

# Cosmologia Física

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# **Introduction**

**A physical model for the Universe**

# Physical Cosmology

describing the physical model of the Universe

## Everything

The universe is all of space and time and their contents.

It comprises all fundamental interactions, physical processes and physical constants, and therefore all forms of energy and matter, and the structures they form.

→ two fundamental constraints in the study of the Universe:

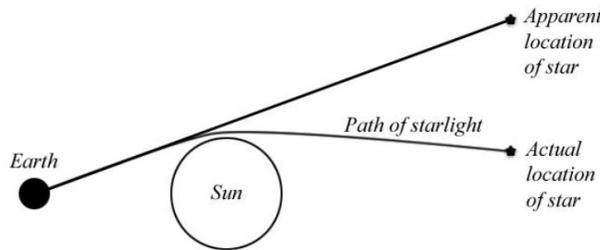
→ finite speed of information propagation → we have only access to part of the Universe - our **lightcone**.

→ we only observe one Universe → we cannot make laboratory experiments, test results in different conditions, or get statistics (possible only under an approximation, the **ergodic hypothesis**) → fundamental limitation – **cosmic variance**.

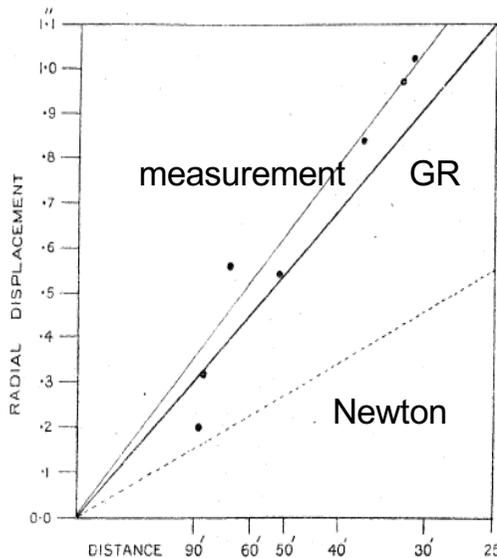
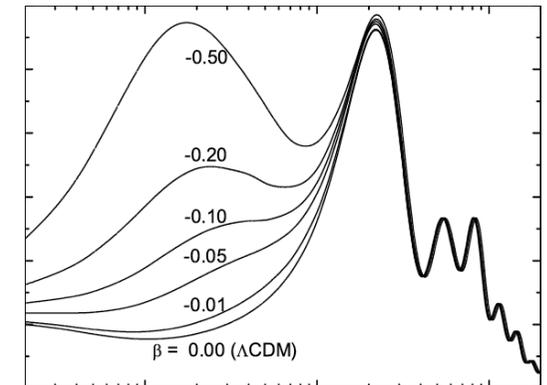
# + Gravity

is the force that drives the cosmological evolution and describes large-scale interactions, because among the 4 fundamental forces strong and weak forces have short range and the Universe is neutral.

It is tested on various scales:



1987 - today



1919

(Eddington)

Gravity is described by **General Relativity**

→ **metric**

→ Einstein equations

Awarded to **Albert Einstein** "for his services to Theoretical Physics, (and especially for his discovery of the law of the photoelectric effect)." (1/1)



1921

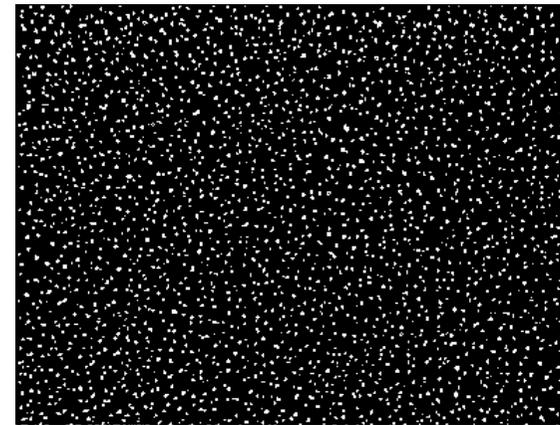
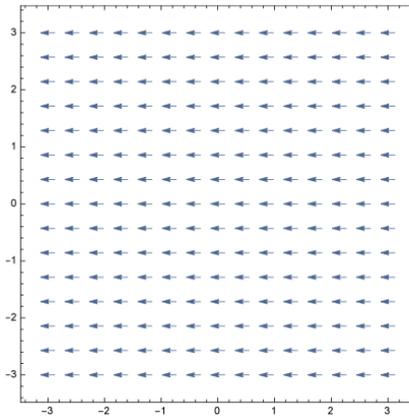
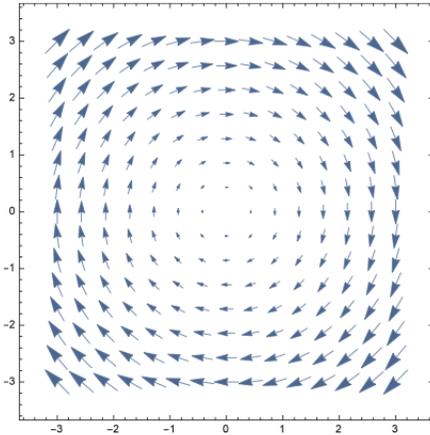
## + Cosmological principle

Isotropy :

the Universe observed in any direction looks the same → rotational invariance

Homogeneity:

the Universe is identical in all points, at each instant → translational invariance

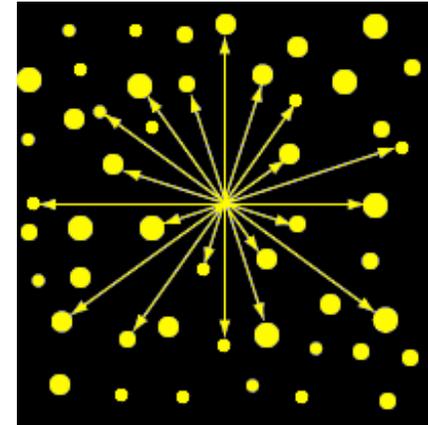


→ metric is Robertson-Walker, **spherically symmetric** with two degrees of freedom:  $a$ ,  $K$  → and two related cosmological parameters:  $H_0$ ,  $\Omega_K$

## + Olbers paradox

→ universal loss of luminosity → **redshift**

→ scale factor “a” must **evolve**



## + Observations of the recession of galaxies

→ Expansion **Big Bang theory**

→ Thermal history

→ Nucleosynthesis

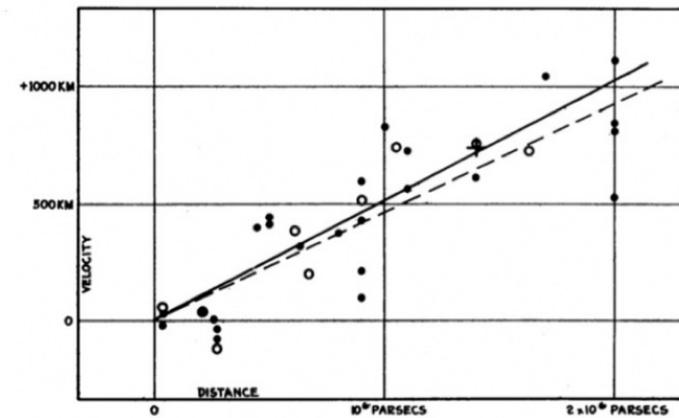


FIGURE 1  
Velocity-Distance Relation among Extra-Galactic Nebulae.

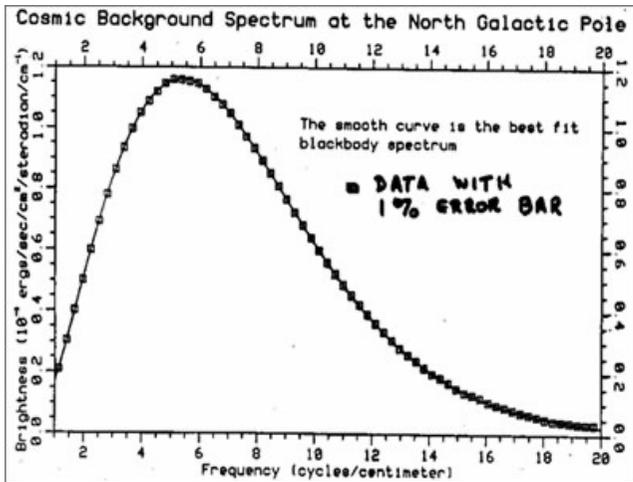
(Hubble)

1929

→ Existence of a universal background radiation:

the **cosmic background radiation, CMB** [ $z \sim 1100$ ]

1965 - 1990



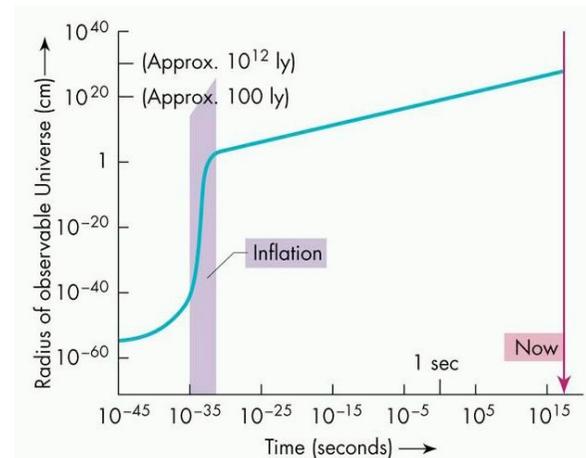
(COBE, 1990)

Awarded to **Arno A. Penzias and Robert W. Wilson** "for their discovery of cosmic microwave background radiation." (1/4 + 1/4)



