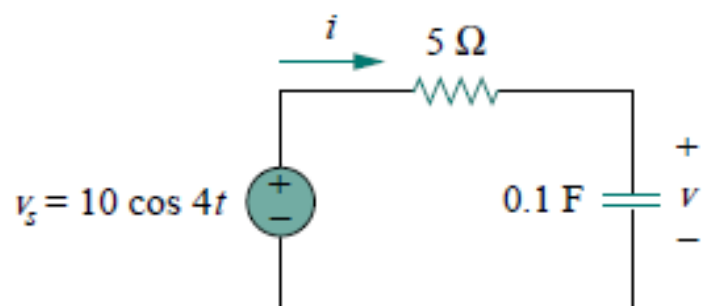
$$\mathbf{Z} = R + jX$$

A parte real de $\mathbf{Z}$ é a resistência

A parte imaginária de $\mathbf{Z}$ é a reatância

$$|\mathbf{Z}| = \sqrt{R^2 + X^2}, \quad \theta = \tan^{-1} \frac{X}{R}$$

$$R = |\mathbf{Z}| \cos \theta, \quad X = |\mathbf{Z}| \sin \theta$$



From the voltage source $10 \cos 4t$, $\omega = 4$,

$$\mathbf{V}_s = 10 \angle 0^\circ \text{ V}$$

The impedance is

$$\mathbf{Z} = 5 + \frac{1}{j\omega C} = 5 + \frac{1}{j4 \times 0.1} = 5 - j2.5 \Omega$$

Hence the current

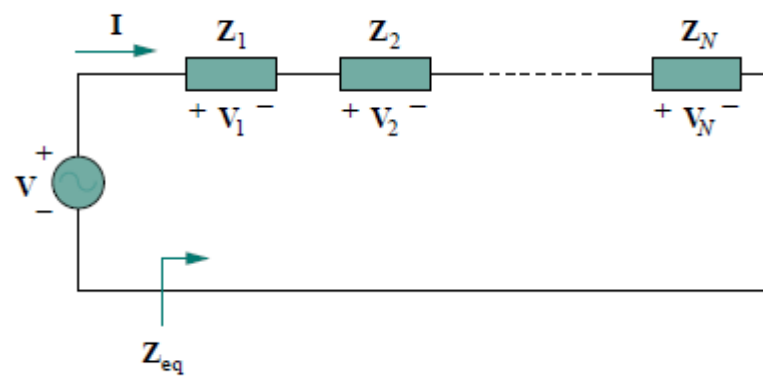
$$\begin{aligned} \mathbf{I} &= \frac{\mathbf{V}_s}{\mathbf{Z}} = \frac{10 \angle 0^\circ}{5 - j2.5} = \frac{10(5 + j2.5)}{5^2 + 2.5^2} \\ &= 1.6 + j0.8 = 1.789 \angle 26.57^\circ \text{ A} \end{aligned}$$

$$\begin{aligned} \mathbf{V} &= \mathbf{I} \mathbf{Z}_C = \frac{\mathbf{I}}{j\omega C} = \frac{1.789 \angle 26.57^\circ}{j4 \times 0.1} \\ &= \frac{1.789 \angle 26.57^\circ}{0.4 \angle 90^\circ} = 4.47 \angle -63.43^\circ \text{ V} \end{aligned}$$

Converting $\mathbf{I}$ and $\mathbf{V}$ in Eqs. (9.9.1) and (9.9.2) to the time domain,

