

UNIVERSO PRIMITIVO
Mestrado em Física Astronomia 2025-2026

Exercise Sheet 2

1. Convert kilogram (kg), second (s), meter (m) and kelvin (K) to giga-electron volt (GeV), assuming natural units $c = \hbar = k_B = 1$. Use your findings to express your weight, age, height and body temperature in GeV.
2. Derive expressions for the number density, energy density and pressure of a gas of ultra-relativistic particles in thermal equilibrium with vanishing chemical potential.
3. Show that the energy density and pressure of non-relativistic particles with vanishing chemical potential is given by $\rho = (m + \frac{3}{2}T) n$ and $P = nT$, respectively. Explain why in these conditions one has $P \ll \rho$.
4. Consider a thermal equilibrium distribution of relativistic particles with non-vanishing chemical potential μ . Compute the number density, energy and pressure for:
 - 4.1. Degenerate fermions with $\mu \gg T$;
 - 4.2. $\mu < 0$ and $|\mu| < T$[Hint: in 4.1 assume that for degenerate fermions all energy states are occupied up to a maximum energy equal to μ .]
5. Consider now the case of the non-relativistic limit with a non-vanishing chemical potential. Prove the expressions below. The overbar denotes densities for anti-particles. Assume that particles and anti-particles are in chemical equilibrium. (Regarding 5.2 and 5.3: note that in general there can be excess of particles over antiparticles):
 - 5.1. $n = g \left(\frac{mT}{2\pi}\right)^{3/2} \exp\left(-\frac{m-\mu}{T}\right)$
 - 5.2. $n - \bar{n} = 2g \left(\frac{mT}{2\pi}\right)^{3/2} \exp\left(-\frac{m}{T}\right) \sinh\left(\frac{\mu}{T}\right)$
 - 5.3. $\rho + \bar{\rho} = 2gm \left(\frac{mT}{2\pi}\right)^{3/2} \exp\left(-\frac{m}{T}\right) \cosh\left(\frac{\mu}{T}\right)$