2nd SERIES OF EXERCISES – RADIOACTIVITY

1 – Consider a nucleus X that undergoes a decay with half-life $t_{1/2} = 2\ h$. What percentage of X nuclei still remain after: (a) 15 minutes; (b) 1 hour; (c) 2 hours; (d) 1 day?

2 – A nucleus X has two possible decays: $X \rightarrow Y$, with $t_{1/2} = 0.8\ h$ and $X \rightarrow Z$, with $t_{1/2} = 10\ min$. Initially there are N_0 nuclei of X.

a) In terms of N_0, how many X nuclei exist after: I) 15 minutes; II) 1 hour; III) 1 day?

b) In terms of N_0, how many Y nuclei exist after: I) 15 minutes; II) 1 hour; III) 1 day?

c) In terms of N_0, how many Z nuclei exist after: I) 15 minutes; II) 1 hour; III) 1 day?

d) Are the results from c) consistent with those from a) and b)? Explain.

3 – *(Krane, ex. 6.1)* Consider three radioactive sources, all with activities equal to $1.0\ \mu Ci$ at time $t = 0$, and with half-lives equal to, respectively, $1.0\ s$, $1.0\ h$ and $1.0\ d$.

a) How many radioactive nuclei of each source are present at $t = 0$?

b) How many nuclei of each source decay between $t = 0$ and $t = 1\ s$?

c) How many decay between $t = 0$ and $t = 1\ h$?

4 – *(Krane, ex. 6.2)* Samarium found in nature includes 15.1% of the radioactive isotope ^{146}Sm, which decays by α emission. One gram of natural Samarium has an activity of about 89 decays per second. Calculate the decay constant and the half-life of ^{146}Sm.

5 – *(Krane, ex. 6.3)* The Chernobyl accident in 1986 released large quantities of the radioactive isotopes ^{131}I (with $t_{1/2} = 8.0\ d$) and ^{137}Cs (with $t_{1/2} = 30\ y$) into the atmosphere. In a Chernobyl-type reactor, five times more Caesium is produced than Iodine.

a) Which of the two isotopes contributed a greater activity to the radiation cloud, right after the accident?

b) How long did it take for the two isotopes to reach equal activities?

6 – A nucleus A decays into a nucleus B with $t_{1/2} = 12\ min$, which in turn decays into a stable nucleus C with $t_{1/2} = 15\ s$.

a) Write the differential equations governing the evolution of the number of nuclei A, B and C, N_A, N_B and N_C, with time.

b) Solve these equations, assuming that at $t = 0$ there are only N_0 nuclei of type A.

c) Assuming that at $t = 0$ the activity of nucleus A is 20 μCi, calculate how many B and C nuclei exist 20 minutes after the start of the experiment.

7 – *(Krane, ex. 6.14)* In the decay chain $^{235}U \rightarrow {}^{231}Th \rightarrow {}^{231}Pa$ (with $t_{1/2} = 7.04\times10^8\ y$ for ^{235}U, and $t_{1/2} = 25.5\ h$ for ^{231}Th), the initially pure sample of ^{235}U has an activity of $1.0\ mCi$.

a) What type are the two decays?

b) In this case, secular equilibrium is reached. Why?

c) What is the maximum activity that ^{231}Th reaches?

8 – *(Krane, ex. 6.15)* One of the decays of ^{238}U ($t_{1/2} = 4.47\times10^9\ y$) leads to ^{234}Th ($t_{1/2} = 24.1\ d$), in secular equilibrium with the uranium.

a) What type of decay is this?

b) Why is secular equilibrium reached?

c) What is the activity of the thorium in each gram of uranium?

9 – *(Krane, ex. 6.17)* The decay of ^{232}Th ($t_{1/2} = 1.41\times10^{10}\ y$) eventually leads to the stable isotope ^{208}Pb. It was determined that a given rock contains 3.65 g of ^{232}Th, and 0.75 g of ^{208}Pb. Assuming that all the lead came from the radioactive decay of the thorium, determine the age of the rock.

