Homework questions from the “Revision…” lectures presentations:

#1. Show that for a one-dimensional system the momentum operator in the coordinate representation is given by $\hat{p} = -i\hbar \frac{d}{dx}$. (Hint: start from the momentum representation for the momentum operator)

#2. Prove that if an operator is Hermitian then it satisfies the following:

1. Its eigenvalues are **always real**
2. Its corresponding **eigenfunctions are orthogonal**
3. Its expectation values **are always real**

#3. Show that the momentum operator $\hat{p}_x = -i\hbar \frac{d}{dx}$ is Hermitian.

#4. Consider following the states: $|\Phi\rangle = 3i|\psi_1\rangle - 7i|\psi_2\rangle$ and $|\Theta\rangle = -|\psi_1\rangle + 2i|\psi_2\rangle$,
Where $|\psi_1\rangle$ and $|\psi_2\rangle$ are orthonormal.

1. Calculate: $|\Phi + \Theta\rangle$ and $\langle\Phi + \Theta|$.
2. Calculate the scalars: $\langle\Theta|\Phi\rangle$ and $\langle\Phi|\Theta\rangle$. How o they relate to each other?
3. Are $|\Phi\rangle$ and $|\Theta\rangle$ normalised? If not, normalised them.

#5. Consider a state which is given in terms of orthonormal vectors $|\psi_1\rangle$, $|\psi_2\rangle$, $|\psi_3\rangle$ as follows:

$$|\Psi\rangle = \frac{1}{\sqrt{15}}|\psi_1\rangle + \frac{1}{\sqrt{3}}|\psi_2\rangle + \frac{1}{\sqrt{5}}|\psi_3\rangle$$

Where $|\psi_n\rangle$ are eigenstates of an operator $\hat{A}$ such that: $\hat{A}\,|\psi_n\rangle = (3n^2 - 1)\,|\psi_n\rangle$.

1. Normalise $|\Psi\rangle$.
2. Find the expectation value of $\hat{A}$ in the state $|\Psi\rangle$.
3. Find the expectation value of $\widehat{A^2}$ in the state $|\Psi\rangle$.

#6. For the simple harmonic oscillator ladder operators $\hat{b}$, $\hat{b}^+$ show that:

$$\hat{b}^+|n\rangle = \sqrt{n+1}\,|n+1\rangle,$$

$$\hat{b}|n\rangle = \sqrt{n}\,|n-1\rangle,$$

where $|n\rangle$ is an eigenvector of the Hamiltonian operators.

#7. Calculate matrix elements $\langle n'|\hat{x}|n\rangle$ and $\langle n'|\hat{p}|n\rangle$, where $|n\rangle$ and $|n'\rangle$ are eigenvectors of the Hamiltonian operator for a simple harmonic oscillator.