

Universo Primitivo

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Mestrado em Física - Astronomia

Chapter 7

7 Recombination and Decoupling

- Initial conditions;
- Equilibrium abundances: the Saha equation;
- Hydrogen recombination;
- Photon - electron decoupling;
- Electron freeze-out

References



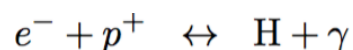
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Recombination and decoupling

Initial conditions

Soon after neutron freeze-out and the production of the light nuclei by Big-Bang nucleosynthesis, $T \approx 0.1$ MeV, the universe:

- consisted of a plasma with **photons, electrons, protons** and **atomic nuclei** (and neutrinos that are decoupled from the plasma).
- **Electrons, protons and atomic nuclei are non-relativistic** (note that $m_e = 0.5\text{MeV}$)
- Electrons are **tightly coupled** to photons due to **Compton scattering** and interact with protons and atomic nuclei via **Coulomb scattering**;
- **Protons, electrons and atomic nuclei** remain in equilibrium, through **electromagnetic interactions** such as:



- **Very little amounts of neutral matter (atoms) exist** because the **lowest ionization energy is only 13.6 eV** (for the Hydrogen atom) **but the plasma is still too hot** for electrons to be totally captured by nuclei;

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Recombination and decoupling

Equilibrium abundances: the Saha equation

Let us now look at this interaction: $e^- + p^+ \leftrightarrow H + \gamma$

At temperatures smaller than the mass of electrons, protons and hydrogen nuclei, these species have non-relativistic equilibrium abundances given by (Ex. 5.1, sheet 2):

$$n_i^{\text{eq}} = g_i \left(\frac{m_i T}{2\pi} \right)^{3/2} \exp \left(\frac{\mu_i - m_i}{T} \right)$$

Where, $i = \{e, p, H\}$ and $\mu_p + \mu_e = \mu_H$ because photons have 0 chemical potential. With these densities we can compute the following ratio:

$$\left(\frac{n_H}{n_e n_p} \right)_{\text{eq}} = \frac{g_H}{g_e g_p} \left(\frac{m_H}{m_e m_p} \frac{2\pi}{T} \right)^{3/2} e^{(m_p + m_e - m_H)/T}$$

That involves the densities of the species in the reaction and is independent of their chemical potentials.

In the exponential pre-factor, it is safe to assume $m_H \approx m_p$, but the very small difference of mass between hydrogen and the proton is crucial. It gives the **binding (or ionization) energy** of the hydrogen atom

$$B_H \equiv m_p + m_e - m_H = 13.6 \text{ eV}$$

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Recombination and decoupling

Equilibrium abundances: the Saha equation

The number of effective degrees of freedom of the hydrogen atom is $g_H = 4$. This arises because the spins of the electron and the proton can be aligned or anti-aligned, giving rise to 4 states. If we assume **the Universe is not electrically charged** electrons and protons should have the same initial density, $n_e = n_p$. So the previous ratio can be simplified as:

$$\left(\frac{n_H}{n_e^2} \right)_{\text{eq}} = \left(\frac{2\pi}{m_e T} \right)^{3/2} e^{B_H/T}$$

The ratio on the left-hand side of this equation can be related to the electron fraction abundance,

$$X_e \equiv \frac{n_e}{n_b}$$

Where the baryon density is, $n_b = \eta n_\gamma = \eta \times \frac{2\zeta(3)}{\pi^2} T^3$ and η is the baryon to photon ration.

Since the majority of baryons is in the form of hydrogen and free protons, one may assume that $n_b \approx n_p + n_H = n_e + n_H$, and one can derive that:

