Properties of the strong force

• It is very strong (it has to overcome the Coulomb repulsion).

• It only acts over short distances.

• Not all particles feel it (electrons and neutrinos do not feel it).

• It is independent of the charges of the particles – it does not distinguish between protons and neutrons.

• It depends on the *spins* of the particles.

• It is described, to first order, by an attractive potential (which binds the nucleons).

• However, at very short distances it becomes repulsive (nuclear density is more or less constant even in nuclei with many nucleons).

The big difference between nuclear physics and, for example, atomic physics is that in one the interaction potential (Coulomb) is known, while in the other the potential is unknown and must be inferred from experiments.

NUCLEAR RADIUS

What is the size of nuclei? Of the order of the fermi = 1 femtometre = 10^{-15} m

Earth radius $\approx 6000 \text{ km} = 6 \times 10^{6} \text{ m}$

Human body $\approx 1.7 \text{ m}$

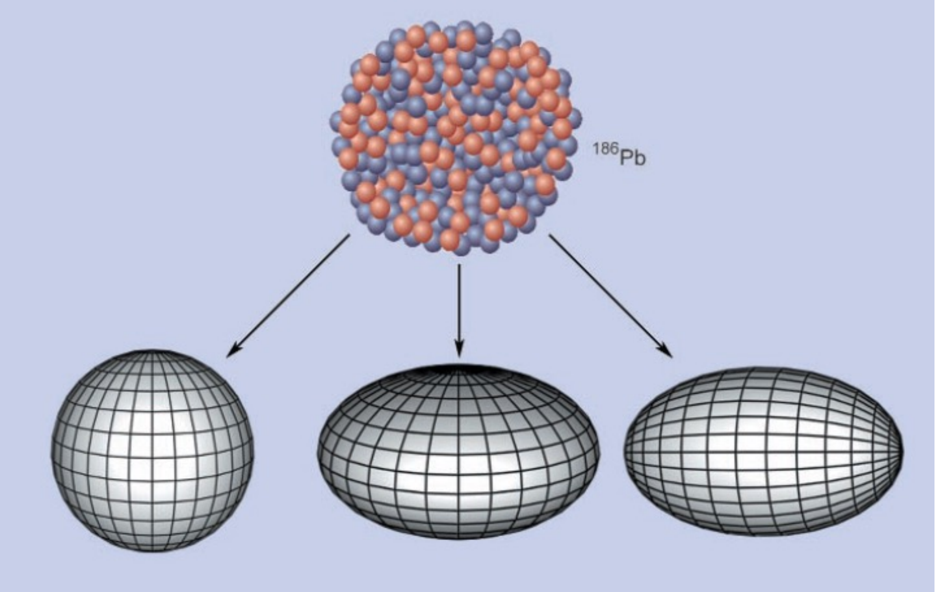
Bacterium $\approx 5\ \mu\text{m} = 5 \times 10^{-6} \text{ m}$

DNA $\approx 2 \text{ nm} = 2 \times 10^{-9} \text{ m}$

Hydrogen atom $\approx 1 \overset{o}{\text{A}} = 10^{-10} \text{ m}$

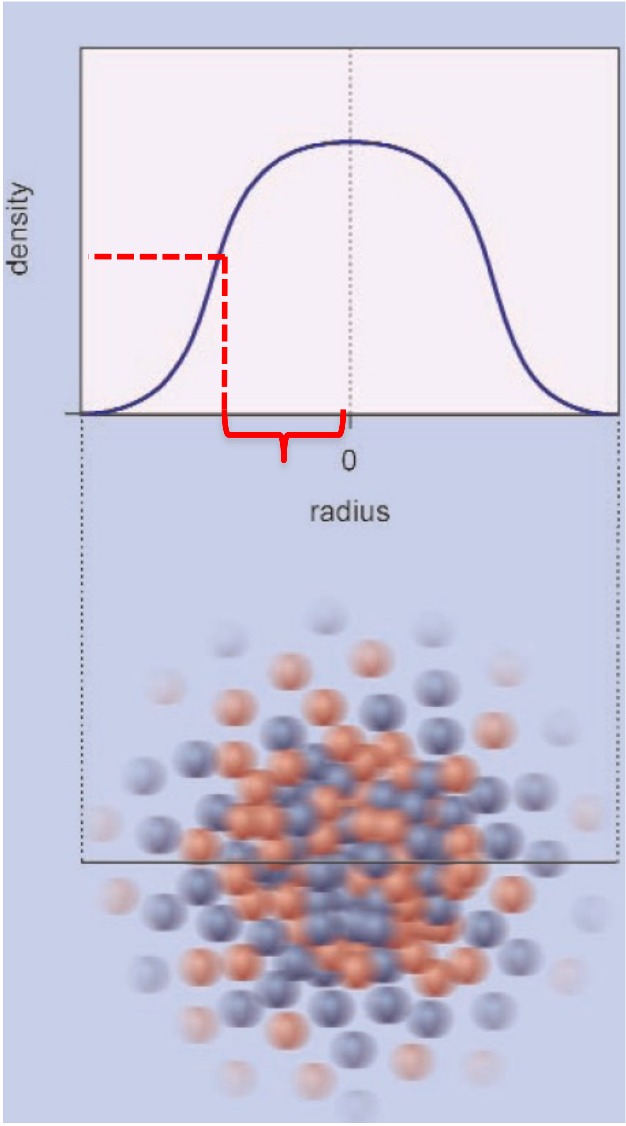
Nucleus $\approx 1 \text{ fm} = 10^{-15} \text{ m}$