→ Horizon, flatness and coincidence problems

Solved by **Inflation**



## + Observation of anisotropies in the CMB

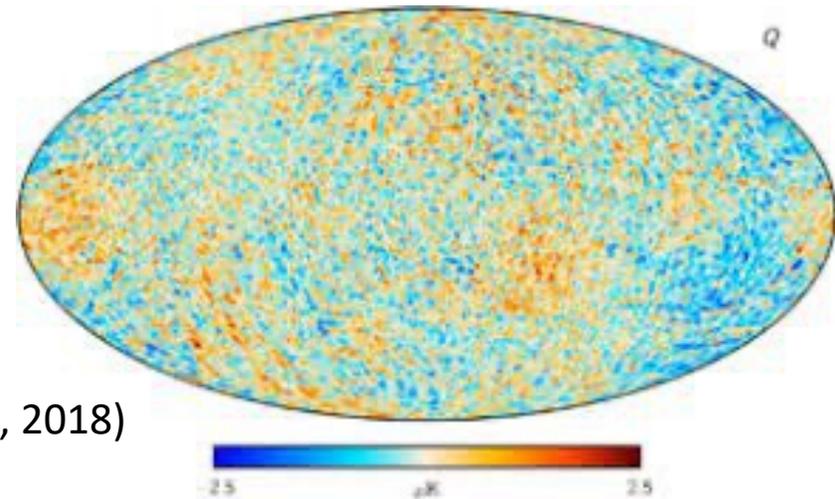
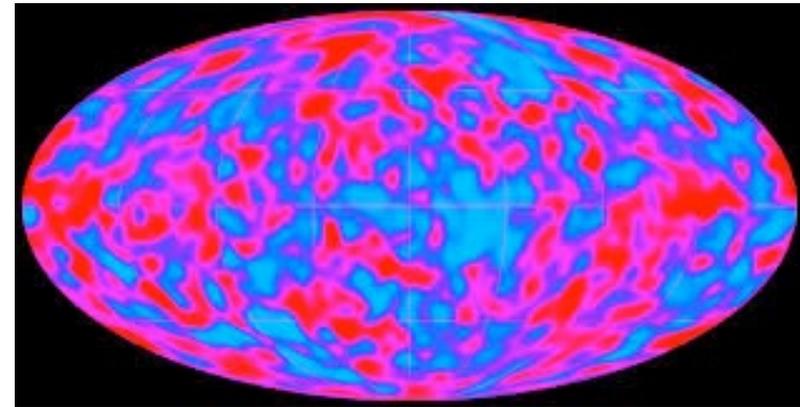
1992 - 2018

(COBE, 1992)

→ Existence of perturbations to the cosmological principle → found the **seeds of structure**

→ Problem of the origin of the seeds of structure

Solved by the mechanism of **quantum fluctuations** + inflation + **gravitational interaction**



(Planck, 2018)

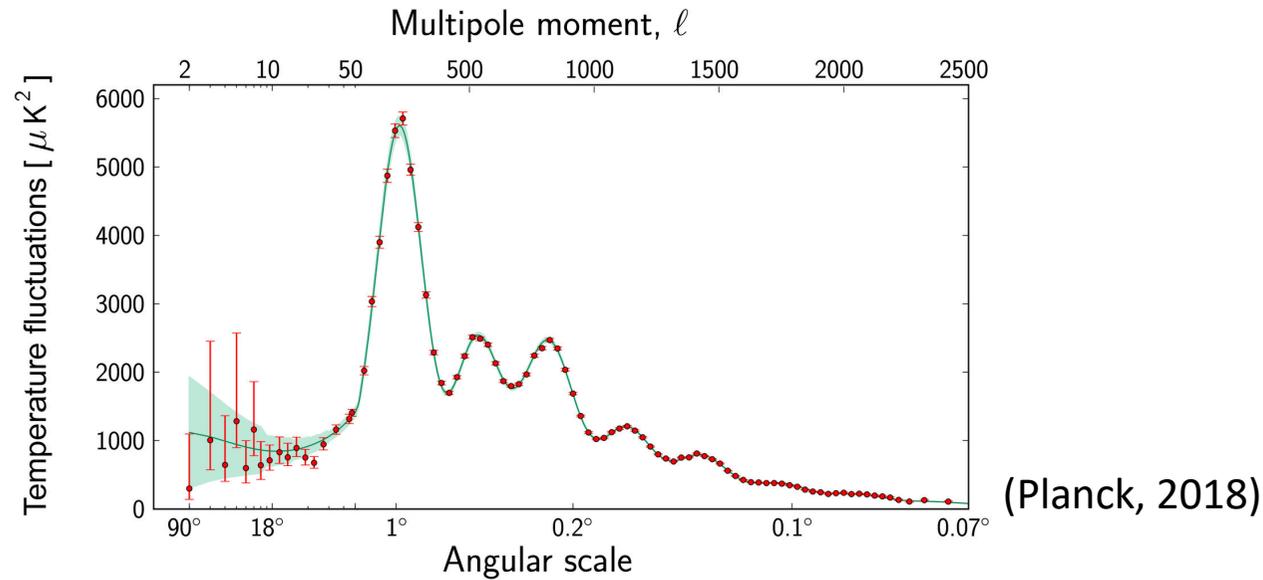
Awarded to **John C. Mather** and **George F. Smoot** "for their discovery of the blackbody form and anisotropy of the cosmic microwave background radiation." (1/2 + 1/2)



## + Measurement of the anisotropies in the CMB

1992 - 2018

Their amplitude is very small  $\delta_T \sim 10^{-5}$



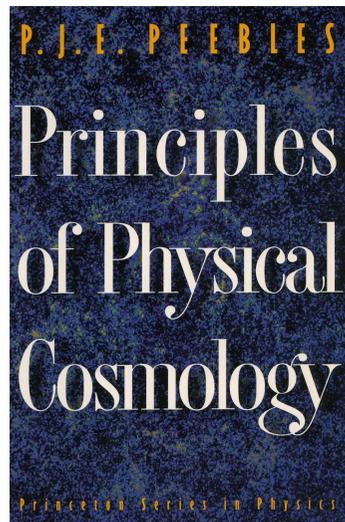
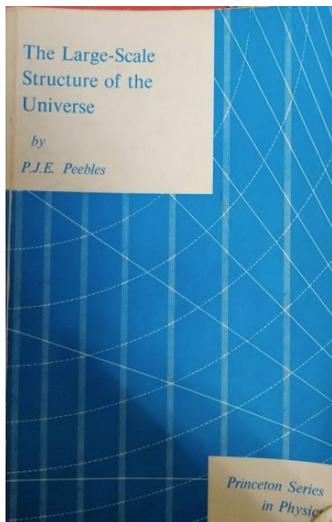
→ **Isotropy** confirmed (wide angular bins, i.e., on large angular scales)

→ indicates very small clustering at  $z=1100 \rightarrow \delta_b(z=1100) \sim 10^{-5}$

- + Gravitational collapse is slow ( $\delta_b$  grows only a factor  $\sim 10^3$  until  $z=0$ )
- + Today there are structures with large density contrast  $\delta$  (large clustering at  $z=0$ )

→ Problem of the mechanism of **structure formation**

Solved by the hypothesis of the existence of an extra component in the cosmological fluid - **Dark matter** → **CDM model**



Awarded to **P. James. E. Peebles** "for theoretical discoveries in physical cosmology" (1/2)



## + Structure formation

### Linear (gravitational clustering)

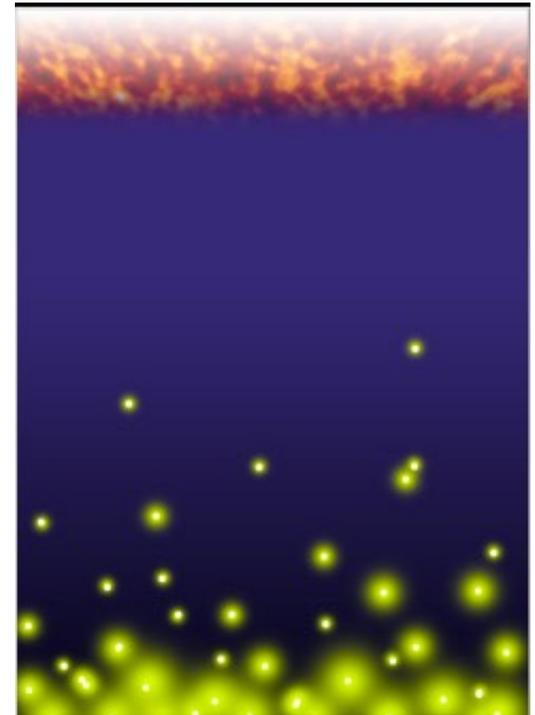
→ There is a very long process of **linear** clustering, during the **dark ages** and beyond. In some points, local gravitational fields start to become strong.

### Non-linear (collapse)

→ Formation of **dark matter halos**

**Non-linear** collapse of baryonic matter on those halos → neutral Hydrogen HI clouds condense and form the **first stars**, ending the dark ages → **Cosmic Dawn** [ $z \sim 20$ ]

Meanwhile, in some parts (or scales) of the Universe linear structure formation continues its slow process.



## + Galaxy formation

→ The gravitational collapse does not describe all aspects of structure formation.

Non-gravitational effects associated to the baryonic matter start to be important at this stage:

**Cooling** - the gas has to cool-down to condense. By losing pressure it falls into the center of the halo where it can form stars. Angular momentum conservation during the fall produces a disk → spiral galaxies

**Feedback** - the quantity of cold gas available decreases by influence of the environment

**Mergers** - frequent interactions between halos may form elliptical galaxies from primitive spiral galaxies.



The first galaxies [ $z \sim 15?$ ] led to the **Reionization** era [ $z \sim 10$ ]

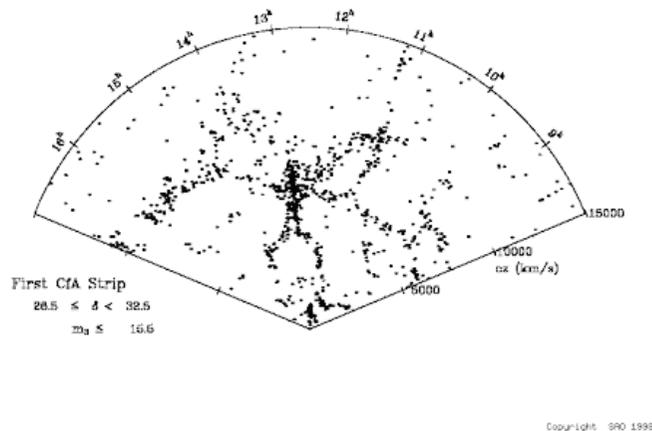
→ New radiation ionizes the HI clouds, forming ionized Hydrogen regions HII - the **reionization** of the Universe

## + Observation of the LSS of luminous matter in 2D and 3D

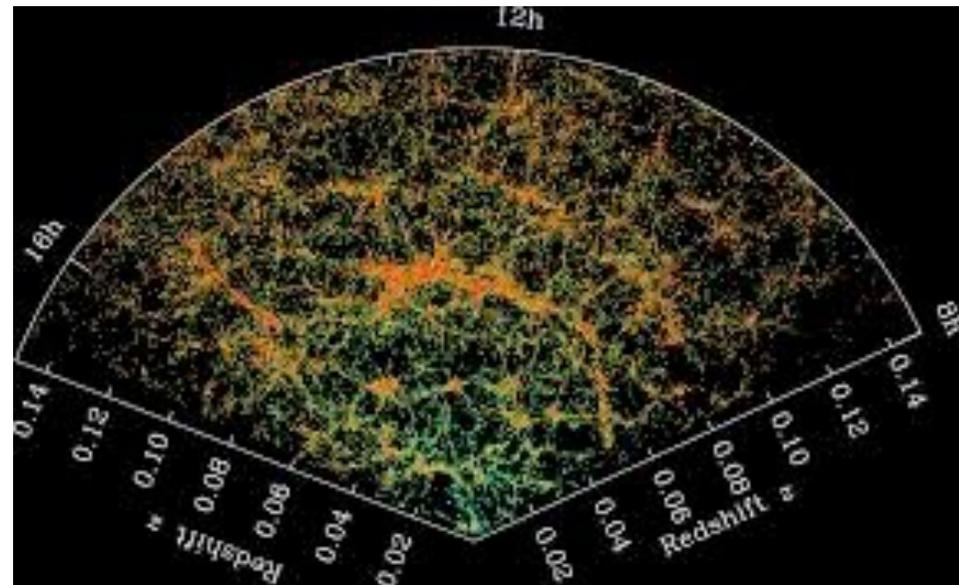
Since the epoch of reionization, galaxies have been forming and evolving, with an intense period of star formation occurring at the **Cosmic Noon** [ $z \sim 2-3$ ]