$$\frac{1 - X_e}{X_e^2} = \frac{n_H}{n_e^2} n_b$$

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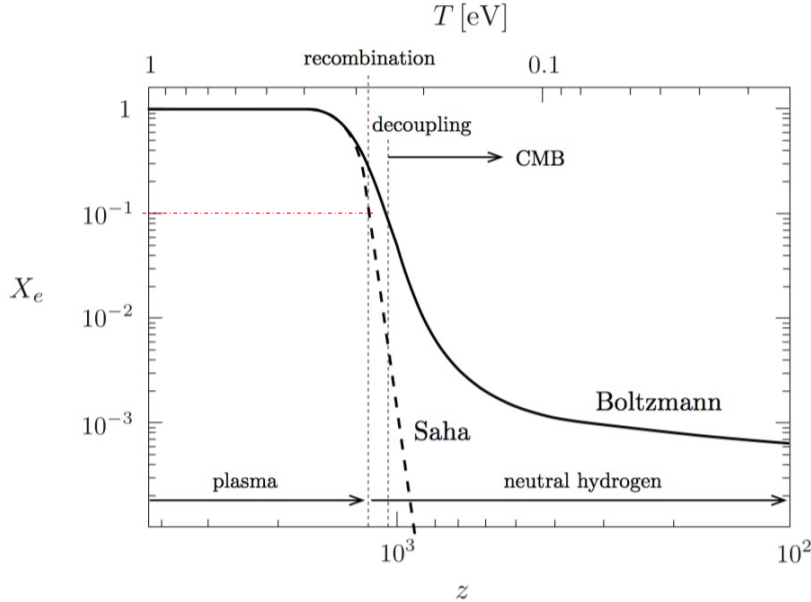
Recombination and decoupling

Equilibrium abundances: the Saha equation

Combining these expressions we obtain the so-called Saha equation.

$$\left(\frac{1 - X_e}{X_e^2} \right)_{\text{eq}} = \frac{2\zeta(3)}{\pi^2} \eta \left(\frac{2\pi T}{m_e} \right)^{3/2} e^{B_H/T}$$

Which allows one to compute the free electrons fraction in equilibrium



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Recombination and decoupling

Hydrogen recombination

A way to **define the epoch of recombination is to consider** the moment when the free electron fraction reduces below a given amount, usually $X_e = 0.1$.

Using this in the Saha equation one can compute the temperature at recombination:

$$\left(\frac{1 - 0.1}{0.1^2} \right)_{\text{eq}} = \frac{2\zeta(3)}{\pi} \eta \left(\frac{2\pi T_{\text{rec}}}{m_e} \right)^{3/2} e^{B_H/T_{\text{rec}}} \Leftrightarrow T_{\text{rec}} \approx 0.3 \text{ eV} \simeq 3600 \text{ K}$$

If one uses the redshift – temperature relation derived in Chapter 3,

$$T_{\text{rec}} = T_0(1 + z_{\text{rec}})$$

one obtains that

$$z_{\text{rec}} \approx 1320$$

This redshift is lower than the redshift of matter radiation equality (exercise in exercise sheet 1), **so recombination occurred during the matter dominated era**, where the time dependence of the scale factor is $a(t) \propto t^{2/3}$. Using this scaling in the temperature – redshift relation one concludes that:

$$t_{\text{rec}} = \frac{t_0}{(1 + z_{\text{rec}})^{3/2}} \sim 290\,000 \text{ yrs}$$

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Recombination and decoupling

Photon – electron decoupling

Photons and electron are strongly coupled mostly due to Compton scattering

$$e^- + \gamma \leftrightarrow e^- + \gamma$$

The **Compton scattering interaction rate** can be approximated by $\Gamma_\gamma \approx n_e \sigma_T$ where the Thompson cross section is $\sigma_T \approx 2 \times 10^{-3} \text{ MeV}^{-2}$.

The epoch of decoupling between photons and electrons can be estimated by equating

$$\Gamma_\gamma(T_{dec}) \sim H(T_{dec})$$

Now:

$$\begin{cases} \Gamma_\gamma(T_{dec}) = n_b X_e(T_{dec}) \sigma_T = \frac{2\zeta(3)}{\pi^2} \eta \sigma_T X_e(T_{dec}) T_{dec}^3 , \\ H(T_{dec}) = H_0 \sqrt{\Omega_m} \left(\frac{T_{dec}}{T_0} \right)^{3/2} . \end{cases}$$

Using these in the previous equation gives:

$$X_e(T_{dec}) T_{dec}^{3/2} \sim \frac{\pi^2}{2\zeta(3)} \frac{H_0 \sqrt{\Omega_m}}{\eta \sigma_T T_0^{3/2}}$$

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Recombination and decoupling

Photon decoupling

Using this result with the Saha equation one obtains:

$$T_{dec} \sim 0.27 \text{ eV}$$

Which is a slightly lower temperature than the recombination temperature.