Although this is not true for all nuclei (especially those with high A – such as uranium, for example) we will assume that nuclei are approximately spherical.



When the isotope of polonium, ^{190}Po, undergoes alpha decay, the lead isotope ^{186}Pb is formed. This nucleus can exist in three forms, all with much the same energy: spherical (left), oblate (centre) and prolate (right).

NUCLEAR RADIUS



The radius of a nucleus is not well defined – it is impossible to define clearly where the nucleus ends.

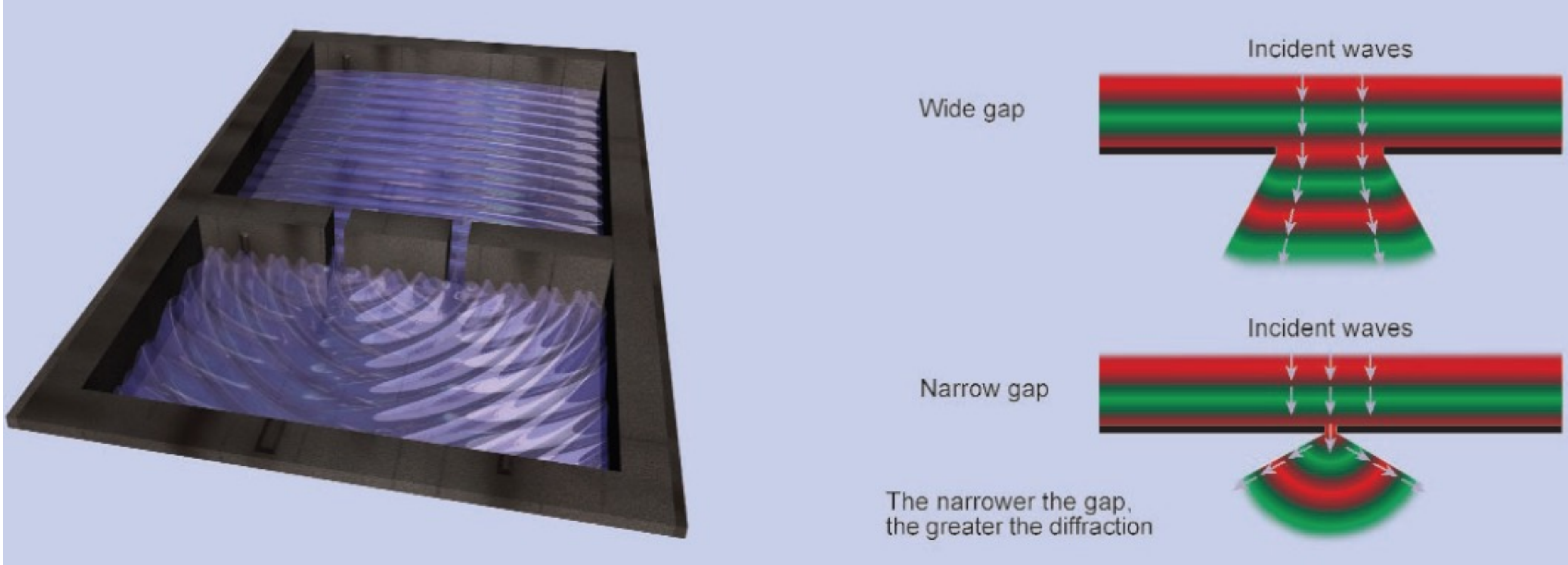
To compare the sizes of different nuclei we need to clearly define the concept of nuclear radius.