Mapping the galaxies positions gives (biased) information on the **cosmic web** - the DM **large-scale structure** of the Universe - by using **galaxy clustering** methods and **redshift space distortions** → find out the details of structure formation

1980 - today



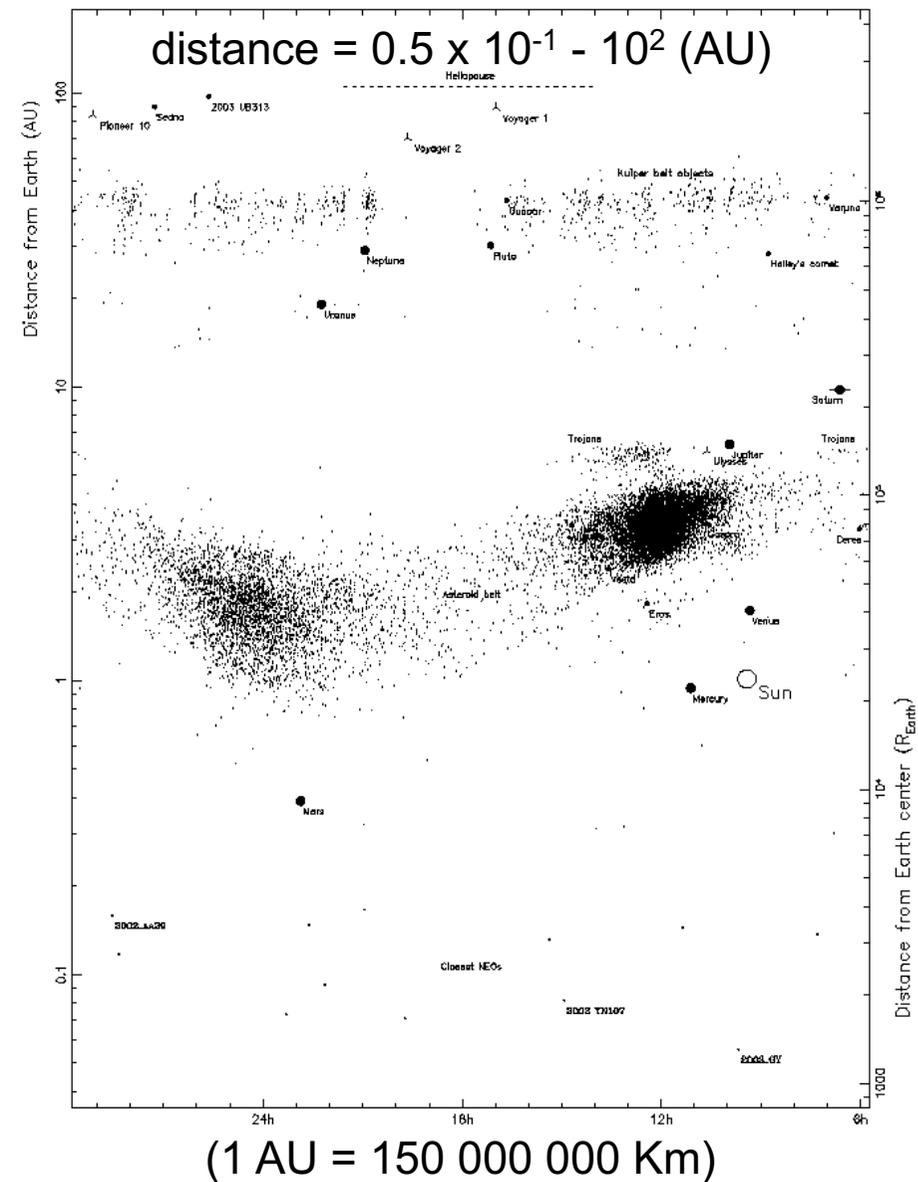
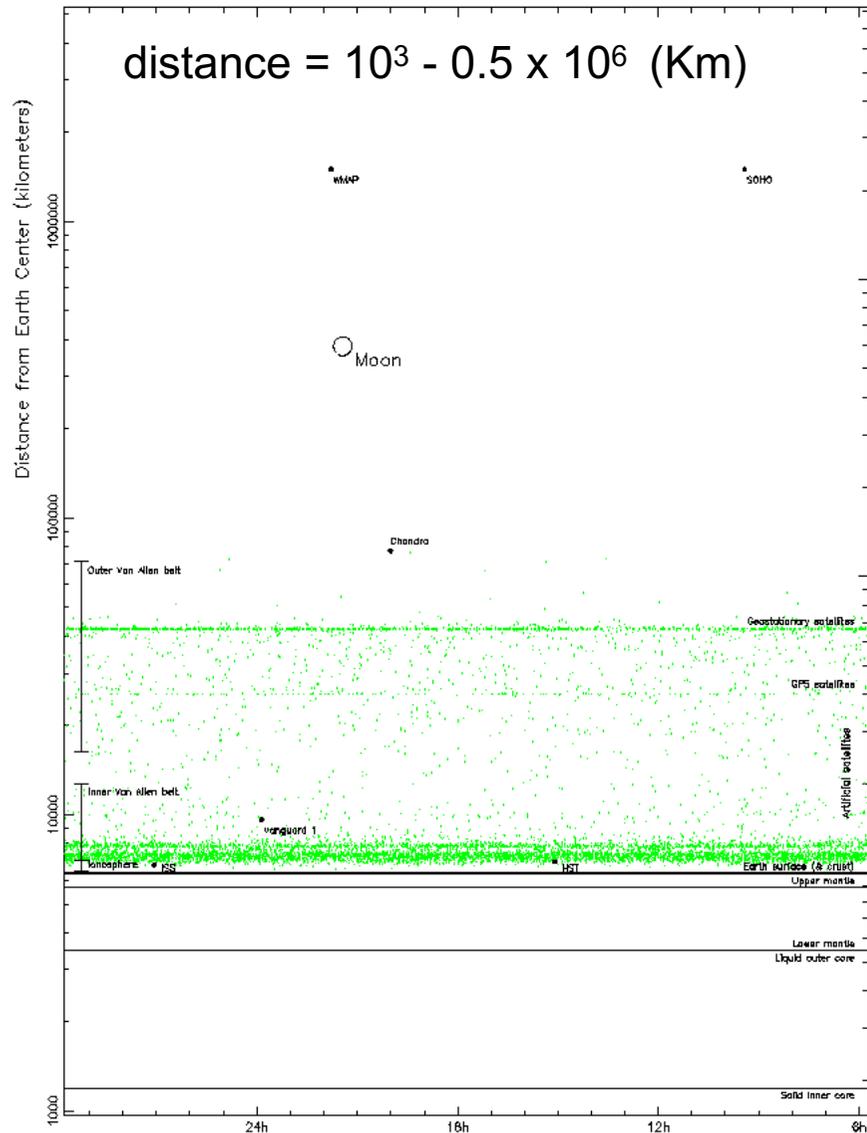
(CfA, 1980)