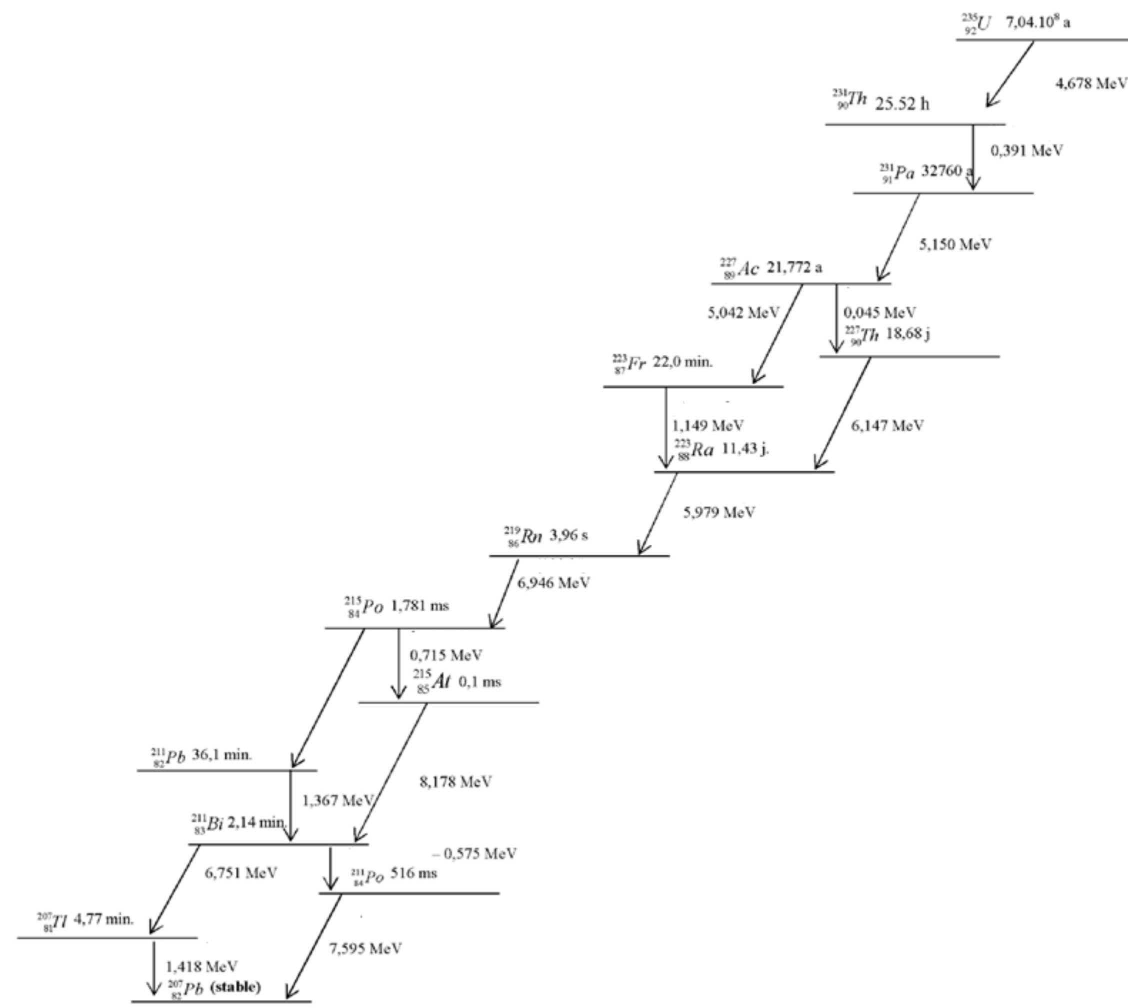
10 - *(Krane, ex. 6.18)* When analysing the content of ^{14}C in a log of wood taken from an old shelter, it was found that a sample of it gave an activity of 2.1 decays per minute. A sample taken from a log of identical size of the same type, but freshly cut from a tree, gives an activity of 5.3 decays per minute. What is the age of the shelter?