How are the radii/density of nuclei measured? In electron scattering experiments off nuclei (like diffraction experiments). We will come back to these experiments.

These experiments tell us that the nuclear density is approximately constant for a large number of nuclei, falling rapidly once the edge of the nucleus is reached.

This fact allows us to define a nuclear radius: the distance from the centre of the nucleus to a point where the density is half the density at the centre.

NUCLEAR RADIUS



To understand how the diffraction experiment works, it is enough to observe matter waves, such as water waves, passing through slits.

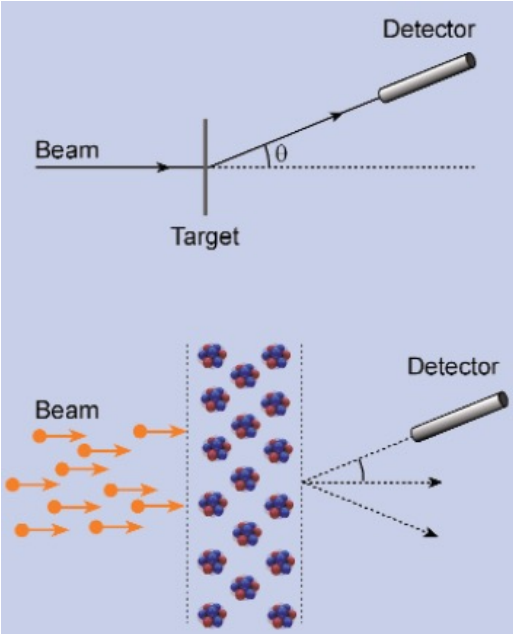
The smaller the slit, the more the waves are diffused or scattered on the other side of the barrier. If we did not know the size of the gap, we could in principle determine it by observing the behaviour of the waves.

If the distance between two waves (the wavelength) is too large, then the diffraction pattern disappears.

The same thing happens with light, creating a diffraction pattern by passing light through very small slits.

NUCLEAR RADIUS

If, in the diffraction experiment, we replace the laser with electrons, the bands will be the number of electrons measured at a given distance.



A proton has positive charge and is distributed in an approximately uniform manner in the nucleus. Since electrons have negative charge, they are deflected by the positive charges. The way they are deflected depends on how the charge is distributed, and therefore on the radius of the nucleus.

Since electrons behave as waves, a diffraction pattern is produced, although it is a diffuse pattern. This diffraction pattern will tell us how the nuclear charge is distributed. If we assume the distribution is uniform, we obtain an estimate of the radius of the nucleus.

NUCLEAR RADIUS

As we have seen, these experiments tell us that the nuclear density is approximately constant for a large number of nuclei, falling rapidly once the edge of the nucleus is reached.

This fact allows us to define a nuclear radius: the distance from the centre of the nucleus to a point where the density is half the density at the centre.

$$\rho_N = \frac{\text{n}^\text{o} \text{ de nucleões}}{\text{volume núcleo}} \approx \frac{A}{4\pi R^3/3} \approx \text{ constante} \quad \Rightarrow \quad \boxed{R = R_0\, A^{1/3}} \qquad R_0 = 1.2 \text{ fm } \text{ (experiência)}$$

$$\rho_N = \frac{A}{4\pi R_0^3 A/3} \approx 1.4 \times 10^{44} \text{ partículas/m}^3 \approx 2.24 \times 10^{17} \text{ kg/m}^3$$

Water $\approx 1000 \text{ kg/m}^3$

Lead $\approx 1.13 \times 10^4 \text{ kg/m}^3$

Centre of the Earth $\approx 1.3 \times 10^4 \text{ kg/m}^3$

Neutron star $\approx 5 \times 10^{17} \text{ kg/m}^3$

Calculate nuclear radii – use the nuclei from problem 3 of problem set 1

MIRROR NUCLEI

Consider two nuclei, nucleus 1 with Z_1 protons and N_1 neutrons, and nucleus 2 with Z_2 protons and N_2 neutrons. The two nuclei are called mirror nuclei if $Z_1 = N_2$ and $Z_2 = N_1$. Note that $A_1 = A_2$.
Examples: ^3H and ^3He; ^{14}C and ^{14}O; ^{15}N and ^{15}O; ^{24}Na and ^{24}Al.

These nuclides are isobars - same A. Isobars can also have the same Z and therefore the same N (isomers). In that case, the difference is whether the nuclide is in the ground state (minimum energy state) or in an excited state. Nuclei that are in an excited state tend to decay to the ground state through an isomeric transition, in which gamma radiation is emitted.