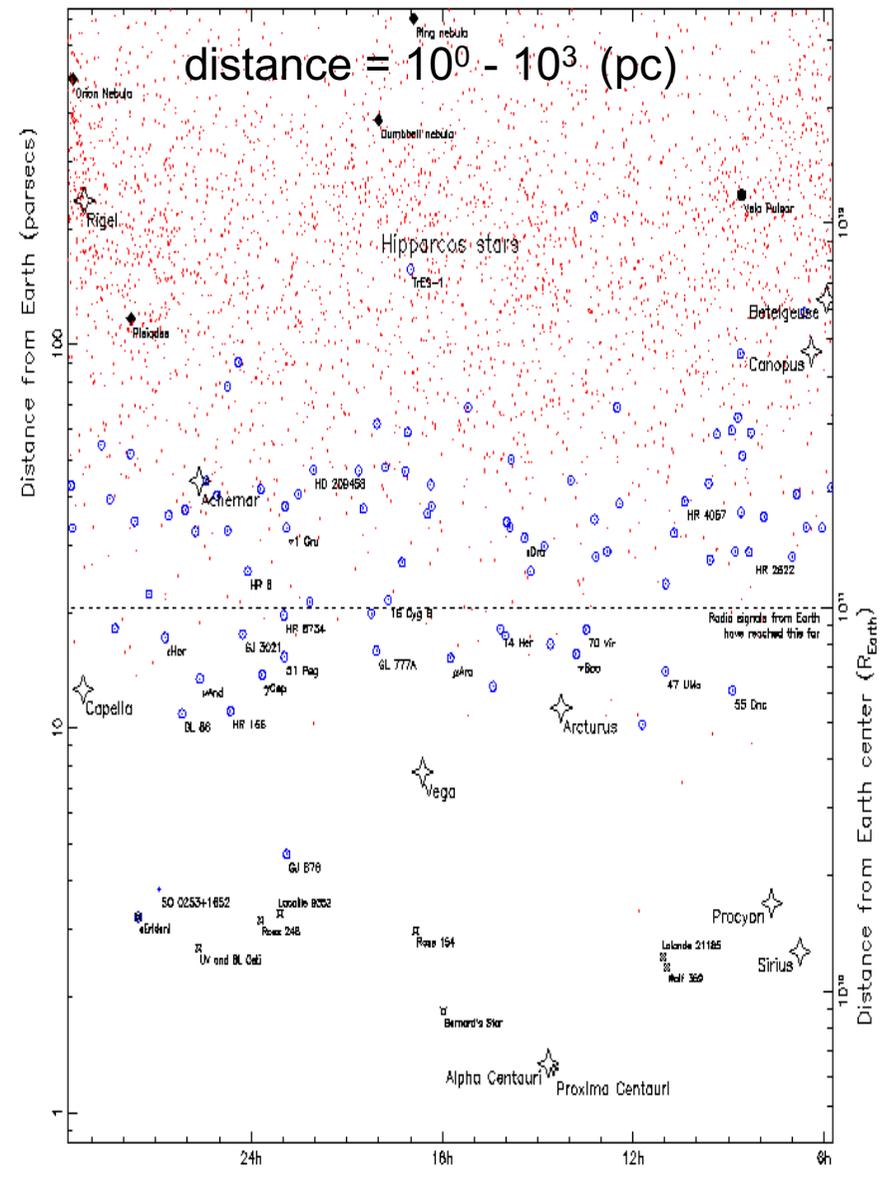
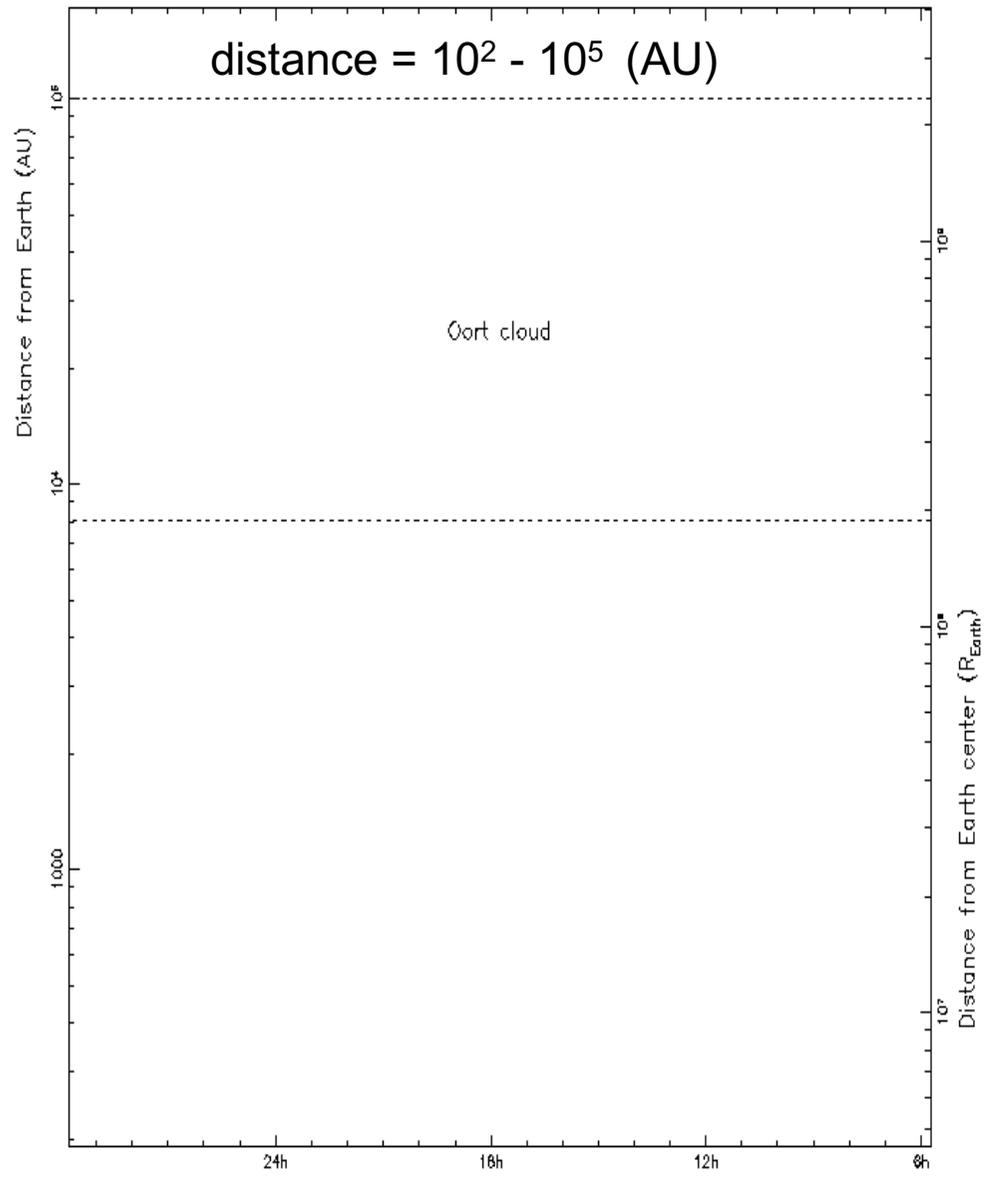
(SDSS, 2008)

Map of the observed Universe - logarithmic scale and showing the astrophysical objects in their actual coordinates.

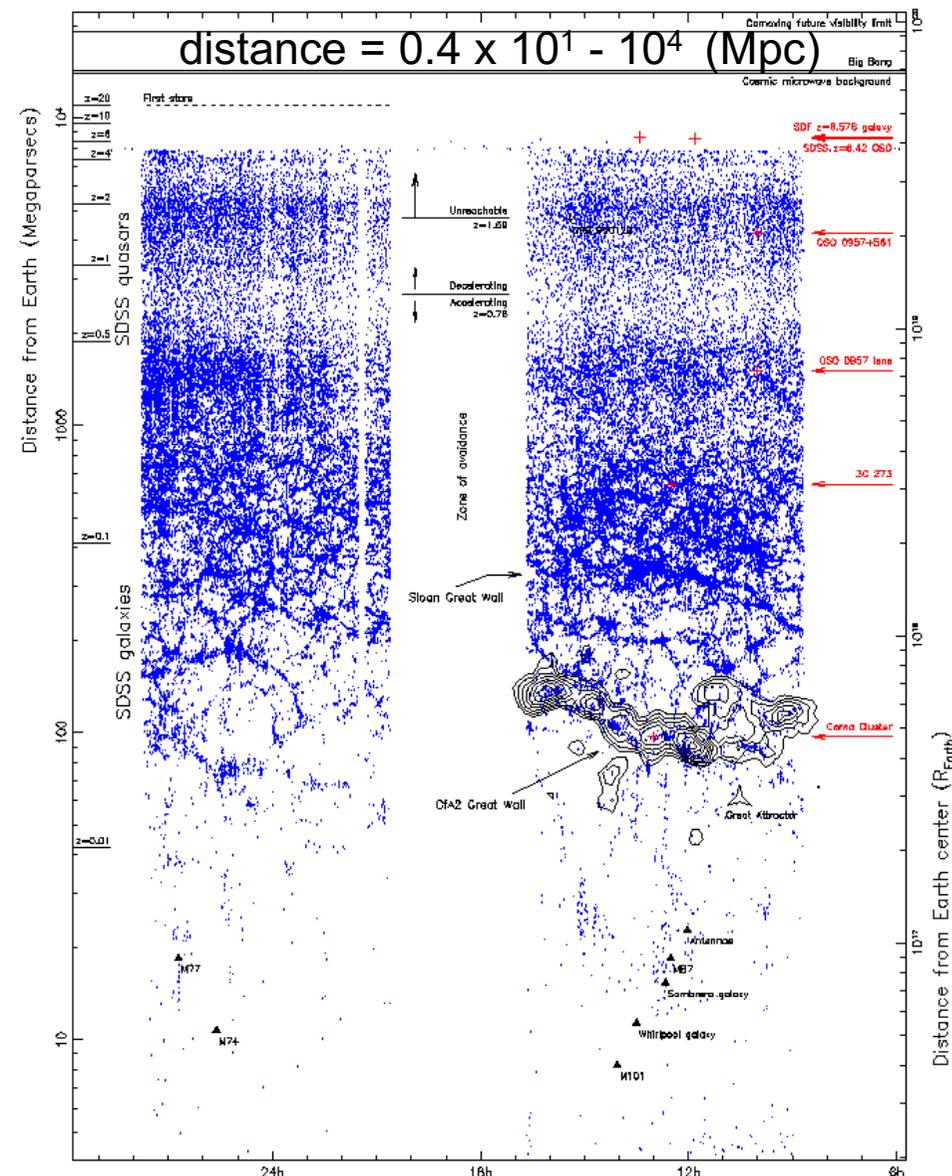
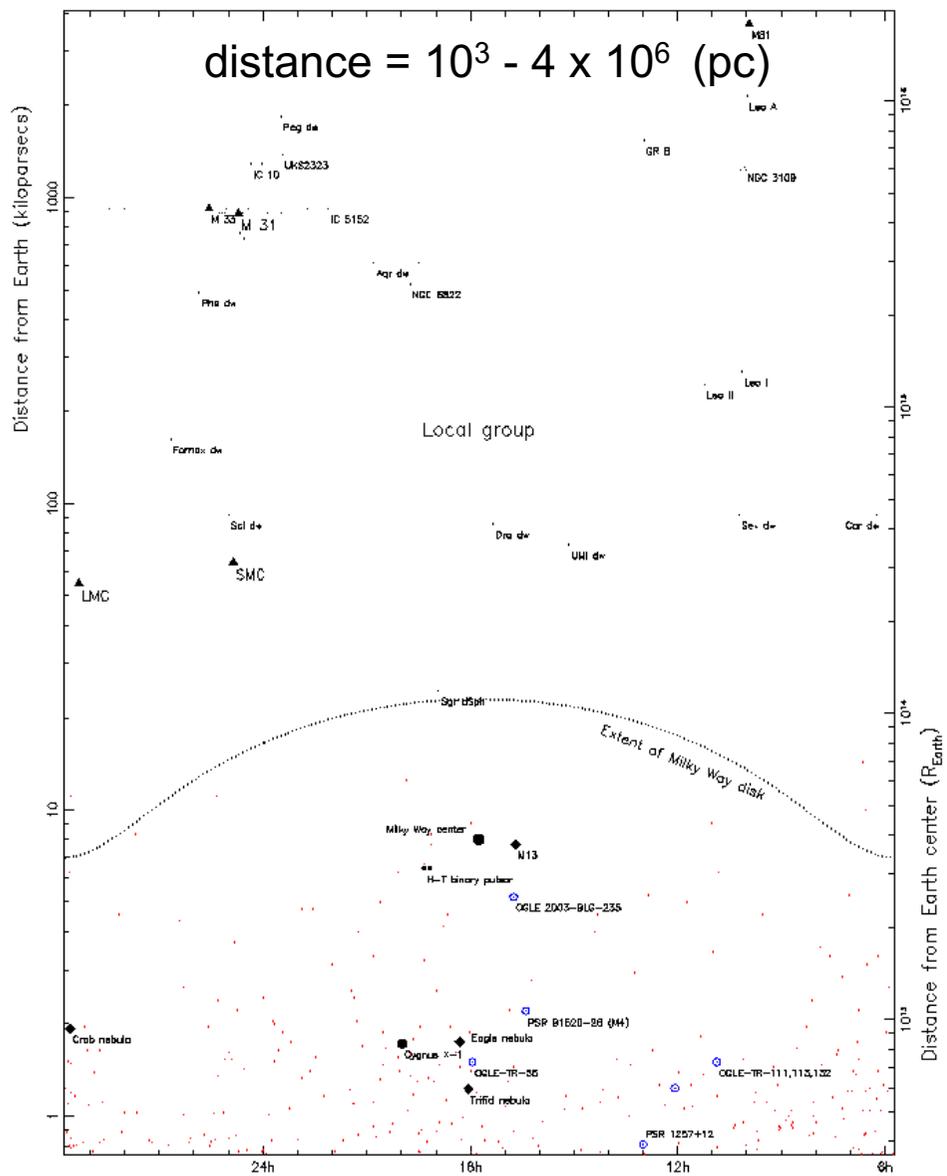
(Gott et al. 2005) <http://www.astro.princeton.edu/universe/>



