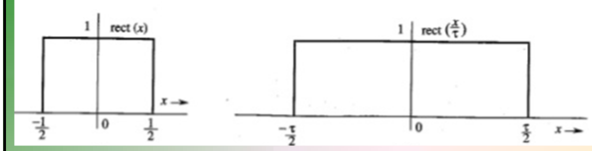
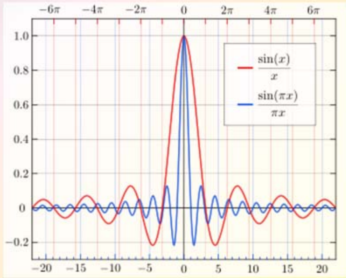
Funções úteis da óptica: rectângulo, sinc

$$\Pi\left(\frac{x}{\tau}\right) = rect\left(\frac{x}{\tau}\right) = \left\{ \begin{array}{ll} 0, & |x| > \frac{\tau}{2} \\ \frac{1}{2}, & |x| = \frac{\tau}{2} \\ 1, & |x| < \frac{\tau}{2} \end{array} \right\}$$



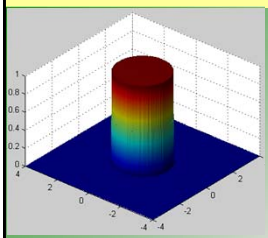


A TF de um rect é a função Seno Cardinal, sinc: $\mathrm{sinc}(x) = \frac{\sin(\pi x)}{\pi x}.$

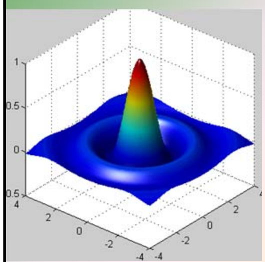
Um rectângulo 2D é um produto de rect:

$$\mathrm{rect}(ax)\,\mathrm{rect}(by) \qquad \frac{1}{|ab|}\,\mathrm{sinc}(f_X/a)\,\mathrm{sinc}(f_Y/b)$$

Funções úteis da óptica: cilindro *(circ), somb*



$$\mathrm{circ}(r) = \begin{cases} 1 & r < 1 \\ \frac{1}{2} & r = 1 \\ 0 & \text{otherwise} \end{cases}$$



A Transformada de Fourier em coordenadas esféricas (ρ,ϕ), para funções simétricas, é conhecida como transformada de Fourier-Bessel. Para a função *circ*:

$$\mathcal{B}\{\mathrm{circ}(r)\} = \frac{1}{2\pi\rho^2}\int_0^{2\pi\rho} r' J_0(r')\,dr' = \frac{J_1(2\pi\rho)}{\rho}$$

A TF de um circ tem a forma de um chapéu mexicano (Sombrero, *somb*)

Difracção de Fraunhofer: abertura rectangular

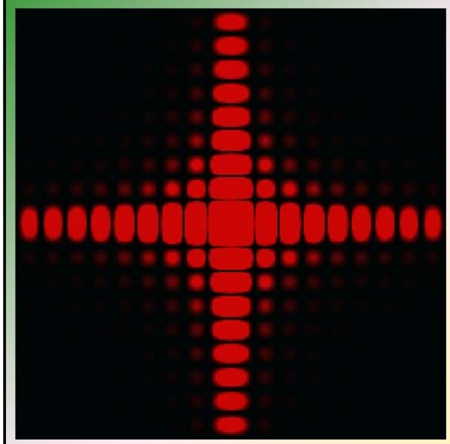
- Abertura, no plano z = 0: $t_A(\xi,\eta) = \mathrm{rect}\left(\frac{\xi}{2w_X}\right)\mathrm{rect}\left(\frac{\eta}{2w_Y}\right)$
- Iluminação: onda plana ao longo de Oz: $U_i(\xi,\eta) = 1$.
- Imediatamente depois da abertura: $U(\xi,\eta) = 1 \,.\, t_A(\xi,\eta)$
- Campo no infinito: $$U(x,y) = \frac{e^{jkz}e^{j\frac{k}{2z}(x^2+y^2)}}{j\lambda z}\mathcal{F}\{U(\xi,\eta)\}\Big|_{\substack{f_X = x/\lambda z \\ f_Y = y/\lambda z}}.$$
- TF: $$\mathcal{F}\{U(\xi,\eta)\} = A\,\mathrm{sinc}(2w_X f_X)\,\mathrm{sinc}(2w_Y f_Y)$$
- Campo no infinito: $$U(x,y) = \frac{e^{jkz}e^{j\frac{k}{2z}(x^2+y^2)}}{j\lambda z}A\,\mathrm{sinc}\left(\frac{2w_X x}{\lambda z}\right)\mathrm{sinc}\left(\frac{2w_Y y}{\lambda z}\right)$$
- Irradiância no infinito: $$I(x,y) = \frac{A^2}{\lambda^2 z^2}\mathrm{sinc}^2\left(\frac{2w_X x}{\lambda z}\right)\mathrm{sinc}^2\left(\frac{2w_Y y}{\lambda z}\right)$$

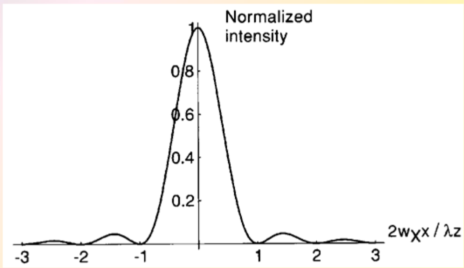
$A = 4w_X w_Y$

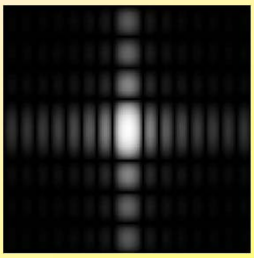
Difracção de Fraunhofer: abertura rectangular

$$t_A(\xi,\eta) = \mathrm{rect}\left(\frac{\xi}{2w_X}\right)\mathrm{rect}\left(\frac{\eta}{2w_Y}\right)$$

$$I(x,y) = \frac{A^2}{\lambda^2 z^2}\mathrm{sinc}^2\left(\frac{2w_X x}{\lambda z}\right)\mathrm{sinc}^2\left(\frac{2w_Y y}{\lambda z}\right)$$







Difracção de Fraunhofer: abertura circular

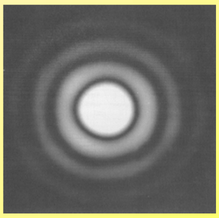
- Abertura de raio *w* (simetria radial): $\boxed{t_A(q) = \mathrm{circ}\left(\frac{q}{w}\right)}$