We will assume that nuclei are spheres with a uniform charge distribution $Q = Ze$. Using electromagnetism, the energy of a spherical charge distribution can be written as

$$E = \frac{3}{5}\frac{Q^2}{4\pi\varepsilon_0 R} = \frac{3}{5}\frac{e^2}{4\pi\varepsilon_0 R}Z^2$$

Then the difference between the binding energies of two mirror nuclei with Z and Z-1 protons will be due only to their different electrostatic energies

$$\Delta E = \frac{3}{5}\frac{e^2}{4\pi\varepsilon_0 R}\left[Z^2-(Z-1)^2\right] = \frac{3}{5}\frac{e^2}{4\pi\varepsilon_0 R}\left[2Z-1\right] = \frac{3}{5}\frac{e^2}{4\pi\varepsilon_0 R}A = \frac{3}{5}\frac{e^2}{4\pi\varepsilon_0 R_0 A^{1/3}}A$$

MIRROR NUCLEI

The final expression for the energy difference (which reproduces the experimental results very well)

$$\Delta E_c = \frac{3}{5}\frac{\mathrm{e}^2}{4\pi\varepsilon_0 R_0} A^{2/3}$$

Example

$^{27}Si_{14}$ **decays to** $^{27}Al_{13}$ **with the emission of a positron with a maximum energy of 3.4 MeV**

$m_p = 1.007276$ u; $m_n = 1.008665$ u; $m_e = 5.485803 \times 10^{-4}$ u; $e = 1.6 \times 10^{-19}$ C; $\varepsilon_0 = 8.85 \times 10^{-12}$ Fm^{-1}

1 u = 931.502 MeV; 1 eV = 1.6×10^{-19} J

Calculation of the energy change (loses a positron, a proton converts to a neutron)

$$E_{\max} = \Delta E_c + \left(m_p - m_n - m_e\right) c^2 \Rightarrow \Delta E_c = E_{\max} - \left(m_p - m_n - m_e\right) c^2 = 8.32776 \times 10^{-13} \text{ J}$$

Calculation of R_0

$$\Delta E_c = \frac{3}{5}\frac{\mathrm{e}^2}{4\pi\varepsilon_0 R_0} A^{2/3} \Rightarrow R_0 = \frac{3}{5}\frac{\mathrm{e}^2}{4\pi\varepsilon_0 \Delta E_c} A^{2/3} = 1.49 \times 10^{-15} \text{ m}$$

SEMI-EMPIRICAL MASS FORMULA

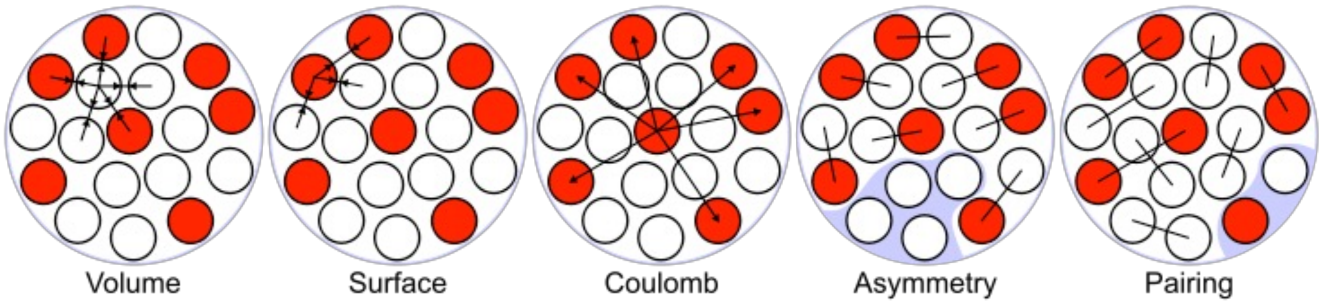
- **Based on the *liquid drop model* of Gamow.**

- **Proposed in 1935 by Weizsäcker.**

This formula is called semi-empirical because it is based on theoretical principles but its parameters are adjusted experimentally. Another reason is that its terms arise out of necessity and not from first principles.

Despite its name, the formula gives us the binding energy of a nucleus, not its mass.

