7th Series of Exercises

Nuclear fission and reactors

Use the data tables at the end of the series. Unless stated otherwise: $1\,\mathrm{b} = 10^{-24}\,\mathrm{cm}^2$, $N_A = 6.022 \times 10^{23}\,\mathrm{mol}^{-1}$, $1\,\mathrm{MeV} = 1.602 \times 10^{-13}\,\mathrm{J}$, energy per fission $\approx 200\,\mathrm{MeV}$. In Table 2, σ_a is the thermal cross section for absorption *without* fission (radiative capture), so that the total absorption cross section is $\sigma_a + \sigma_f$. The multiplication factor of a finite reactor is

$$k = k_\infty P_{\mathrm{NL}}, \qquad k_\infty = \eta\,\varepsilon\,p\,f, \qquad P_{\mathrm{NL}} = \frac{1}{1 + B^2 M^2},$$

where M is the migration length (Table 3) and B the geometric buckling (Table 4).

Problem 1

a) Explain the concepts of *fissile* and *fertile* material.

b) Calculate the excitation energy of the compound nucleus formed when each of the isotopes in Table 1 captures a thermal neutron (take the neutron at rest). Using the activation energies given, say, with justification, which of these isotopes are fissile.

Problem 2 The average number of neutrons per fission (ν), the thermal cross section for absorption without fission (σ_a) and the thermal fission cross section (σ_f) are given in Table 2. Calculate the average number of fast neutrons produced per thermal neutron absorbed in the fuel, η, for:

a) natural uranium (0.72% ^{235}U);

b) uranium enriched to 3%;

c) uranium enriched to 5%;

d) pure ^{235}U;

e) pure ^{239}Pu.

Problem 3 The four-factor formula gives the multiplication factor of the neutron population in one fission cycle of an infinite reactor (no leakage): $k_\infty = \eta\,\varepsilon\,p\,f$.

a) Explain the meaning of each factor.

b) Obtain the criticality condition of a reactor by analysing the differential equation for the time evolution of the number of neutrons.

c) For each case of Problem 2, calculate k_∞. Use the typical values $\varepsilon = 1.03$, $p = 0.9$, $f = 0.9$.

Problem 4 Including leakage due to the finite size of the reactor, the multiplication factor becomes $k = k_\infty P_{\mathrm{NL}}$, where the non-leakage probability depends on the migration length M of the moderator (Table 3) and on the geometric buckling B of the core (Table 4). Using the results of Problem 3:

a) Calculate the minimum radius of a spherical natural-uranium reactor moderated by graphite. And if the moderator were heavy water?

b) The same for a PWR with uranium enriched to 5%.

c) Consider a PWR with uranium enriched to 3%, moderated by water. What is the minimum size of a cubic reactor? Comment.

d) And if the reactor used pure ^{239}Pu, with the same moderator?

Problem 5 Consider now more realistic values for the factors p and f: 0.7 and 0.8, respectively (keep $\varepsilon = 1.03$).

a) Show that criticality cannot be reached with natural uranium in a reactor with these parameters.

b) Calculate the minimum uranium enrichment needed to reach criticality.

c) For a cylindrical PWR of radius 1 m, with uranium enriched to 5%, calculate the minimum height needed for criticality.

Problem 6 Consider a nuclear reactor fuelled with uranium enriched to 4%. Using the data in the tables at the end:

a) Show that the average number of fast neutrons produced per thermal neutron absorbed in the fuel, η, is about 1.89.

b) 5% extra fast neutrons are produced by fast fission. While being moderated, 15% of the neutrons are absorbed in resonances. After thermalisation, 14% of the thermal neutrons are absorbed by the moderator. Show that $k_\infty \approx 1.45$.

c) What is the critical size of a cubic, graphite-moderated reactor with this composition?

d) A spherical reactor with this composition has a radius of about 8.15 m and operates in the critical regime. Which moderator was used?

Problem 7 Consider the following typical values of the coefficients (ε, p, f):

Reactor	PWR	CANDU	AGR
ε	1.01	1.08	1.20
p	0.80	0.84	0.62
f	0.79	0.97	0.98

a) Calculate the minimum uranium enrichment needed for the PWR to reach criticality. Assume $P_{\mathrm{NL}} = 0.9$.

b) For a CANDU reactor with natural uranium (0.72% ^{235}U), calculate the minimum edge of a cubic core.

c) The fuel of a PWR with uranium enriched to 3% was contaminated with ^{135}Xe, whose thermal absorption cross section is 2.6×10^6 b. Estimate the xenon contamination (Xe atoms per uranium atom) above which the reactor cannot reach criticality (use the table and $P_{\mathrm{NL}} = 0.9$).

d) For an AGR with uranium enriched to 5%, calculate the dimensions of a square-prism core, with height equal to twice the edge of its base, needed for criticality.

e) Repeat d) for a fuel with 4.9% ^{235}U, 0.1% ^{239}Pu, the rest ^{238}U.