$q = \sqrt{\xi^2 + \eta^2}$

- Iluminação por onda plana: $U_i(\xi,\eta) = 1$.
- Campo no infinito: $U(r) = \frac{e^{jkz}}{j\lambda z} \exp\left(j\frac{kr^2}{2z}\right) \mathcal{B}\{U(q)\}\Big|_{\rho = r/\lambda z}$

$A = \pi w^2$

- Raio no plano de Fourier: $\rho = \sqrt{f_X^2 + f_Y^2}$
- Campo no infinito: $\boxed{U(r) = e^{jkz} e^{j\frac{kr^2}{2z}} \frac{A}{j\lambda z}\left[2\frac{J_1(kwr/z)}{kwr/z}\right]}$
- Irradiância no infinito: $\boxed{I(r) = \left(\frac{A}{\lambda z}\right)^2 \left[2\frac{J_1(kwr/z)}{kwr/z}\right]^2}$

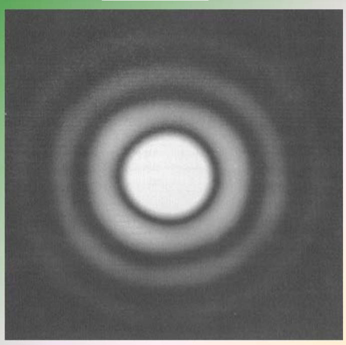


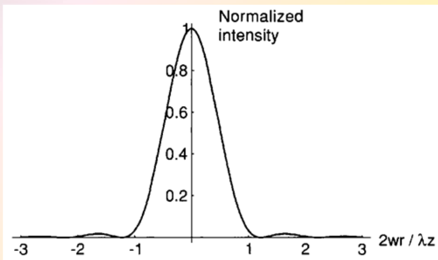
Difracção de Fraunhofer: abertura circular

$$\boxed{t_A(q) = \mathrm{circ}\left(\frac{q}{w}\right)}$$

$A = \pi w^2$

$$\boxed{I(r) = \left(\frac{A}{\lambda z}\right)^2 \left[2\frac{J_1(kwr/z)}{kwr/z}\right]^2}$$



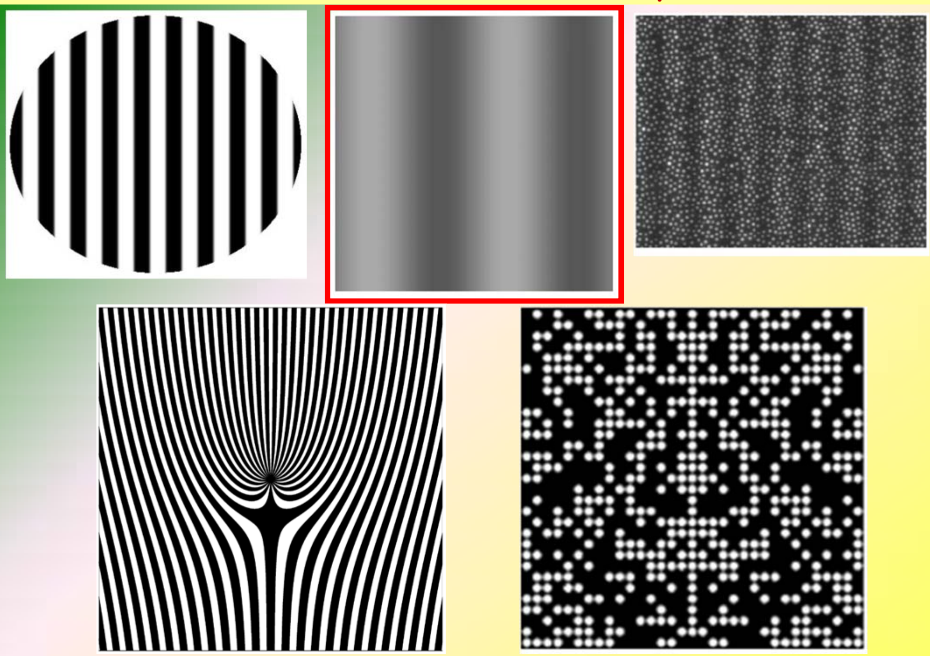


- **Diâmetro** do lobo central:

$$\boxed{d = 1.22\frac{\lambda z}{w}}$$

x	$\left[2\frac{J_1(\pi x)}{\pi x}\right]^2$	max, min
0	1	max
1.220	0	min
1.635	0.0175	max
2.233	0	min
2.679	0.0042	max
3.238	0	min
3.699	0.0016	max

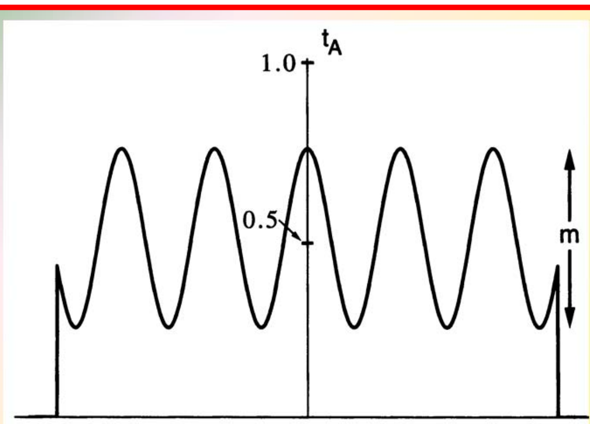
Redes de difracção de amplitude



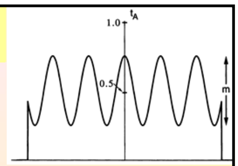
Redes de difracção de amplitude

- Rede periódica linear, segundo o eixo ξ, 2D, de frequência espacial f_0, modulação m, inscrita num quadrado de semi-largura w.
- Função de transmissão em amplitude:

$$t_A(\xi,\eta) = \left[\frac{1}{2} + \frac{m}{2}\cos(2\pi f_0 \xi)\right] \mathrm{rect}\left(\frac{\xi}{2w}\right) \mathrm{rect}\left(\frac{\eta}{2w}\right)$$



Redes de difracção de amplitude



Rede de difracção 1D $\quad t(\xi) = (1+\cos f_0\xi)\,\mathrm{rect}(\xi)$

Transformada de Fourier

$$T(f_X) = \left[\delta(f_X) + \frac{1}{2}\delta(f_X - f_0) + \frac{1}{2}\delta(f_X + f_0)\right] * \mathrm{sinc}(f_X)$$

$$T(f_X) = \mathrm{sinc}(f_X) + \frac{1}{2}\mathrm{sinc}(f_X - f_0) + \frac{1}{2}\mathrm{sinc}(f_X + f_0)$$

$$\cos(2\pi f_0 x) = \frac{e^{+i2\pi f_0 x} + e^{-i2\pi f_0 x}}{2}$$

$$\frac{1}{2}[\delta(f_X - f_0) + \delta(f_X + f_0)]$$

$$g \star\star\, \delta = \iint_{-\infty}^{\infty} g(x,y)\,\delta(\xi - x, \eta - y)\,dxdy$$

$$= g(\xi,\eta)$$

Usa-se um valor preciso da frequência espacial

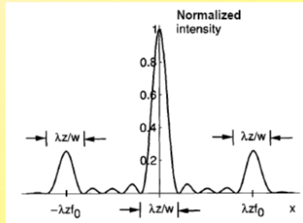
$f_X = \frac{x}{\lambda z} \to x \quad$ (se $\lambda z = 1$)