(1 AU = 150 000 000 Km)



(1 pc = 200 000 AU)

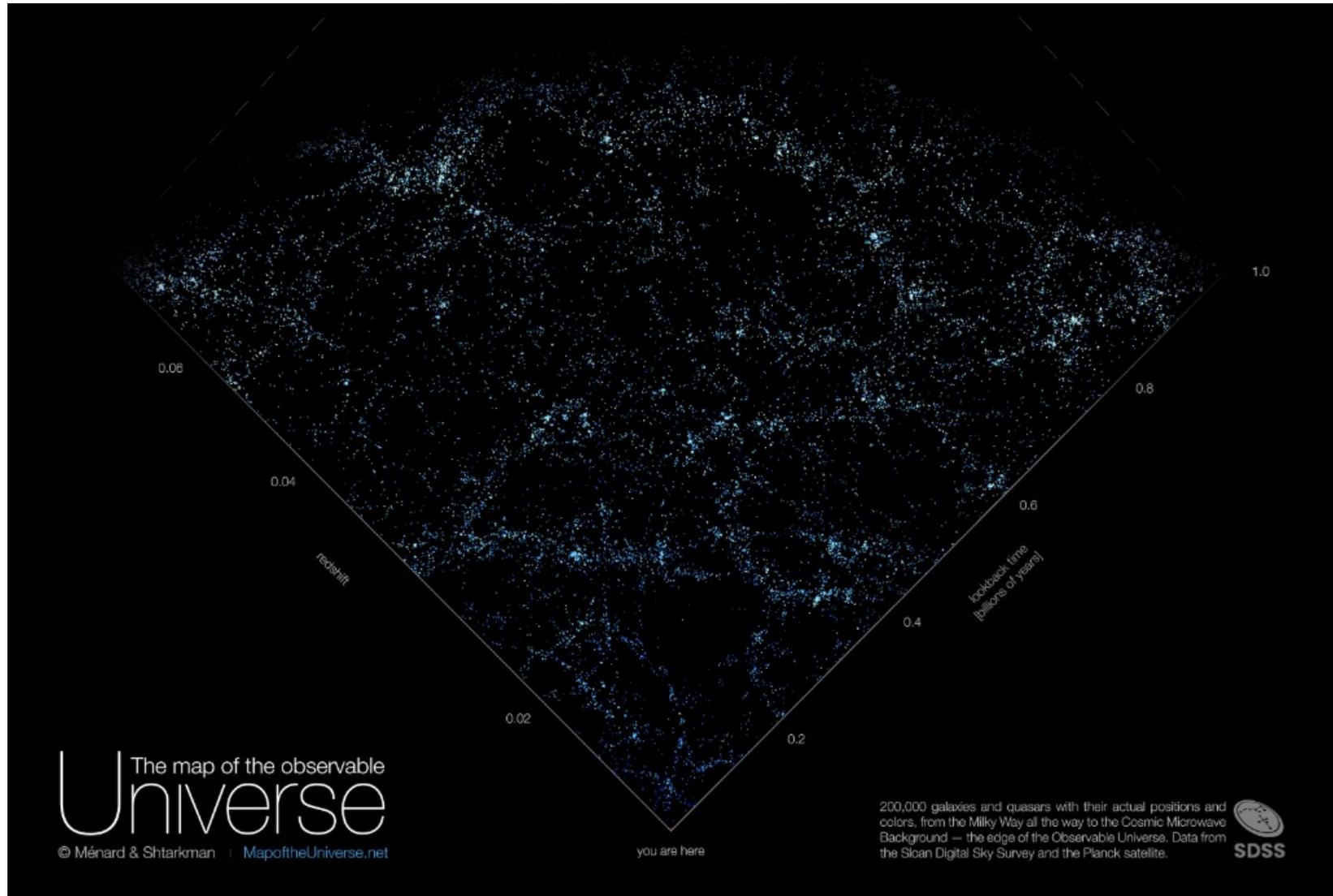


[New map of the observed Universe](#) - logarithmic scale, showing 200 000 galaxies from the SDSS archive in their actual coordinates