$$i(t) = 1.789 \cos(4t + 26.57^\circ) \text{ A}$$

$$v(t) = 4.47 \cos(4t - 63.43^\circ) \text{ V}$$

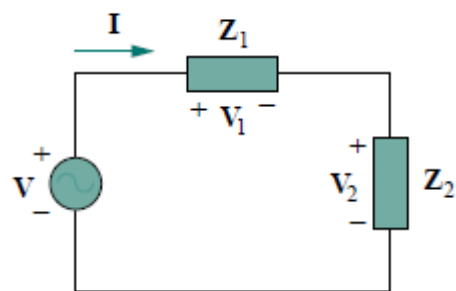


$$V = V_1 + V_2 + \cdots + V_N = I(Z_1 + Z_2 + \cdots + Z_N)$$

$$Z_{eq} = \frac{V}{I} = Z_1 + Z_2 + \cdots + Z_N$$

$$Z_{eq} = Z_1 + Z_2 + \cdots + Z_N$$

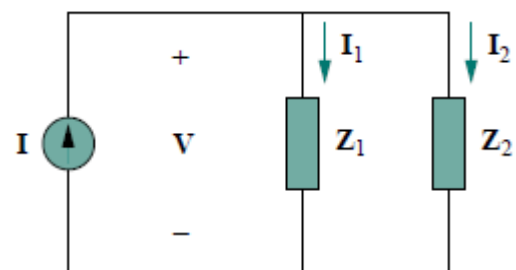
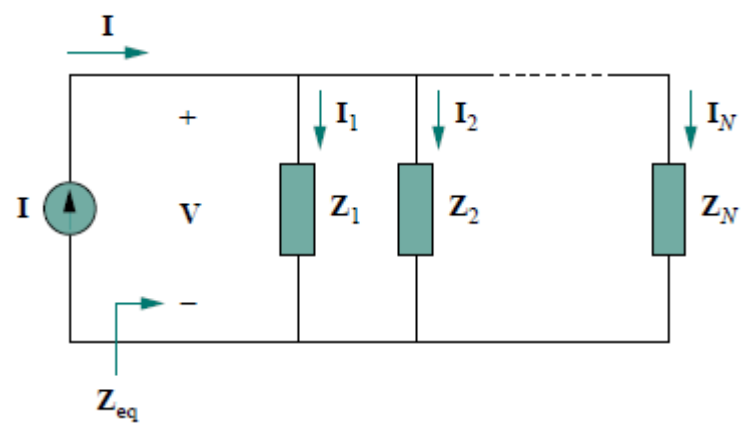
- as expressões deduzidas para as associações de resistência em série e paralelo podem ser generalizadas para as impedâncias.
- a Lei dos Nós e das Malhas mantêm a sua validade;
- as Leis dos Nós e das Malhas verificam-se vectorialmente!



$$\mathbf{I} = \frac{\mathbf{V}}{\mathbf{Z}_1 + \mathbf{Z}_2}$$

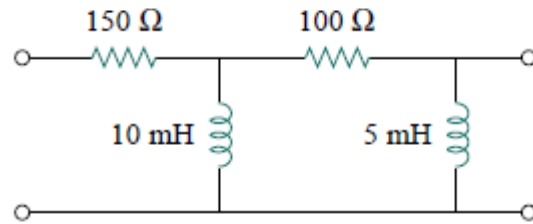
Since $\mathbf{V}_1 = \mathbf{Z}_1 \mathbf{I}$ and $\mathbf{V}_2 = \mathbf{Z}_2 \mathbf{I}$, then

$$\mathbf{V}_1 = \frac{\mathbf{Z}_1}{\mathbf{Z}_1 + \mathbf{Z}_2} \mathbf{V}, \quad \mathbf{V}_2 = \frac{\mathbf{Z}_2}{\mathbf{Z}_1 + \mathbf{Z}_2} \mathbf{V}$$

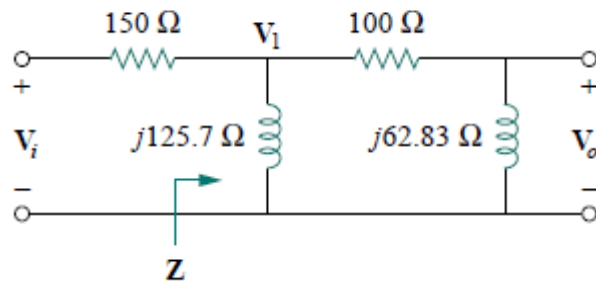


$$I_1 = \frac{Z_2}{Z_1 + Z_2} I, \quad I_2 = \frac{Z_1}{Z_1 + Z_2} I$$

Calcular a diferença de fase entre entrada e saída para 2 kHz



(a)



$$10 \text{ mH} \implies X_L = \omega L = 2\pi \times 2 \times 10^3 \times 10 \times 10^{-3} = 40\pi = 125.7 \Omega$$

$$5 \text{ mH} \implies X_L = \omega L = 2\pi \times 2 \times 10^3 \times 5 \times 10^{-3} = 20\pi = 62.83 \Omega$$

Consider the circuit in Fig. 9.35(b). The impedance Z is the parallel combination of $j125.7 \Omega$ and $100 + j62.83 \Omega$. Hence,

$$Z = j125.7 \parallel (100 + j62.83)$$

$$= \frac{j125.7(100 + j62.83)}{100 + j188.5} = 69.56 \angle 60.1^\circ \Omega \quad (9.14.1)$$

Using voltage division,

$$V_1 = \frac{Z}{Z + 150} V_i = \frac{69.56 \angle 60.1^\circ}{184.7 + j60.3} V_i$$

$$= 0.3582 \angle 42.02^\circ V_i \quad (9.14.2)$$

and

$$V_o = \frac{j62.832}{100 + j62.832} V_1 = 0.532 \angle 57.86^\circ V_1 \quad (9.14.3)$$

Combining Eqs. (9.14.2) and (9.14.3),

$$V_o = (0.532 \angle 57.86^\circ)(0.3582 \angle 42.02^\circ) V_i = 0.1906 \angle 100^\circ V_i$$