Amplitude complexa

$$U(x) = \frac{A}{i\lambda z}\left[\mathrm{sinc}(x) + \frac{1}{2}\mathrm{sinc}(x - f_0) + \frac{1}{2}\mathrm{sinc}(x + f_0)\right]$$

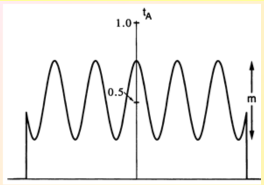
Irradiância, E=|U|² - sem sobreposição dos sinc's

$$I(x) = \left(\frac{A}{\lambda z}\right)^2\left[\mathrm{sinc}^2(x) + \frac{1}{4}\mathrm{sinc}^2(x - f_0) + \frac{1}{4}\mathrm{sinc}^2(x + f_0)\right]$$



Redes de difracção de amplitude

$$\boxed{t_A(\xi,\eta) = \left[\frac{1}{2} + \frac{m}{2}\cos(2\pi f_0\xi)\right]\mathrm{rect}\left(\frac{\xi}{2w}\right)\mathrm{rect}\left(\frac{\eta}{2w}\right)}$$



- Iluminação por onda plana: $U_i(\xi,\eta) = 1$.
- Campo no plano da rede (ξ,η), imediatamente depois da rede ($z=0^+$)
- Teorema da Convolução: $\quad \mathcal{F}\{gh\} = G(f_X,f_Y) \star\star\, H(f_X,f_Y)$
- Transformadas de Fourier dos dois factores:

$$\mathcal{F}\left\{\frac{1}{2} + \frac{m}{2}\cos(2\pi f_0\xi)\right\} = \frac{1}{2}\delta(f_X, f_Y) + \frac{m}{4}\delta(f_X + f_0, f_Y) + \frac{m}{4}\delta(f_X - f_0, f_Y)$$

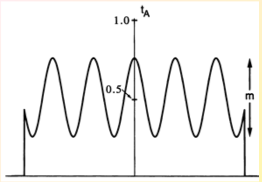
$$\mathcal{F}\left\{\mathrm{rect}\left(\frac{\xi}{2w}\right)\mathrm{rect}\left(\frac{\eta}{2w}\right)\right\} = A\,\mathrm{sinc}(2wf_X)\,\mathrm{sinc}(2wf_Y)$$

- O δ-Dirac é o **elemento neutro** da convolução:

$$g \star\star\, \delta = \iint_{-\infty}^{\infty} g(x,y)\,\delta(x-\xi, y-\eta)\,dxdy = g(\xi,\eta)$$

Redes de difracção de amplitude

$$t_A(\xi,\eta) = \left[\frac{1}{2} + \frac{m}{2}\cos(2\pi f_0 \xi)\right] \mathrm{rect}\left(\frac{\xi}{2w}\right) \mathrm{rect}\left(\frac{\eta}{2w}\right)$$



- Como o δ-Dirac é o elemento neutro da convolução, o campo no infinito é:

$$U(x,y) = \frac{A}{j2\lambda z} e^{jkz} e^{j\frac{k}{2z}(x^2+y^2)} \mathrm{sinc}\left(\frac{2wy}{\lambda z}\right) \left\{ \mathrm{sinc}\left(\frac{2wx}{\lambda z}\right) + \frac{m}{2}\mathrm{sinc}\left[\frac{2w}{\lambda z}(x + f_0\lambda z)\right] + \frac{m}{2}\mathrm{sinc}\left[\frac{2w}{\lambda z}(x - f_0\lambda z)\right] \right\}$$

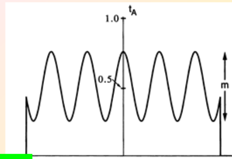
- Para se calcular a Irradiância no infinito, $I(x,y) \sim |U(x,y)|^2$. Se:

$f_0 \gg 1/w$ *(muitos ciclos de variação no interior da abertura)*

os três sinc's não se sobrepõem. Os produtos cruzados serão desprezáveis…

Redes de difracção de amplitude

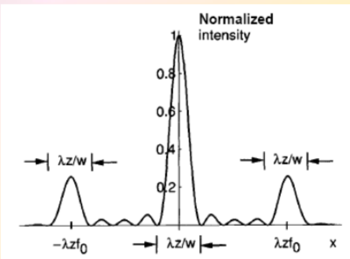
$$t_A(\xi,\eta) = \left[\frac{1}{2} + \frac{m}{2}\cos(2\pi f_0 \xi)\right] \mathrm{rect}\left(\frac{\xi}{2w}\right) \mathrm{rect}\left(\frac{\eta}{2w}\right)$$



- A Irradiância no infinito é:

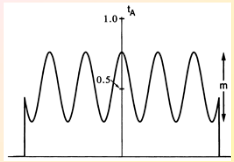
$$I(x,y) \approx \left[\frac{A}{2\lambda z}\right]^2 \mathrm{sinc}^2\left(\frac{2wy}{\lambda z}\right) \left\{ \mathrm{sinc}^2\left(\frac{2wx}{\lambda z}\right) + \frac{m^2}{4}\mathrm{sinc}^2\left[\frac{2w}{\lambda z}(x + f_0\lambda z)\right] + \frac{m^2}{4}\mathrm{sinc}^2\left[\frac{2w}{\lambda z}(x - f_0\lambda z)\right] \right\}$$

- Perfil segundo x:

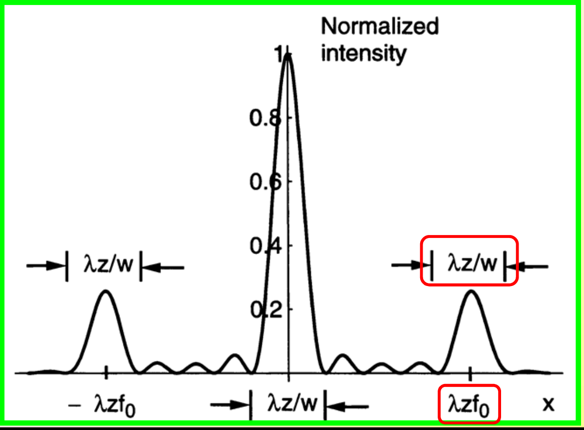


Redes de difracção de amplitude

$$t_A(\xi,\eta) = \left[\frac{1}{2} + \frac{m}{2}\cos(2\pi f_0 \xi)\right] \mathrm{rect}\left(\frac{\xi}{2w}\right) \mathrm{rect}\left(\frac{\eta}{2w}\right)$$



$$I(x,y) \approx \left[\frac{A}{2\lambda z}\right]^2 \mathrm{sinc}^2\left(\frac{2wy}{\lambda z}\right)\left\{\mathrm{sinc}^2\left(\frac{2wx}{\lambda z}\right) + \frac{m^2}{4}\mathrm{sinc}^2\left[\frac{2w}{\lambda z}(x + f_0\lambda z)\right] + \frac{m^2}{4}\mathrm{sinc}^2\left[\frac{2w}{\lambda z}(x - f_0\lambda z)\right]\right\}$$