(Ménard & Shtarkman, Nov 2022) <http://mapoftheuniverse.net/>

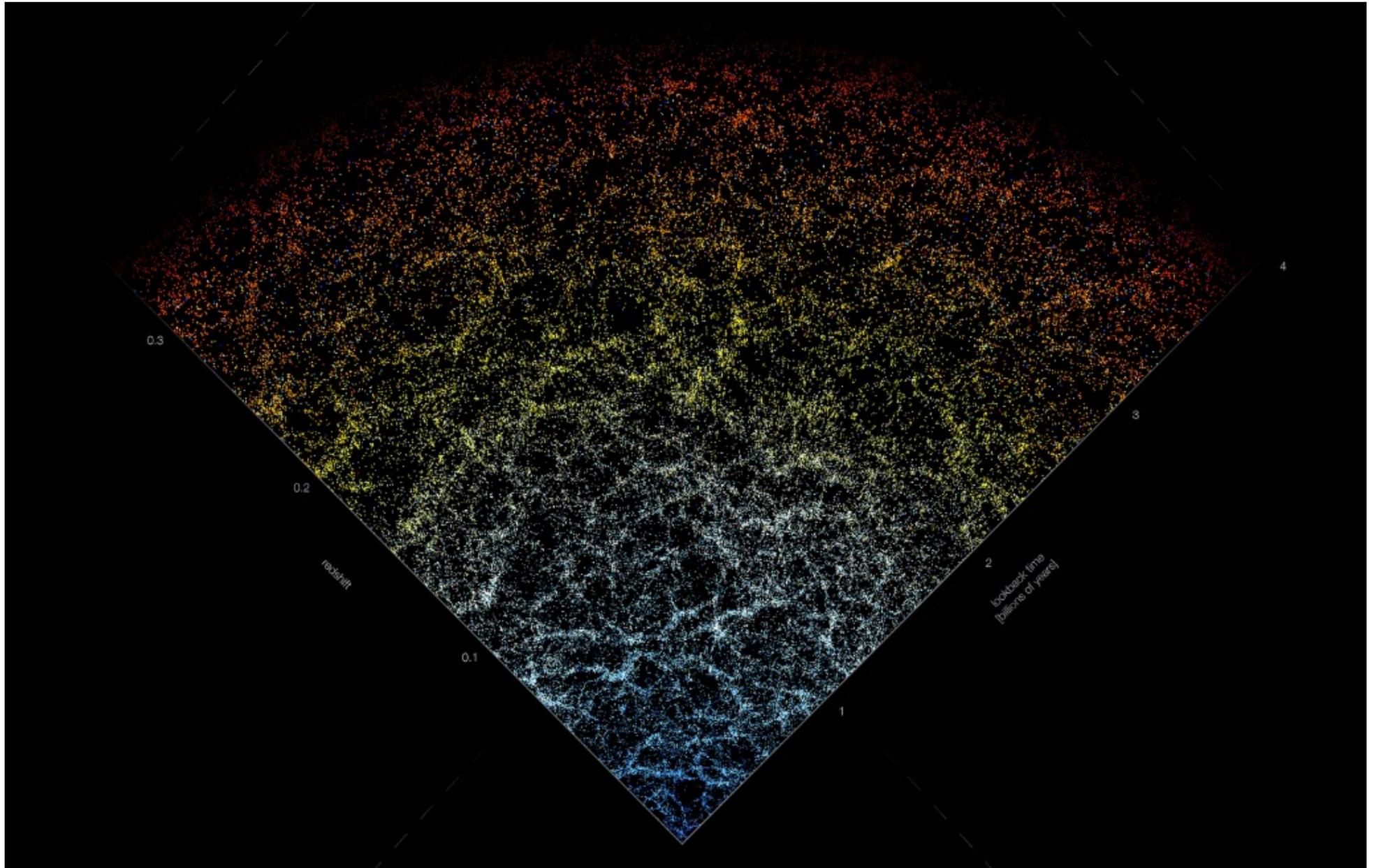
$z = 0 - 0.07$

0 - 1 Gyr



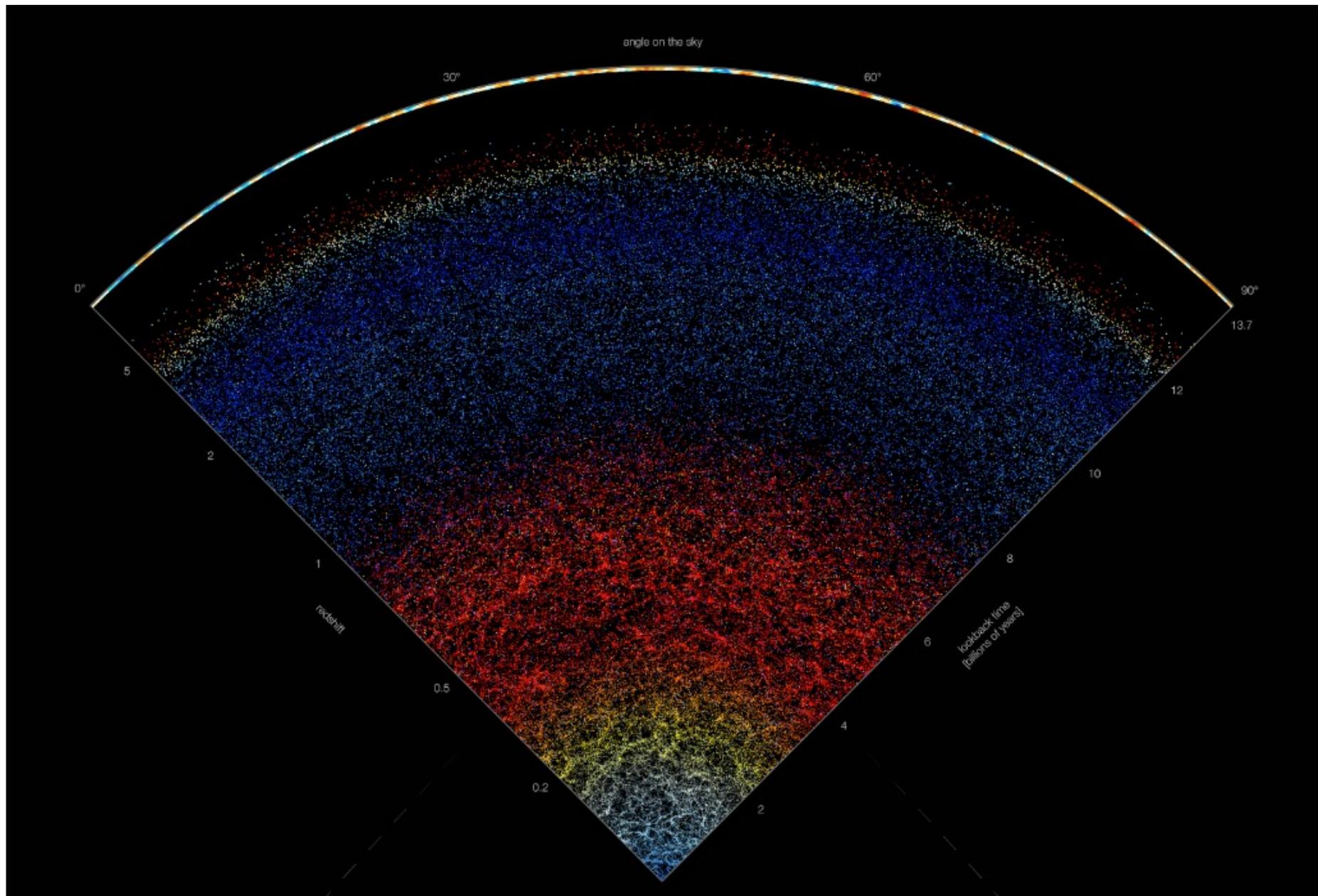
$z = 0 - 0.3$

0 - 4 Gyr



$z = 0 - 1000$

0 – 13.7 Gyr

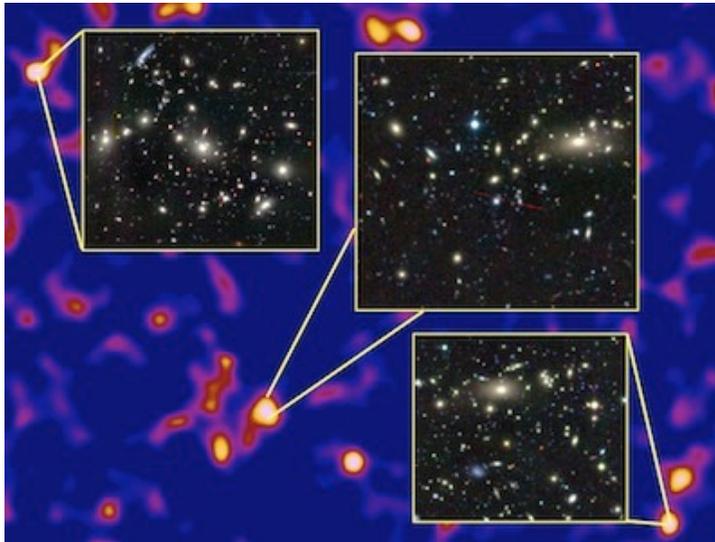


→ **Homogeneity** confirmed (on large scales)

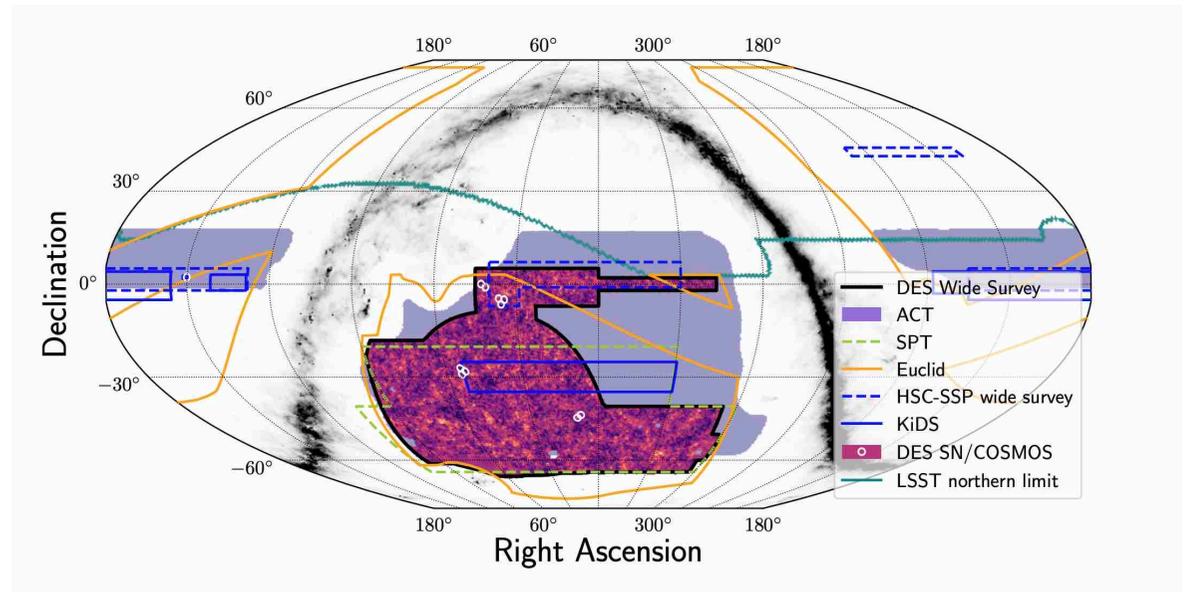
## + Observation of the LSS of dark matter in 2D and 3D

Mapping the galaxies shapes gives (less biased) information on the **cosmic web** – the DM **large-scale structure** of the Universe - by using **weak gravitational lensing** methods → find out the details of structure formation

2000 - today



(CFHTLenS, 2012)



(DES, 2026)

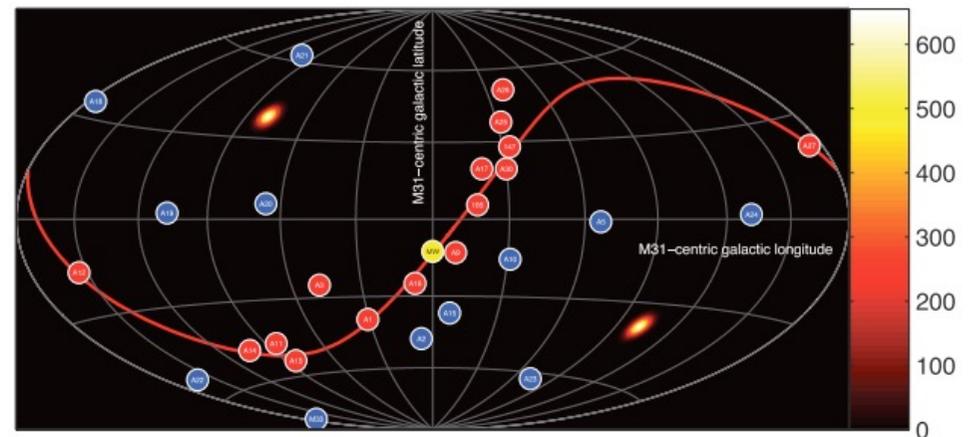
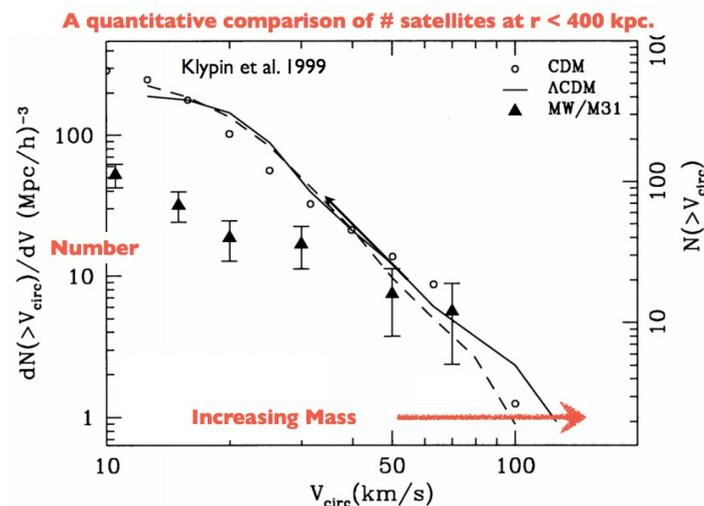
## + Observations of properties of small-scale structure (kpc)

→ Problem of the radial density profile of structures (**cusps/core**)

→ Problem of lack of structures (**satellite galaxies**)

→ Problem of the **satellite orbital plane** - possibly solved in 2022 with new simulations and Gaia 6-dim data (Sawalla et al, arXiv: 2205.02860)

Several problems not yet solved, leading to hypothesis of existence of other types of dark matter (Warm Dark Matter, Interacting DM), interacting DM/baryons in dense environments (Baryon feedback), hypothesis of modifications of GR on galactic scales (MOND)



## + Measurements of distances to SN

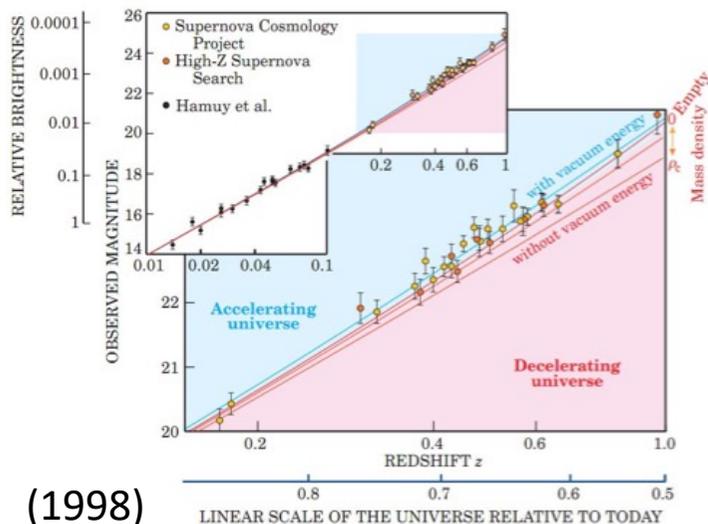
**Supernovae** at all redshifts are fainter (more distant?) than expected from the  $d_L(z)$  predicted by the CDM cosmological model

The Universe changed from a decelerated expansion to an accelerated one

→ Problem of the **acceleration** mechanism

Solved by assuming the existence of an extra component in the cosmological fluid - **Dark energy**

1998 - today



(1998)

Awarded to **Saul Perlmutter, Brian P. Schmidt** and **Adam G. Riess** "for the discovery of the accelerating expansion of the Universe through observations of distant supernovae." (1/2 + 1/4 + 1/4)



→ Problem of the **nature of dark energy**

There are dozens of different models of dark energy (e.g. review arXiv 2103.01183). The simplest one has passed many observational tests. In this model, dark energy is GR's cosmological constant →  **$\Lambda$ CDM model**

→ Alternatively, the observations can be explained by a Universe that is not accelerating but where the theory of gravitation on large scales is not GR, and the measured distances are compatible with the ones expected in that theory