The semi-empirical formula has 5 terms, which we will now describe



SEMI-EMPIRICAL MASS FORMULA

VOLUME TERM: for most nucleons, B/A is, to within 10%, about 8 MeV/nucleon. So the first contribution is a constant term for B/A. This term is called the volume term because it is equivalent to a constant energy distribution throughout the volume.

$$\boxed{B_V = \text{const.} \times V_{\text{núcleo}} = \text{const.} \times \frac{4}{3}\pi \left[R_0 A^{1/3}\right]^3 = a_V A}$$

a_V is determined experimentally.

SURFACE TERM: the larger the surface, the more unstable the nucleus will tend to be (as with a soap bubble). The surface term is therefore proportional to the surface area of the sphere and reduces B.

$$\boxed{B_S = -\,\text{const.} \times S_{\text{núcleo}} = -\,\text{const.} \times 4\pi \left[R_0 A^{1/3}\right]^2 = -\,a_S A^{2/3}}$$

a_S is determined experimentally.

SEMI-EMPIRICAL MASS FORMULA

COULOMB TERM: corresponds to the contribution from the repulsion of the protons, which reduces B. Each proton "sees" Z-1 protons, and the energy of a proton in a sphere with (Z-1) other protons is given by

$$\boxed{B_C = -\frac{3}{5}\frac{\mathrm{e}^2}{4\pi\varepsilon_0 R} Z(Z-1) = -\frac{3}{5}\frac{\mathrm{e}^2}{4\pi\varepsilon_0 R_0 A^{1/3}} Z(Z-1) = -\,a_C\, Z(Z-1)A^{-1/3}}$$

a_C is determined experimentally.

SYMMETRY TERM: important for light nuclei, the most stable of which have Z approximately equal to A/2. For heavier nuclei this term is less important (greater Coulomb repulsion requires the presence of more neutrons).

$$\boxed{B_{SIM} = -\,a_{SIM}\frac{(A-2Z)^2}{A}}$$

a_{SIM} is determined experimentally.

SEMI-EMPIRICAL MASS FORMULA

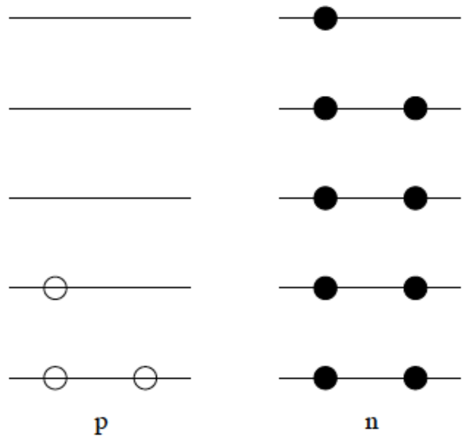
PAIRING TERM: term that describes the tendency of nucleons to pair up and give rise to stable configurations (pairing force)

- **Z even and N even – more stable nuclei**

- **Z odd and N odd – less stable nuclei (lower binding energy)**

- **A odd – does not apply**

$$\boxed{\begin{cases} +a_P A^{-3/4}, \text{ se N e Z pares} \\ -a_P A^{-3/4}, \text{ se N e Z ímpares} \\ 0, \text{ se A ímpar} \end{cases}}$$



a_P is determined experimentally.