Ordens de difracção

Separação entre ordens

Divisão do fluxo luminoso

Sensibilidade a λ → **Espectometria**

→ …

Redes de difracção de perfil rectangular (1D)

Rede

$$t(\xi) = \left[\mathrm{rect}\frac{\xi}{a} * \sum_{n=-\infty}^{\infty}\delta(\xi - nb)\right]\mathrm{rect}\frac{\xi}{c} = \left[\mathrm{rect}\frac{\xi}{a} * \frac{1}{b}\mathrm{comb}\frac{\xi}{b}\right]\mathrm{rect}\frac{\xi}{c}$$

Transformada de Fourier

$$T(f_X) = [a\,\mathrm{sinc}(af_X)\ \ \mathrm{comb}(bf_X)] * c\,\mathrm{sinc}(cf_X)$$

$$f(x)\delta(x-y) = f(y)(x-y)$$

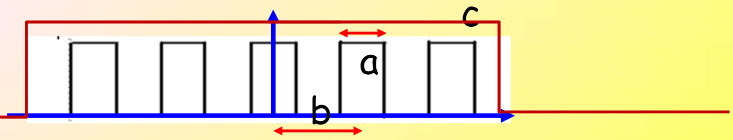
$$T(f_X) = \left[a\,\mathrm{sinc}(af_X)\ \frac{1}{b}\sum_{n=-\infty}^{\infty}\delta\left(f_X - \frac{n}{b}\right)\right] * c\,\mathrm{sinc}(cf_X)$$

$$\delta\left(\frac{x-y}{b}\right) = b\delta(x-y)$$

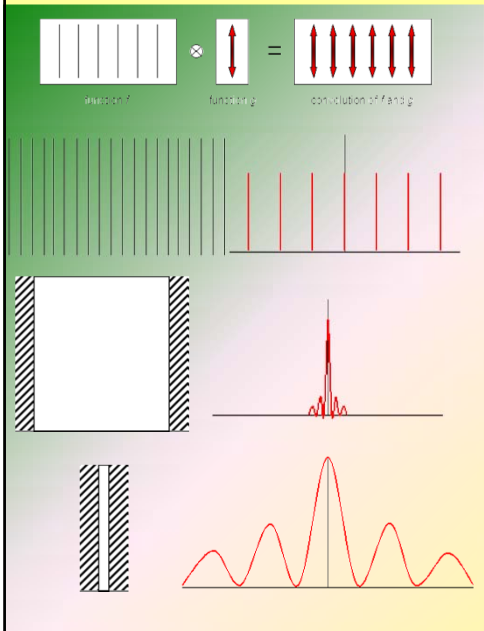
$$T(f_X) = \left[a\sum_{n=-\infty}^{\infty}\mathrm{sinc}\left(\frac{a}{b}n\right)\delta\left(f_X - \frac{n}{b}\right)\right] * c\,\mathrm{sinc}(cf_X)$$

$$T(f_X) = a\sum_{n=-\infty}^{\infty}\mathrm{sinc}\left(\frac{a}{b}n\right)\delta\left(f_X - \frac{n}{b}\right) * c\,\mathrm{sinc}(cf_X)$$

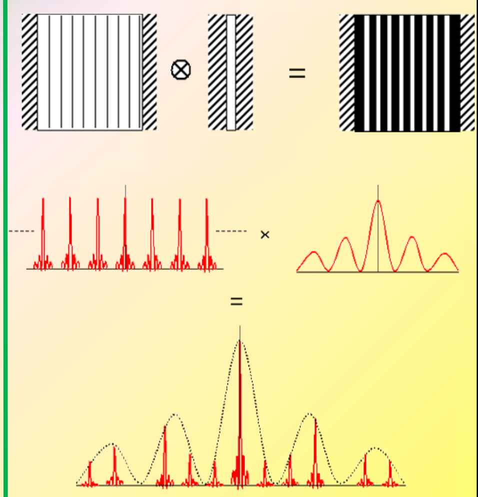
$$T(f_X) = ac\sum_{n=-\infty}^{\infty}\mathrm{sinc}\left(\frac{a}{b}n\right)\mathrm{sinc}\left[c\left(f_X - \frac{n}{b}\right)\right]$$



Redes de difracção de perfil rectangular (2D)

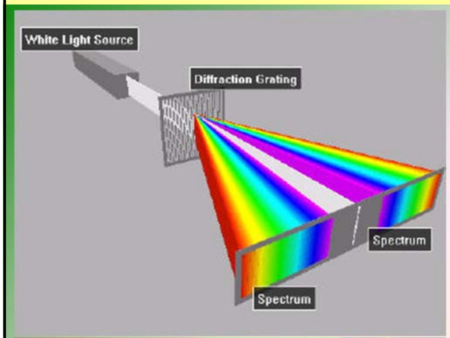


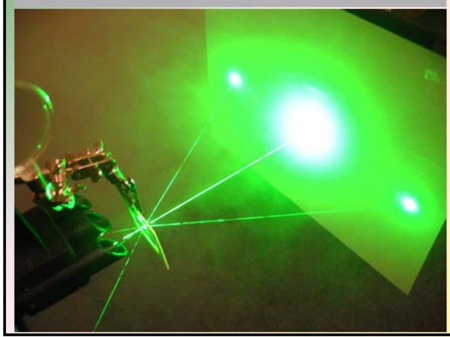
https://www.doitpoms.ac.uk/tlplib/diffraction/convolution.php

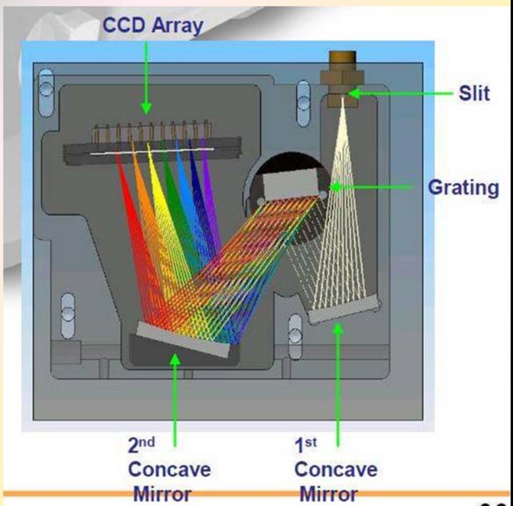


Diffraction pattern from a one dimensional grating with finite width slits. The pattern is comprised of a 'comb' function (from the array of slits), onto which a narrow sinc function is convoluted (from the finite width of the grating). The whole pattern is enveloped by a wide sinc function, shown as the dotted line, which arises as a result of the finite slit width.