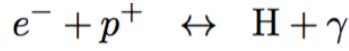
Using this value in the temperature-redshift relation and the time-redshift (or time-temperature) relations one obtains the following estimates for the redshift and time of decoupling

$$\begin{aligned} z_{dec} &\sim 1100 , \\ t_{dec} &\sim 380\,000 \text{ yrs} . \end{aligned}$$

Recombination and decoupling

Electron Freeze-out

To compute the non-equilibrium abundance of free electrons one should rely on the Boltzmann equation for the reaction that describes the capture of free electrons by protons:



To a reasonable approximation one may assume that:

- The hydrogen density is $n_H \approx n_H^{\text{eq}}$
- The universe is neutral, i.e. $n_e = n_p$

Under these assumptions, the Boltzmann equation derived in Chapter 4 (slides 26-28) for the above interaction reduces to:

$$\frac{1}{a^3} \frac{d(n_e a^3)}{dt} = -\langle \sigma v \rangle [n_e^2 - (n_e^{\text{eq}})^2]$$

The thermally averaged cross recombination section $\langle \sigma v \rangle$ can be approximated by:

$$\langle \sigma v \rangle \simeq \sigma_T \left(\frac{B_H}{T} \right)^{1/2}$$

Where B_H is the binding energy of the hydrogen atom.

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Recombination and decoupling

Electron Freeze-out

Using this result, writing $n_e = n_B X_e$ and using the fact that $n_b a^3 = \text{constant}$, one can approximate the Boltzmann equation as:

$$\frac{dX_e}{dx} = -\frac{\lambda}{x^2} [X_e^2 - (X_e^{\text{eq}})^2]$$

(same solution as the Ricatti equation)

Where $x \equiv B_H/T$, and

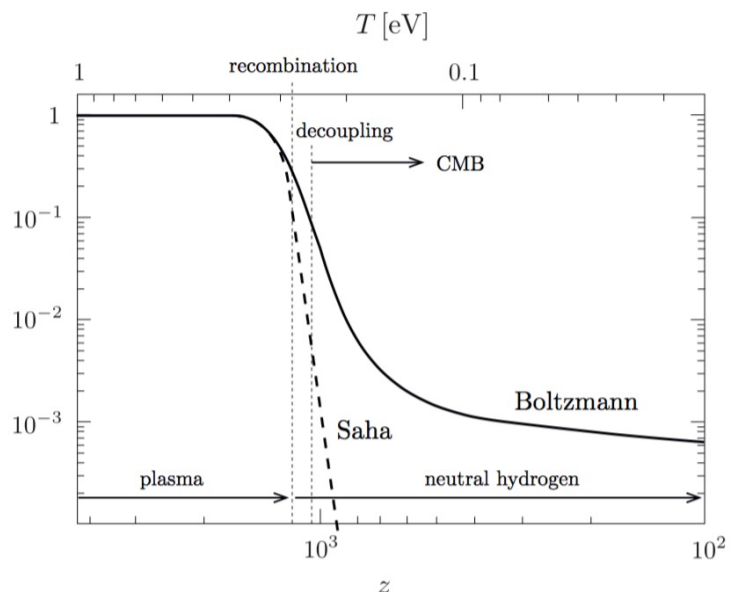
$$\lambda \equiv \left[\frac{n_b \langle \sigma v \rangle}{xH} \right]_{x=1} = 3.9 \times 10^3 \left(\frac{\Omega_b h}{0.03} \right)$$

This equation has the same type of (approximate) solution as the Ricatti equation (see Chap. 5):

$$X_e^\infty \simeq \frac{x_f}{\lambda} = 0.9 \times 10^{-3} \left(\frac{x_f}{x_{\text{rec}}} \right) \left(\frac{0.03}{\Omega_b h} \right)$$

Which is a good approximation to the full Boltzmann Integration (the solid line in the figure). This yields

a present (global) free electron fraction in the Universe of about 0.1% ($x_f/x_{\text{rec}} \sim 1$).



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