SEMI-EMPIRICAL MASS FORMULA

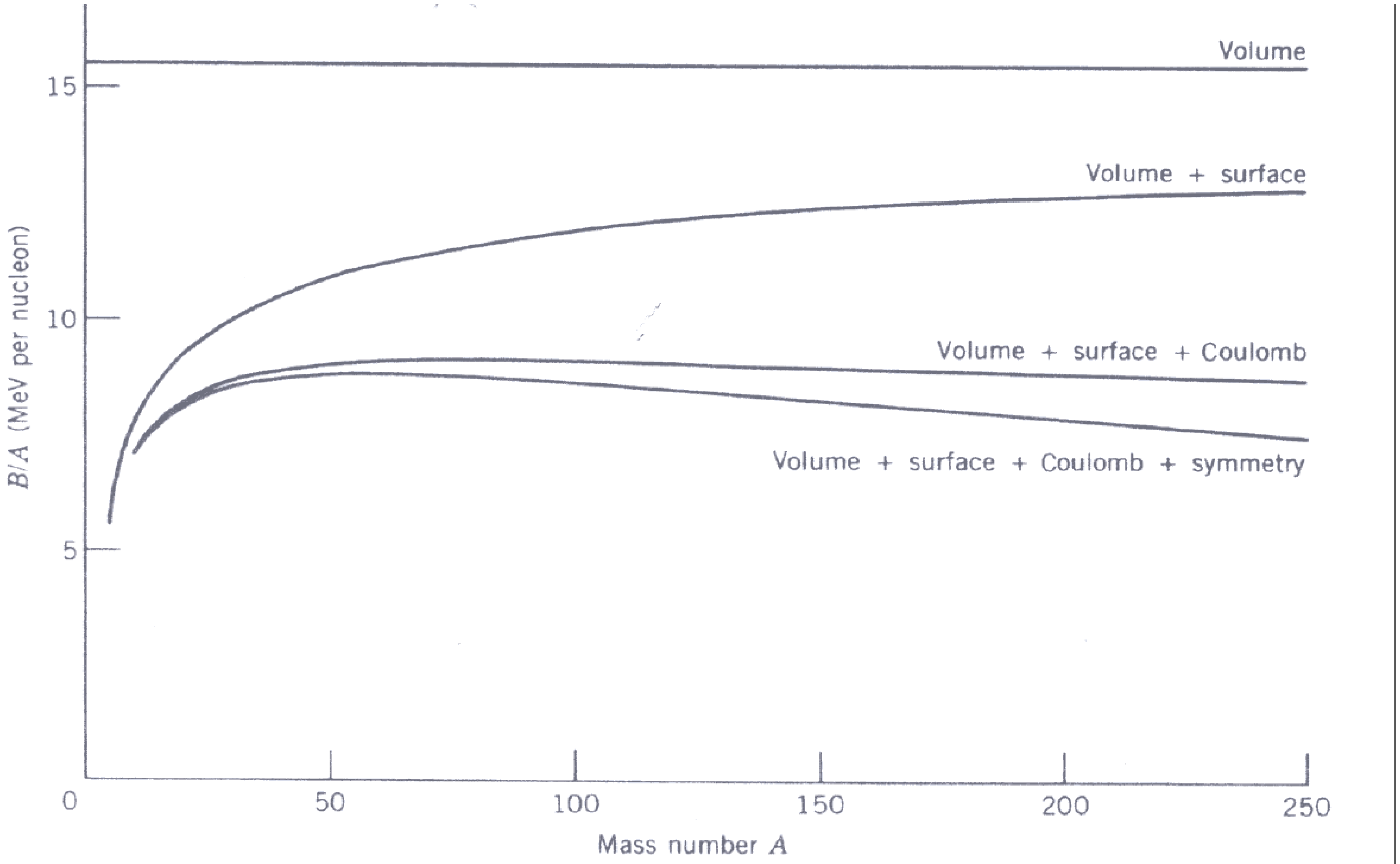
There are still other terms that could be added, but the description with these terms is already good enough. The formula can then be written as

$$\boxed{B(Z,A) = a_V A - a_S A^{2/3} - a_C Z(Z-1)A^{-1/3} - a_{SIM}\frac{(A-2Z)^2}{A} + \delta}$$

$$\delta = \begin{cases} +a_P A^{-3/4}, \text{ se N e Z pares} \\ -a_P A^{-3/4}, \text{ se N e Z ímpares} \\ 0, \text{ se A ímpar} \end{cases}$$

$a_V = 15.5$ MeV ; $a_S = 16.8$ MeV ; $a_C = 0.72$ MeV ; $a_{Sim} = 23$ MeV ; $a_P = 34$ MeV

Relevance of the Different Terms of the Semi-Empirical Mass Formula



Krane, p. 69

Example

$$\boxed{{}^{18}C \qquad Z = 6;\ N = 12; A = 18}$$

$$B(Z,A) = a_V A - a_S A^{2/3} - a_C Z(Z-1)A^{-1/3} - a_{SIM}\frac{(A-2Z)^2}{A} + \delta$$

$$B(Z,A) = 15.5\times 18 - 16.8\times 18^{2/3} - 0.72\times 6\times(6-1)\times 18^{-1/3} - 23\times\frac{(18-2\times 6)^2}{18} + 34\times 18^{-3/4}$$

$$B(Z,A) = 279 - 115.387 - 8.242 - 46 + 3.891$$

$$B(Z,A) = 113.262\ MeV$$

$$\delta = \begin{cases} +a_P A^{-3/4}, \text{ se N e Z pares} \\ -a_P A^{-3/4}, \text{ se N e Z ímpares} \\ 0, \text{ se A ímpar} \end{cases}$$

$a_V = 15.5\ MeV$; $a_S = 16.8\ MeV$; $a_C = 0.72\ MeV$; $a_{Sim} = 23\ MeV$; $a_P = 34\ MeV$

NUCLEAR RADIUS IV