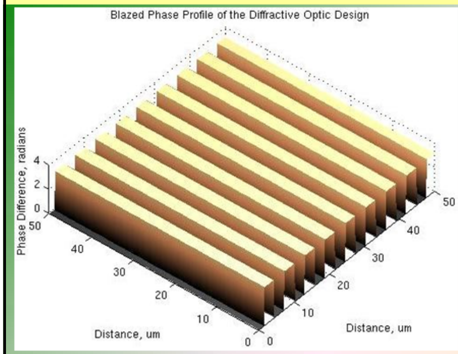
Redes de difracção e espectrómetros

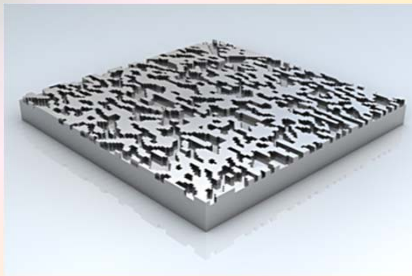


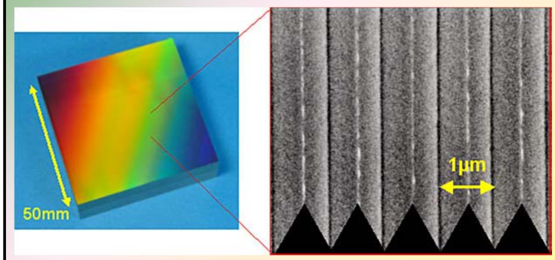


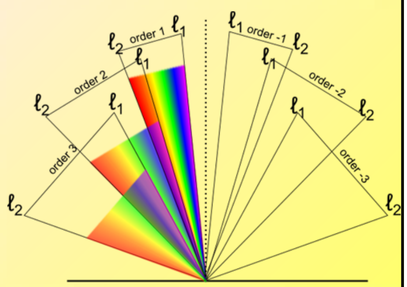


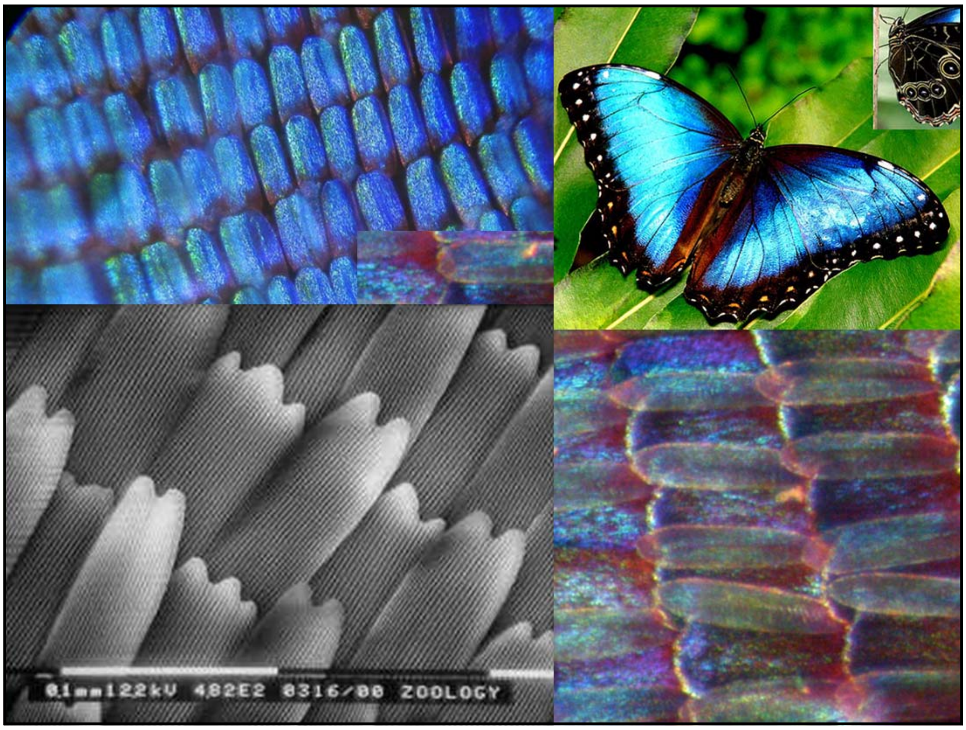
Outras redes de difracção de fase



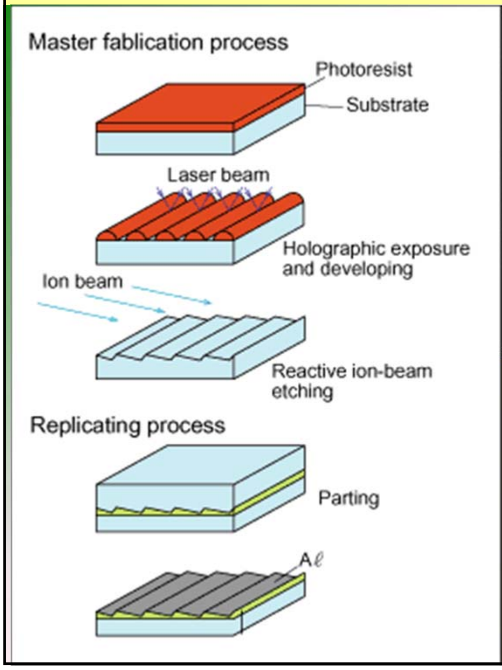


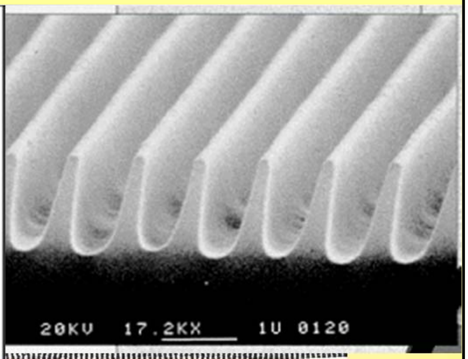






Redes de difracção de fase

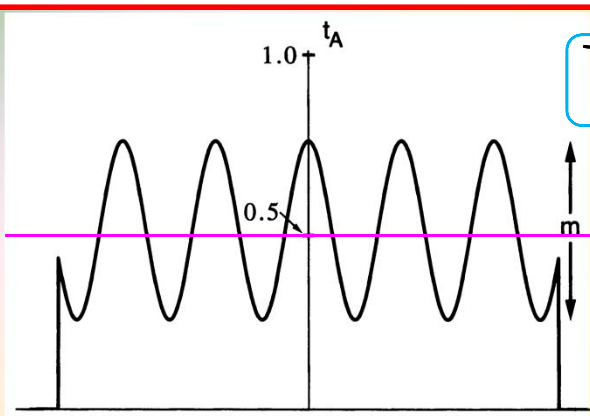




Redes de difracção de fase

- Rede de fase, periódica, linear, segundo o eixo ξ, 2D, de frequência espacial f_0, modulação m, inscrita num quadrado de semi-largura w.
- Função de transmissão em amplitude (complexa):