Problem 8 Consider two reactors with the same moderator, the same fuel and the same relative proportions of moderator and fuel, both operating at criticality. One is spherical, the other cubic. Show, with calculations, which of the two needs less fuel.

Problem 9 Consider a nuclear power station fuelled with uranium enriched to 4%, with an electrical power of 900 MW. The average energy per fission is about 200 MeV.

a) Assuming the reactor operates two thirds of the time, and that the efficiency of conversion of thermal into electrical energy is 33%, estimate the mass of natural uranium (0.72% ^{235}U) needed to keep the reactor running for one year.

b) Each fission produces two highly radioactive fragments with mass numbers, on average, of 92 and 104. Estimate the total mass of these fission products produced per year.

Problem 10 *Thermal utilisation in a homogeneous mixture.* For a homogeneous mixture of fuel and moderator,

$$f = \frac{\Sigma_a^{\text{fuel}}}{\Sigma_a^{\text{fuel}} + \Sigma_a^{\text{mod}}} = \frac{N_{\text{U}}\,\bar{\sigma}_a^{\text{U}}}{N_{\text{U}}\,\bar{\sigma}_a^{\text{U}} + N_{\text{mod}}\,\sigma_a^{\text{mod}}}.$$

a) Calculate the average thermal absorption cross section (fission plus capture) per atom of natural uranium.

b) Calculate f for natural uranium mixed with (i) graphite, 400 carbon atoms per uranium atom ($\sigma_a(\text{C}) = 0.003\,4$ b); (ii) light water, 4 molecules per uranium atom ($\sigma_a(\text{H}_2\text{O}) = 0.664$ b); (iii) heavy water, 100 molecules per uranium atom ($\sigma_a(\text{D}_2\text{O}) = 0.001\,3$ b).

c) With $\eta = 1.333$ and $\varepsilon = 1.03$, what value of p would each mixture need for $k_\infty = 1$? Comment on why natural uranium cannot be used with light water.

Problem 11 *Decay heat.* After shutdown, the power produced by the decay of the fission products is approximately (Way–Wigner formula)

$$\frac{P(t)}{P_0} \approx 0.066\left[t^{-0.2} - (t + T_0)^{-0.2}\right],$$

with t (time after shutdown) and T_0 (operating time) in seconds.

a) A 3 000 MW_{th} reactor is shut down after one year of operation. Calculate the decay power after 1 s, 1 h, 1 day and 30 days.

b) If cooling is lost, this heat boils off water. Calculate the mass of water evaporated per second at each of those times (latent heat 2.26 MJ/kg; ignore heating to the boiling point).

c) Relate your results to the Fukushima accident.

Problem 12 *Breeding plutonium.* In the reactor of Problem 9, the ratio between fissile nuclei produced (^{239}Pu, from neutron capture in ^{238}U) and fissile nuclei consumed (^{235}U, by fission and capture) is the conversion ratio, $C \approx 0.6$ for a PWR.

a) Estimate the mass of ^{239}Pu produced in one year (use the number of fissions of Problem 9 and the ratio σ_a/σ_f of ^{235}U).

b) Considering only thermal neutrons, estimate the ratio between captures in ^{238}U and fissions in ^{235}U for 3% fuel. Why is the real conversion ratio much larger?

c) What value of C defines a breeder reactor, and why do breeders use fast neutrons and plutonium?

Problem 13 *Natural reactors at Oklo.* Two billion years ago, natural chain reactions took place in a uranium deposit at Oklo (Gabon).

a) Calculate the ^{235}U abundance two billion years ago ($t_{1/2}(^{235}\mathrm{U}) = 7.04 \times 10^8$ yr, $t_{1/2}(^{238}\mathrm{U}) = 4.47 \times 10^9$ yr; today 0.72%).

b) Calculate η for that uranium, and k_∞ with $\varepsilon = 1.0$, $p = 0.9$, $f = 0.9$ (water as moderator).

c) Why can no natural water-moderated reactor operate today?