There are also many candidates for a “**modified gravity**” theory

**+ Measurements of the BAO scale** (by DESI 2025)

→ Possibly inconsistent with  $\Lambda$ CDM (not confirmed yet)

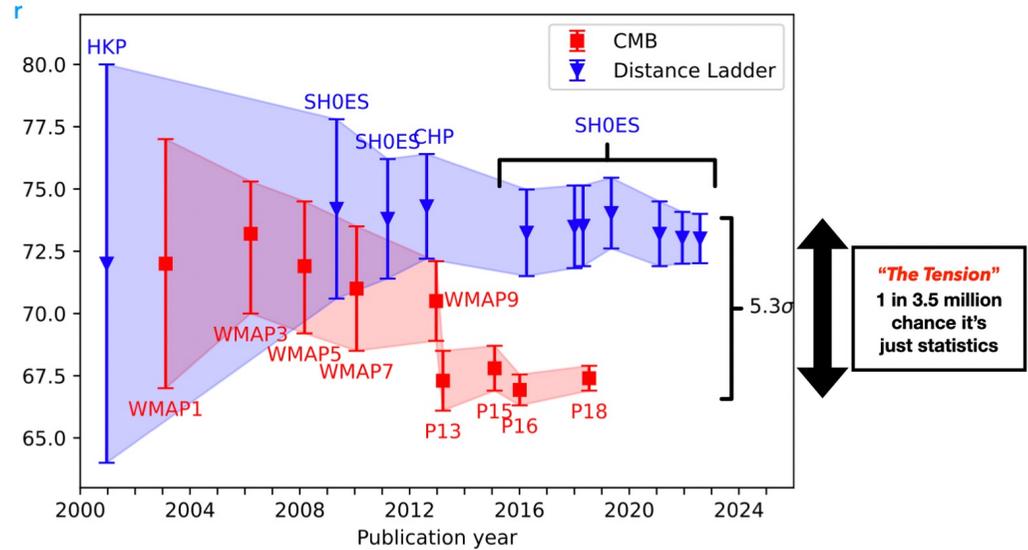
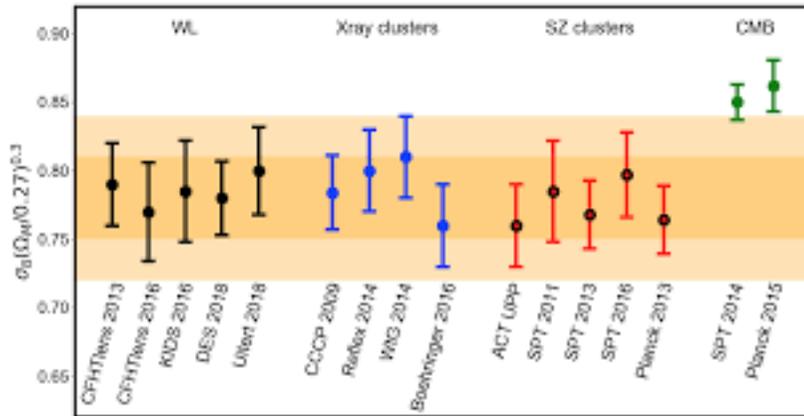
Could be solved by a dynamic dark energy model.

$\Lambda$ CDM still considered the standard model of cosmology

# + Joint results from all cosmological probes in tension

→ Hubble tension problem

→ Sigma 8 tension problem



**"The Tension"**  
1 in 3.5 million  
chance it's  
just statistics

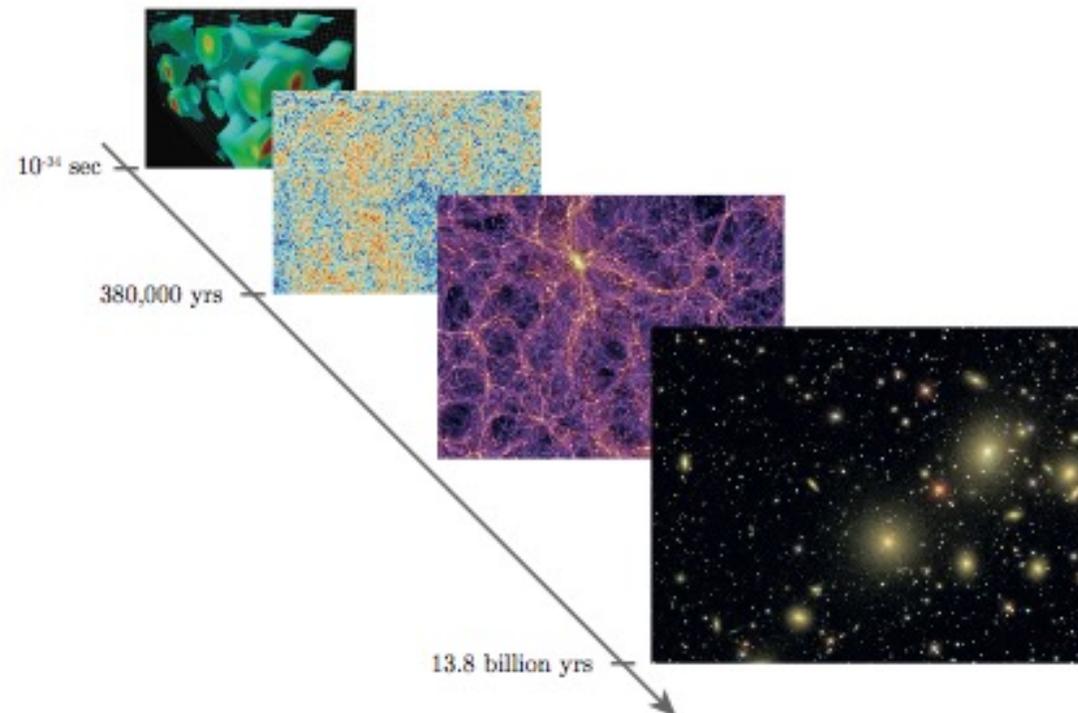
Problems not yet solved. Can be evidence for alternatives to  $\Lambda$ CDM or could be due to systematics in the data.

# $\Lambda$ CDM

the standard physical model of the Universe

General Relativity + Big Bang + Inflation + Gravitational clustering + cosmological fluid that includes dark matter of the type cold and dark energy of the type cosmological constant.

**This physical model has been the standard model of the Universe since the beginning of the XXI<sup>st</sup> century and it is known as  $\Lambda$ CDM.**



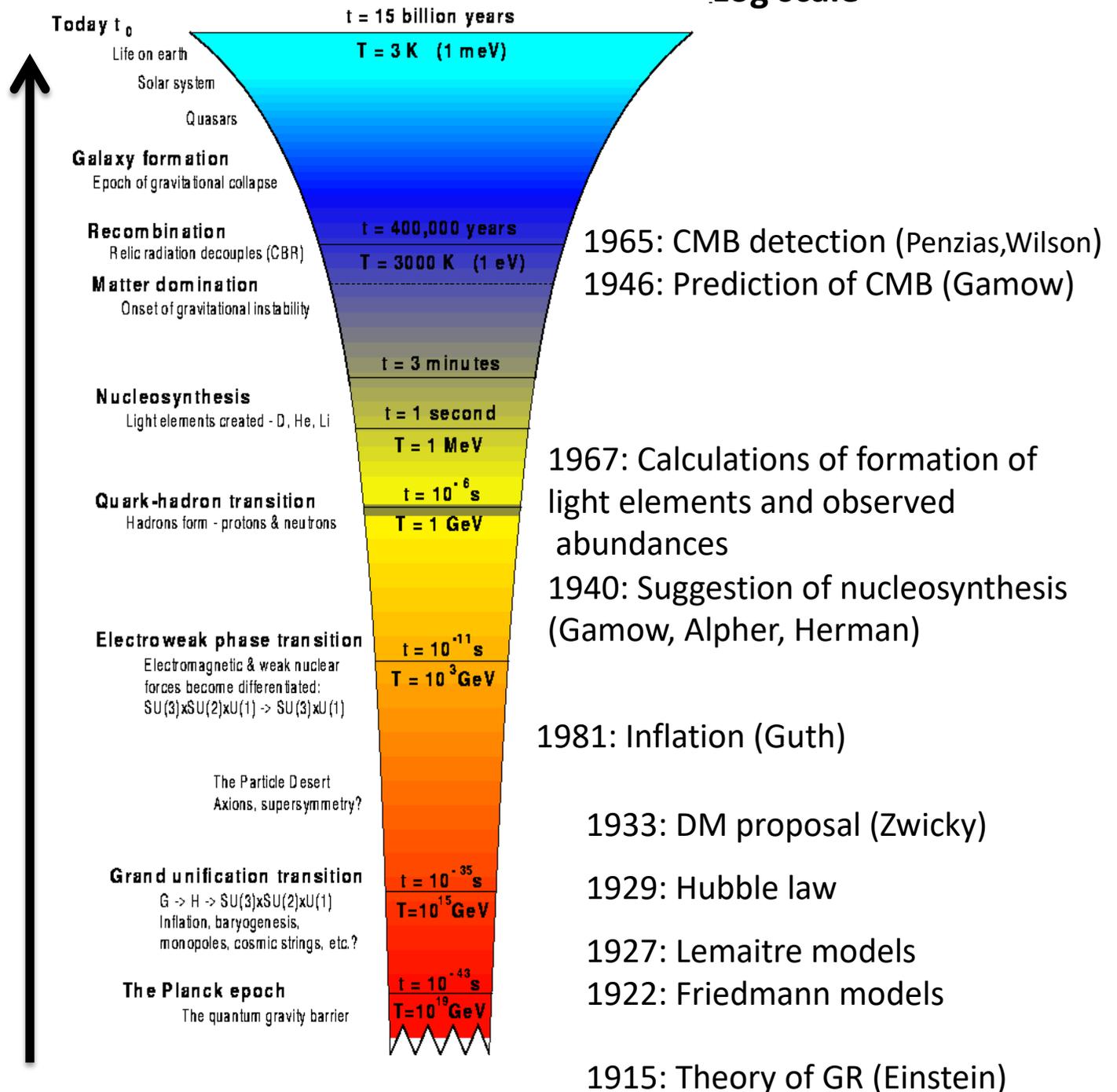
# Cosmic timeline

Log scale

Stable particles are the only ones left: photons, neutrinos, protons, neutrons, electrons, DM particles.