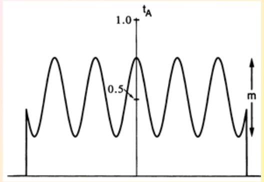
$$\boxed{t_A(\xi,\eta) = \exp\left[j\frac{m}{2}\sin(2\pi f_0 \xi)\right] \mathrm{rect}\left(\frac{\xi}{2w}\right) \mathrm{rect}\left(\frac{\eta}{2w}\right)}$$



Topografia da rede

Redes de difracção de fase

$$t_A(\xi,\eta) = \exp\left[j\frac{m}{2}\sin(2\pi f_0\xi)\right]\mathrm{rect}\left(\frac{\xi}{2w}\right)\mathrm{rect}\left(\frac{\eta}{2w}\right)$$



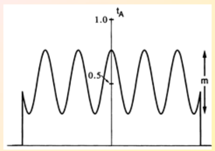
- Iluminação por onda plana: $U_i(\xi,\eta) = 1$.
- Campo no plano da rede (ξ,η), imediatamente depois da rede ($z=0^+$)
- Teorema da Convolução: $\mathcal{F}\{gh\} = G(f_X,f_Y) \star\star H(f_X,f_Y)$
- Reformulação conveniente: $\exp\left[j\frac{m}{2}\sin(2\pi f_0\xi)\right] = \sum_{q=-\infty}^{\infty} J_q\left(\frac{m}{2}\right)\exp(j2\pi q f_0\xi)$
- Transformadas de Fourier dos dois factores:

$$\mathcal{F}\left\{\mathrm{rect}\left(\frac{\xi}{2w}\right)\mathrm{rect}\left(\frac{\eta}{2w}\right)\right\} = A\,\mathrm{sinc}(2wf_X)\,\mathrm{sinc}(2wf_Y)$$

$$\mathcal{F}\left\{\exp\left[j\frac{m}{2}\sin(2\pi f_0\xi)\right]\right\} = \sum_{q=-\infty}^{\infty} J_q\left(\frac{m}{2}\right)\delta(f_X - qf_0, f_Y)$$

Redes de difracção de fase

$$t_A(\xi,\eta) = \exp\left[j\frac{m}{2}\sin(2\pi f_0\xi)\right]\mathrm{rect}\left(\frac{\xi}{2w}\right)\mathrm{rect}\left(\frac{\eta}{2w}\right)$$



- Combinando:

$$\begin{aligned}\mathcal{F}\{U(\xi,\eta)\} &= \mathcal{F}\{t_A(\xi,\eta)\}\\ &= [A\,\mathrm{sinc}(2wf_X)\,\mathrm{sinc}(2wf_Y)] \otimes \left[\sum_{q=-\infty}^{\infty} J_q\left(\frac{m}{2}\right)\delta(f_X - qf_0, f_Y)\right]\\ &= \sum_{q=-\infty}^{\infty} A J_q\left(\frac{m}{2}\right)\mathrm{sinc}\,[2w(f_X - qf_0)]\,\mathrm{sinc}(2wf_Y).\end{aligned}$$

$$U(x,y) = \frac{A}{j\lambda z}e^{jkz}e^{j\frac{k}{2z}(x^2+y^2)}\sum_{q=-\infty}^{\infty} J_q\left(\frac{m}{2}\right)\mathrm{sinc}\left[\frac{2w}{\lambda z}(x - qf_0\lambda z)\right]\mathrm{sinc}\left(\frac{2wy}{\lambda z}\right)$$

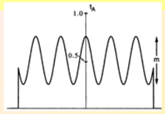
- Para se calcular a Irradiância no infinito, $I(x,y) \sim |U(x,y)|^2$. assume-se novamente que:

  $f_0 \gg 1/w$ *(muitos ciclos de variação no interior da abertura)*

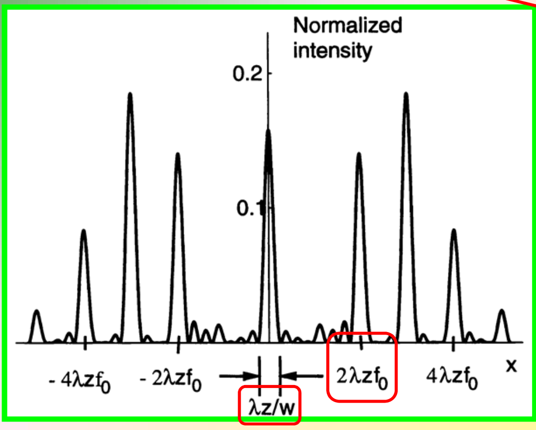
  e os inúmeros sinc's não se sobrepõem. Os produtos cruzados são desprezáveis…
- Logo, a Irradiância será:

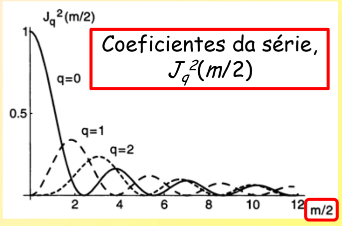
Redes de difracção de fase

$$t_A(\xi,\eta) = \left[\frac{1}{2} + \frac{m}{2}\cos(2\pi f_0 \xi)\right] \mathrm{rect}\left(\frac{\xi}{2w}\right) \mathrm{rect}\left(\frac{\eta}{2w}\right)$$



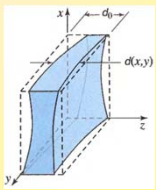
$$I(x,y) \approx \left(\frac{A}{\lambda z}\right)^2 \sum_{q=-\infty}^{\infty} J_q^2\left(\frac{m}{2}\right) \mathrm{sinc}^2\left[\frac{2w}{\lambda z}(x - q f_0 \lambda z)\right] \mathrm{sinc}^2\left(\frac{2wy}{\lambda z}\right)$$





Função de Transmissão em Amplitude (FTA)

1. FONTE GERA UMA ONDA QUE SE PROPAGA, EM **ZZ,** ATÉ AO **OBJECTO**, QUE SE ENCONTRA EM **Z = 0:** $U^-(x,y) = |U^-| \exp(i\varphi^-)$
2. O OBJECTO ALTERA A ONDA INCIDENTE **($Z=0^-$)**, GERANDO-SE UMA ONDA EMERGENTE **($Z=0^+$)**, $U^+(x,y) = |U^+| \exp(i\varphi^+)$
3. A ONDA EMERGENTE CONTINUA A PROPAGAR-SE, DE ACORDO COM O PRINCÍPIO DE H-F, ALTERADA COM AS CARACTERÍSTICAS DO OBJECTO.