Problem 14 *Chemical shim.* PWRs dissolve boric acid in the coolant to absorb the excess reactivity of fresh fuel. Consider a PWR with 3% fuel and $f = 0.79$ without boron, in which fresh fuel gives $k = 1.10$.

a) Show that adding a thermal absorber reduces f to $f' = \dfrac{\Sigma_F}{\Sigma_F + \Sigma_{\text{other}} + \Sigma_B}$, and that k is reduced by the factor f'/f.

b) How large must Σ_B be, relative to $\Sigma_F + \Sigma_{\text{other}}$, to bring k to 1?

c) Using the average absorption cross section per uranium atom of 3% fuel and $\sigma_a(^{10}\mathrm{B}) = 3\,840$ b, estimate the number of ^{10}B atoms needed per uranium atom.

d) Why is this method convenient during a fuel cycle?

Data

Table 1 — Activation energies (of the compound nucleus formed by thermal-neutron capture)

Target	^{230}Th	^{233}Pa	^{233}U	^{235}U	^{238}U	^{239}Pu	^{242}Am
E_a (MeV)	8.3	7.1	6.5	6.2	6.6	6.0	6.2

Table 2 — Thermal-neutron data

Isotope	ν	σ_a (b)	σ_f (b)
^{238}U	0	2.75	0
^{235}U	2.42	97	584
^{239}Pu	2.86	267	742

Table 3 — Migration lengths

Moderator	M (cm)
Water	5.84
Water, BWR	7.43
Water, PWR	7.62
Heavy water	174
Heavy water, CANDU	19.4
Graphite	62
Graphite, AGR	20.2

Table 4 — Geometric buckling

Geometry	B^2
Sphere, radius R	$(\pi/R)^2$
Box, $h \times l \times e$	$(\pi/h)^2 + (\pi/l)^2 + (\pi/e)^2$
Cylinder (R, h)	$(\pi/h)^2 + (2.405/R)^2$

Table 5 — Atomic mass excesses (AME2020), MeV

n	8.0713	^{238}U	47.3077
^{230}Th	30.8630	^{239}U	50.5727
^{231}Th	33.8169	^{239}Pu	48.5884
^{233}Pa	37.4900	^{240}Pu	50.1259
^{234}Pa	40.3413	^{242}Am	55.4697
^{233}U	36.9194	^{243}Am	57.1760
^{234}U	38.1459		
^{235}U	40.9188		
^{236}U	42.4446		

Answers

1. a) See solution. b) E^* (MeV): ^{230}Th 5.12; ^{233}Pa 5.22; ^{233}U 6.84; ^{235}U 6.55; ^{238}U 4.81; ^{239}Pu 6.53; ^{242}Am 6.37. Fissile: ^{233}U, ^{235}U, ^{239}Pu, ^{242}Am.

2. a) 1.333 b) 1.836 c) 1.927 d) 2.075 e) 2.103

3. a) See solution. b) $N(t) = N_0 e^{(k-1)t/\tau}$: critical for $k = 1$. c) 1.112; 1.531; 1.608; 1.731; 1.755.

4. a) 5.8 m (graphite); 16.3 m (heavy water) b) 31 cm c) edge 57 cm d) edge 48 cm

5. a) $k_\infty = 0.769 < 1$ b) 2.0% c) 86 cm

6. a) 1.892 b) $1.892 \times 1.05 \times 0.85 \times 0.86 = 1.452$ c) edge 5.0 m d) $M = 174$ cm: heavy water.

7. a) 2.06% b) 2.54 m c) $N_{\mathrm{Xe}}/N_{\mathrm{U}} \approx 4.8 \times 10^{-7}$ d) base 1.50 m, height 2.99 m e) base 1.49 m, height 2.98 m

8. The sphere: its volume is 0.81 of the cube's.

9. a) 699 kg of ^{235}U fissioned $\Rightarrow \approx 97$ t of natural uranium (minimum) b) ≈ 580 kg

10. a) 7.63 b b) (i) 0.849; (ii) 0.742; (iii) 0.983 c) $p \geq 0.86$; 0.98 (impossible); 0.74.

11. a) 192 MW; 32 MW; 14 MW; 4.2 MW b) 85 kg/s; 14 kg/s; 6.3 kg/s; 1.8 kg/s

12. a) ≈ 500 kg b) 0.15; resonance captures during slowing down c) $C > 1$; requires $\eta > 2$.

13. a) 3.7% b) $\eta = 1.88$, $k_\infty = 1.52$ c) Today's 0.72% is too little for light water (Problems 5 and 10).

14. b) $\Sigma_B = 0.10\,(\Sigma_F + \Sigma_{\mathrm{other}})$ c) $\approx 7.6 \times 10^{-4}$ ^{10}B atoms per U atom d) Concentration can be reduced gradually and uniformly.