11 – Identify the types of the following decays:

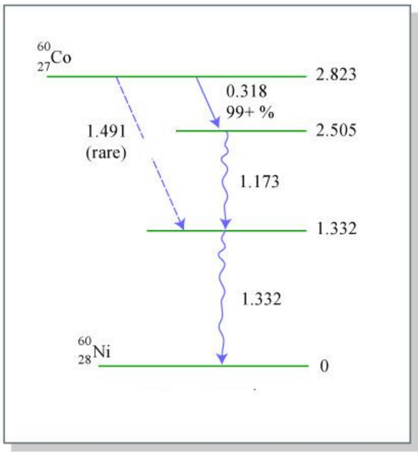
a) $^{137}Cs \rightarrow {}^{137}Ba$ b) $^{234}Th \rightarrow {}^{230}Ra$ c) $^{228}Th \rightarrow {}^{224}Ra$

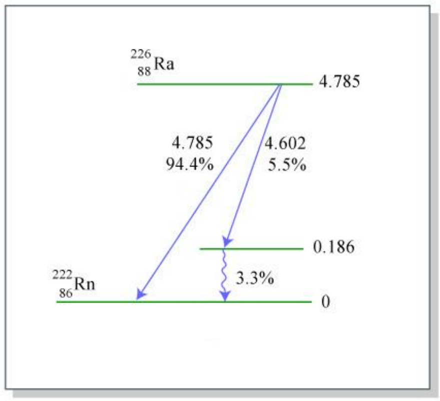
d) $^{22}Na \rightarrow {}^{22}Ne$ e) $^{201}Hg \rightarrow {}^{201}Au$ f) $^{18}O \rightarrow {}^{18}F$

12 – The decay series of Uranium, until it reaches a stable isotope of Lead, is shown in the figure below. Indicate the type of each of the 17 decays shown.



13 – Consider the decay processes shown in the figures below. Indicate the types of each of the decays taking place.





14 – A given nucleus **X** undergoes an *alpha* decay, giving rise to a nucleus **Y,** with half-life $t_{1/2} = 14\ h$. The initial activity of nucleus **X** is $7.1\ \mu Ci$, and initially there are no nuclei of **Y**.

a) What is the initial number of **X** nuclei?
b) How many X nuclei are present after 5 hours have passed?
c) Calculate how many **Y** nuclei are present at the end of one day.
d) What is the activity of **X** at the end of one day?
e) What is the mass of helium gas present at the end of one day?

15 – Fluorine-18 and gallium-68 are isotopes used in PET scans, as both have β+-type decays (consult tables if necessary).

a) Write the equations for the β+ decays of both isotopes.
b) A given sample of gallium-68 has, at a certain moment, an activity equal to $7\ mCi$. How many gallium-68 nuclei are present?
c) What is the activity of this sample after 3 hours? If a sample of fluorine-18 had the same initial activity, what would its activity be after 3 hours?

16 – A given nucleus **X** undergoes a decay giving rise to a nucleus **Y,** with half-life $t_{1/2} = 4\ h$, and another decay, in which **X** gives rise to a nucleus **Z**, with $t_{1/2} = 49\ min$. The initial activity of nucleus **X** is $3.7\ \mu Ci$, and initially there are no nuclei of **Y** or **Z**.

a) What is the initial number of **X** nuclei?
b) Calculate how many **X, Y** and **Z** nuclei are present at the end of one day.

17 – Consider the production of Fluorine-18 ($t_{1/2} = 110\ min$) in a cyclotron in Coimbra. The isotope produced is to be used in a PET scan in Lisbon (assume that three hours are needed to get the product to Lisbon and start the scan), with an activity at the time of the scan of $2\ mCi$. The production rate of F-18 in the cyclotron is $R = 2.5\times10^8\ particles/s$. For how long must the cyclotron be running to obtain the desired final result?

18 – A scan with Iodine-123 ($t_{1/2} = 13.2\ h$) requires isotopes with an activity of *30 mCi*. These isotopes are produced in a cyclotron, taking about *5 h* to reach the medical department for which they are intended. Given that the cyclotron took 11 h to produce the isotopes used in a given scan, determine the minimum reaction rate of the cyclotron.

1 Ci = 3.7×10^{10} decays/second