Como se descreve o objecto, de forma a calcular a onda emergente?

Considera-se o objecto inscrito num paralelepípedo, com faces planas.

Define-se a **Função de Transmissão em Amplitude**, $t(x,y)$, [FTA] tal que:

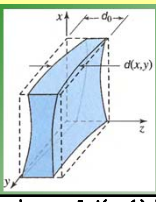
$$U^+(x,y) = t(x,y)\, U^-(x,y)$$

$$|U^+(x,y)| = |t(x,y)|\ |U^-(x,y)| \quad \text{e} \quad \varphi^+(x,y) = \phi_0(x,y) + \varphi^-(x,y)$$

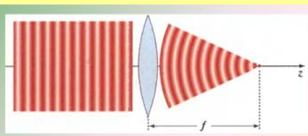
A FTA pode ser de:

Fase	$\lvert t\rvert = t_0$	Apenas varia a fase de U^-
Amplitude	$\phi = \phi_0$	Apenas varia o módulo de U^-
Híbrida	$t(x,y) = \lvert t(x,y)\rvert\, e^{i\phi(x,y)}$	

Exemplos de FTA's

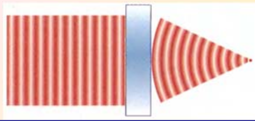


$t(x,y) = h_0 \exp[-i(n-1)\, k_0\, d(x,y)]$

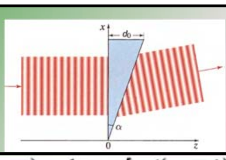


$t(x,y) \approx h_0 \exp\left[-jk_o \frac{x^2+y^2}{2f}\right]$

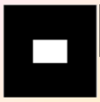
$\frac{1}{f} = (n-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$



$t(x,y) = h_0 \exp[-i\, n(x,y)\, k_0\, d_0]$

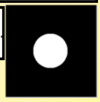


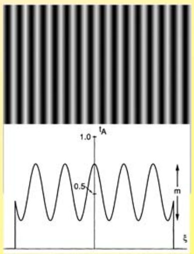
$t(x,y) = h_0 \exp[-j(n-1)k_o \alpha x]$



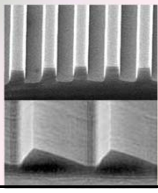
$t_A(\xi,\eta) = \mathrm{rect}\left(\frac{\xi}{2w_X}\right)\mathrm{rect}\left(\frac{\eta}{2w_Y}\right)$

$t_A(q) = \mathrm{circ}\left(\frac{q}{w}\right)$



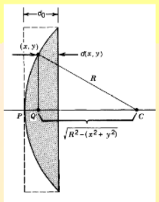


$t_A(\xi,\eta) = \left[\frac{1}{2} + \frac{m}{2}\cos(2\pi f_0 \xi)\right]\mathrm{rect}\left(\frac{\xi}{2w}\right)\mathrm{rect}\left(\frac{\eta}{2w}\right)$



$t_A(\xi,\eta) = \exp\left[j\frac{m}{2}\sin(2\pi f_0 \xi)\right]\mathrm{rect}\left(\frac{\xi}{2w}\right)\mathrm{rect}\left(\frac{\eta}{2w}\right)$

Lente delgada



Função de transmissão em amplitude (FTA) geral

$t(x,y) = h_0 \exp[-i(n-1)k_0 d(x,y)]$

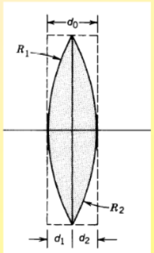
Espessura de uma lente *plano-convexa* é

$$d(x,y) = d_0 - \left\{R - \left[R^2 - (x^2+y^2)\right]^{1/2}\right\}$$

Aproximação paraxial (plano-convexa) $x^2+y^2 \ll R^2$ e espessura

$$\left[R^2-(x^2+y^2)\right]^{1/2} = R\left(1-\frac{x^2+y^2}{R^2}\right)^{1/2} \approx R\left(1-\frac{x^2+y^2}{2R^2}\right)$$

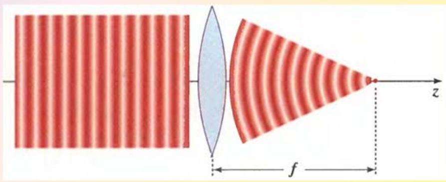
$$d(x,y) \approx d_0 - \frac{x^2+y^2}{2R}$$



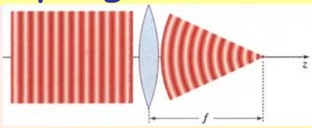
FTA (lente completa) $t(x,y) = h_0 e^{-ik\frac{x^2+y^2}{2f}}$

Com $\frac{1}{f} = (n-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$ (**distância focal** de uma lente delgada)

Uma lente positiva de distância focal **f** transforma ondas planas em ondas paraboloidais convergentes para o foco.



Resolução: critério de Rayleigh



A lente (de raio ρ) difracta a onda incidente:

- Iluminação por onda plana: $U_i(\xi,\eta) = 1$.
- FTA da lente: $t(\xi,\eta) = h_0 e^{-ik\frac{\xi^2+\eta^2}{2f}} . \mathrm{circ}\left(\frac{\sqrt{\xi^2+\eta^2}}{\rho}\right)$
- Campo imediatamente depois da lente ($z=0^+$): $U(\xi,\eta) = U_i(\xi,\eta) \times t(\xi,\eta)$
- Propagação para o plano focal da lente ($z = f$) (condições de Fresnel):

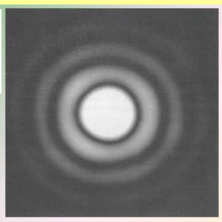
$$U(x,y) = \frac{e^{jkz}}{j\lambda z} e^{j\frac{k}{2z}(x^2+y^2)} \iint_{-\infty}^{\infty} \left\{ U(\xi,\eta) e^{j\frac{k}{2z}(\xi^2+\eta^2)} \right\} e^{-j\frac{2\pi}{\lambda z}(x\xi+y\eta)} d\xi\, d\eta,$$

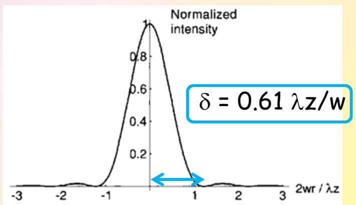
$$\boxed{U(x,y) = \frac{e^{ikz} e^{-i\frac{k}{2f}(x^2+y^2)}}{i\lambda f} \mathcal{F}\{circ\}_{f_X=\frac{x}{\lambda f},\, f_Y=\frac{y}{\lambda f}}}$$

- Situação normal de formação de imagem num telescópio!
- No plano focal, a irradiância é descrita pelo quadrado do módulo da TF da pupila:
  - Uma estrela → um padrão de Airy
  - Duas estrelas angularmente próximas: dois padrões de Airy, porventura sobrepostos.
- Duas estrelas são resolvidas se a separação entre os dois padrões de Airy satisfizer o **critério de Rayleigh**!

Resolução: critério de Rayleigh

$$I(r) = \left(\frac{A}{\lambda z}\right)^2 \left[2\frac{J_1(kwr/z)}{kwr/z}\right]^2$$





Raio w, Diâmetro $D = 2w$

Diâmetro do lobo central:

$$\boxed{d = 1.22\frac{\lambda z}{w}}$$

- **Critério de resolução de Rayleigh: duas fontes incoerentes podem ser resolvidas por um sistema limitado por difracção e com uma pupila circular quando o centro do padrão de irradiância de Airy de uma coincidir com o primeiro zero do padrão de Airy da outra.**
- O 1º zero de J_1 ocorre para $\pi x = 1.22$.
- A separação mínima radial no plano imagem ($z=f$) é metade da largura do lobo central (d) do padrão de Airy:

  $\delta = 0.61\ \lambda f/w \rightarrow 1.22\ \lambda f/D$

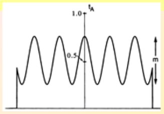
- A separação angular (no espaço objecto) será

  $\theta = 1.22\ \lambda/D$ (rad)

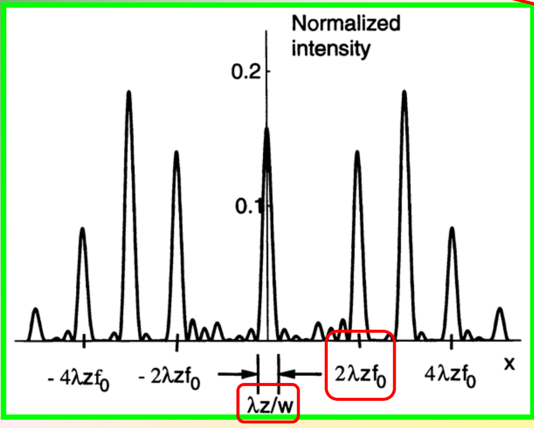
x	$\left[2\frac{J_1(\pi x)}{\pi x}\right]^2$	max, min
0	1	max
1.220	0	min
1.635	0.0175	max
2.233	0	min
2.679	0.0042	max
3.238	0	min
3.699	0.0016	max

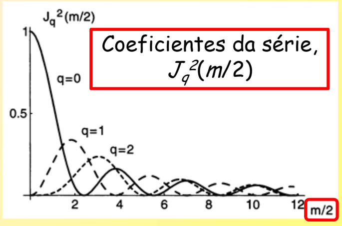
Redes de difracção de fase

$$t_A(\xi,\eta) = \left[\frac{1}{2} + \frac{m}{2}\cos(2\pi f_0 \xi)\right] \mathrm{rect}\left(\frac{\xi}{2w}\right) \mathrm{rect}\left(\frac{\eta}{2w}\right)$$



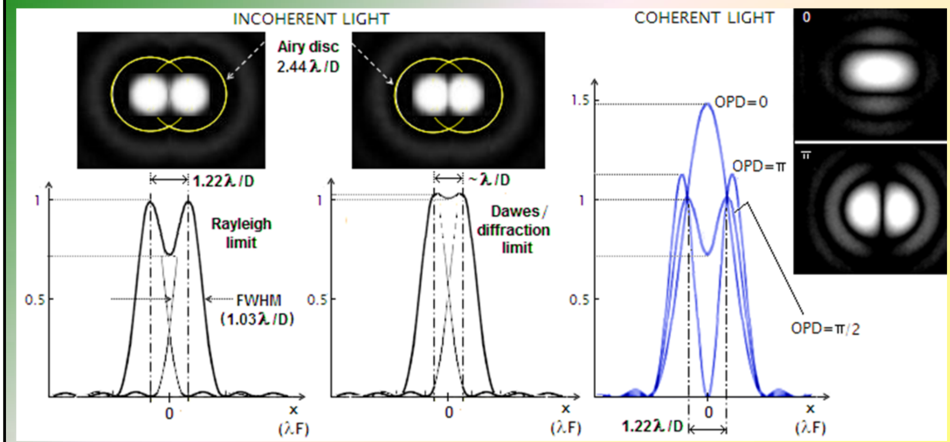
$$I(x,y) \approx \left(\frac{A}{\lambda z}\right)^2 \sum_{q=-\infty}^{\infty} J_q^2\left(\frac{m}{2}\right) \mathrm{sinc}^2\left[\frac{2w}{\lambda z}(x - q f_0 \lambda z)\right] \mathrm{sinc}^2\left(\frac{2wy}{\lambda z}\right)$$

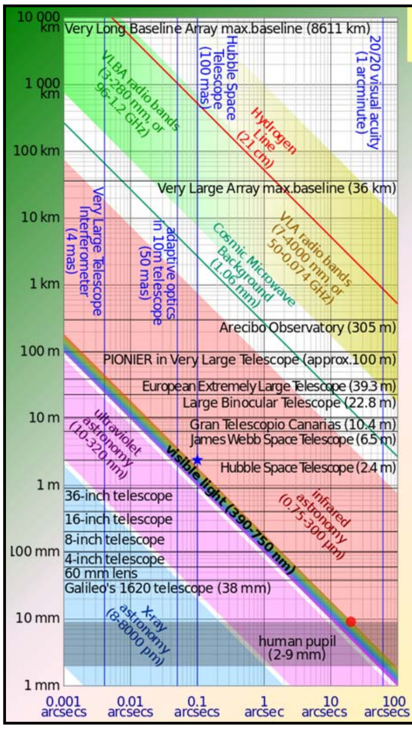




Resolução: casos incoerente e coerente

➢ .





Resolução: critério de Rayleigh

Aperture diameter *vs* **angular resolution** at diffraction limit, for various λ.

Examples:

★ - Hubble Space Telescope is almost diffraction-limited in the visible, at **0.1”**

● - human eye should have a resolving power of **20”** in theory (60”, in practice).