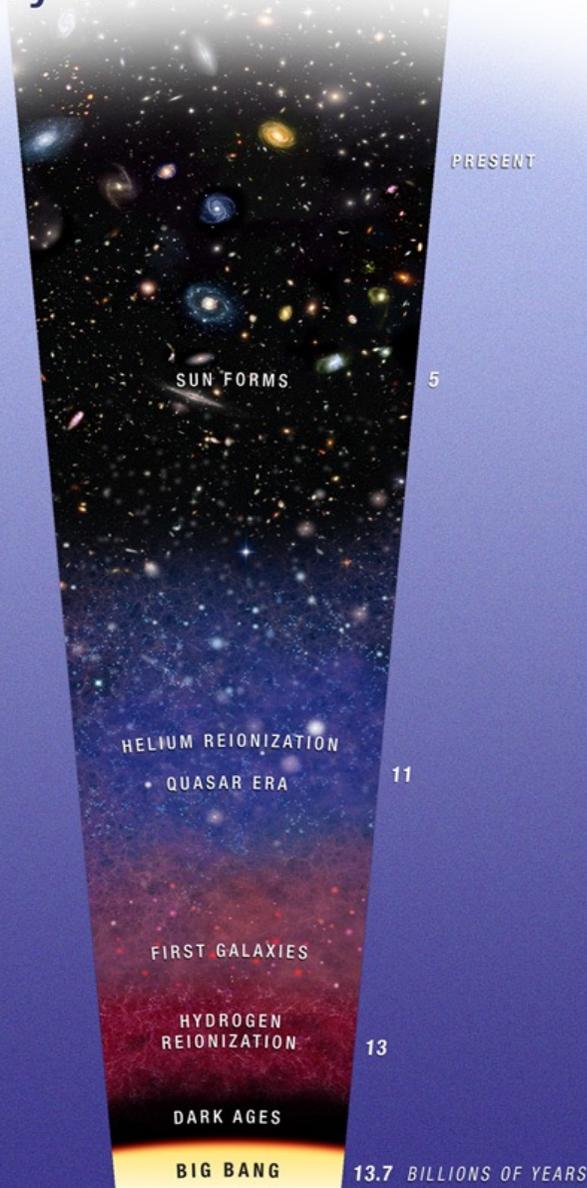
During the thermal history, the various species gradually decouple (leave the equilibrium) as their reaction rates become smaller than the expansion rate.

Inflation - mechanism introduced to solve some of the problems of the Big Bang model. It also provides the inhomogeneities initial conditions from quantum fluctuations.



# Linear scale

## History of the Universe



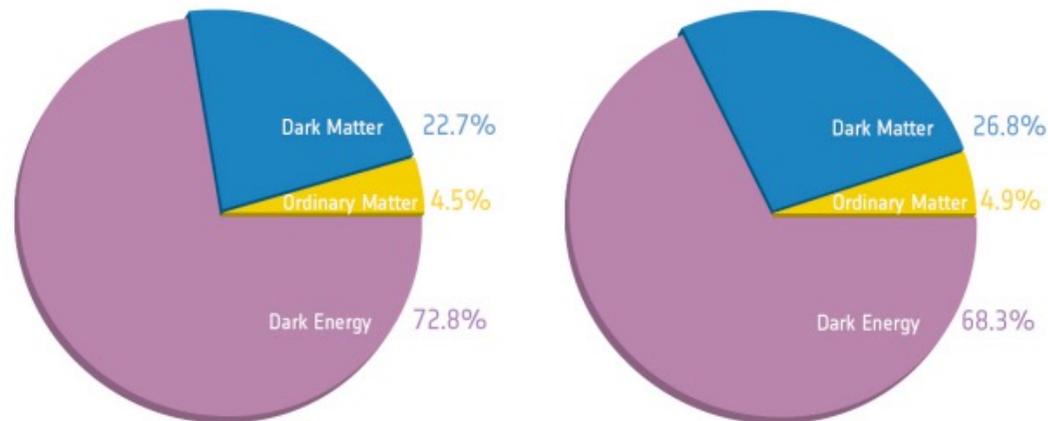
<p>2001: H<sub>0</sub> distance ladder (HST Key Project) (W. Freedman)</p> <p>1998: Accelerated expansion (SNIa)</p>	<p><b>Homogeneity</b></p>
<p>2005: Detection of the BAO peak (SDSS)</p> <p>2001: LSS map (SDSS, 2dFGRS) → SDSS IV (2019) → DESI (2024)</p> <p>2000: Weak lensing (LSS of DM) → DES (2021) → Euclid (2027)</p> <p>1986: The Great Wall (scale of homogeneity?)</p> <p>1970: Large-scale structure (first z-surveys of galaxies)</p>	<p><b>LSS</b></p>
<p>2006: Bullet Cluster (Chandra, Lensing) (DM observed?)</p> <p>1996: Nbody simulations (Virgo) (Universal profile NFW)</p> <p>1993: M<sub>b</sub> from clusters is 15% of M<sub>tot</sub> (S. White) (DE?)</p> <p>1982: X-ray cluster mass (Einstein satellite)</p> <p>1933: Cluster dynamics: DM needed (F. Zwicky)</p>	<p><b>Clusters</b></p>
<p>1996: z-evolution of Star-formation rate (HDF) (P. Madau)</p> <p>1986: First LSB galaxy</p> <p>1979: First gravitational lens system</p> <p>1974: Mass function (Press, Schechter) (NL collapse)</p> <p>1970: Rotation curves (DM also needed in galaxies) (V. Rubin)</p>	<p><b>Cosmic Noon</b></p>
<p>2024: Luminous early galaxies (record z=14.2) (JWST)</p> <p>2010: Cosmological HI 21cm (Pen) → SKA (&gt; 2027)</p> <p>1970s: Discovery of Ly-α forest</p> <p>1965: Gunn-Peterson test (the universe is highly ionized)</p> <p>1963: Discovery of the first quasar (first high-z source)</p>	<p><b>Cosmic Dawn</b></p>
<p>2013: CMB high precision and polarization (Planck)</p> <p>2003: CMB small scales (WMAP)</p> <p>2000: CMB 1st peak (Boomerang, Maxima) (Universe flat)</p> <p>1992: Anisotropies of CMB (COBE) (DM also needed)</p> <p>1990: CMB Black-body (COBE) (Big Bang)</p>	<p><b>Dark ages</b></p>
<p>2016: Gravitational waves (LIGO) → LISA (2037)</p> <p>2002: Neutrino oscillations</p>	<p><b>Primordial</b></p>

## $\Lambda$ CDM parameters

$\Lambda$ CDM is a complex model. It is a theoretical construction supported by observations.

It includes a variety of physical processes that occur in a variety of epochs, in a variety of scales and contains a large number of free parameters – the **cosmological parameters** - that need to be fixed by the observations.

The values of the cosmological parameters determine the details of the expansion of the Universe and the evolution and formation of its large-scale structures → **they determine the "cosmology"**.



Before Planck

After Planck

Parameter	Planck+WP		Planck+WP+highL		Planck+lensing+WP+highL		Planck+WP+highL+BAO	
	Best fit	68% limits	Best fit	68% limits	Best fit	68% limits	Best fit	68% limits
$\Omega_b h^2$	0.022032	$0.02205 \pm 0.00028$	0.022069	$0.02207 \pm 0.00027$	0.022199	$0.02218 \pm 0.00026$	0.022161	$0.02214 \pm 0.00024$
$\Omega_c h^2$	0.12038	$0.1199 \pm 0.0027$	0.12025	$0.1198 \pm 0.0026$	0.11847	$0.1186 \pm 0.0022$	0.11889	$0.1187 \pm 0.0017$
$100\theta_{MC}$	1.04119	$1.04131 \pm 0.00063$	1.04130	$1.04132 \pm 0.00063$	1.04146	$1.04144 \pm 0.00061$	1.04148	$1.04147 \pm 0.00056$
$\tau$	0.0925	$0.089^{+0.012}_{-0.014}$	0.0927	$0.091^{+0.013}_{-0.014}$	0.0943	$0.090^{+0.013}_{-0.014}$	0.0952	$0.092 \pm 0.013$
$n_s$	0.9619	$0.9603 \pm 0.0073$	0.9582	$0.9585 \pm 0.0070$	0.9624	$0.9614 \pm 0.0063$	0.9611	$0.9608 \pm 0.0054$
$\ln(10^{10} A_s)$	3.0980	$3.089^{+0.024}_{-0.027}$	3.0959	$3.090 \pm 0.025$	3.0947	$3.087 \pm 0.024$	3.0973	$3.091 \pm 0.025$
$A_{100}^{PS}$	152	$171 \pm 60$	209	$212 \pm 50$	204	$213 \pm 50$	204	$212 \pm 50$
$A_{143}^{PS}$	63.3	$54 \pm 10$	72.6	$73 \pm 8$	72.2	$72 \pm 8$	71.8	$72.4 \pm 8.0$
$A_{217}^{PS}$	117.0	$107^{+20}_{-10}$	59.5	$59 \pm 10$	60.2	$58 \pm 10$	59.4	$59 \pm 10$
$A_{143}^{CIB}$	0.0	$< 10.7$	3.57	$3.24 \pm 0.83$	3.25	$3.24 \pm 0.83$	3.30	$3.25 \pm 0.83$
$A_{217}^{CIB}$	27.2	$29^{+6}_{-9}$	53.9	$49.6 \pm 5.0$	52.3	$50.0 \pm 4.9$	53.0	$49.7 \pm 5.0$
$A_{143}^{SZ}$	6.80	...	5.17	$2.54^{+1.1}_{-1.9}$	4.64	$2.51^{+1.2}_{-1.8}$	4.86	$2.54^{+1.2}_{-1.8}$
$r_{143 \times 217}^{PS}$	0.916	$> 0.850$	0.825	$0.823^{+0.069}_{-0.077}$	0.814	$0.825 \pm 0.071$	0.824	$0.823 \pm 0.070$
$r_{143 \times 217}^{CIB}$	0.406	$0.42 \pm 0.22$	1.0000	$> 0.930$	1.0000	$> 0.928$	1.0000	$> 0.930$
$\gamma^{CIB}$	0.601	$0.53^{+0.13}_{-0.12}$	0.674	$0.638 \pm 0.081$	0.656	$0.643 \pm 0.080$	0.667	$0.639 \pm 0.081$
$\xi^{tSZ \times CIB}$	0.03	...	0.000	$< 0.409$	0.000	$< 0.389$	0.000	$< 0.410$
$A^{kSZ}$	0.9	...	0.89	$5.34^{+2.8}_{-1.9}$	1.14	$4.74^{+2.6}_{-2.1}$	1.58	$5.34^{+2.8}_{-2.0}$
$\Omega_\Lambda$	0.6817	$0.685^{+0.018}_{-0.016}$	0.6830	$0.685^{+0.017}_{-0.016}$	0.6939	$0.693 \pm 0.013$	0.6914	$0.692 \pm 0.010$
$\sigma_8$	0.8347	$0.829 \pm 0.012$	0.8322	$0.828 \pm 0.012$	0.8271	$0.8233 \pm 0.0097$	0.8288	$0.826 \pm 0.012$
$z_{re}$	11.37	$11.1 \pm 1.1$	11.38	$11.1 \pm 1.1$	11.42	$11.1 \pm 1.1$	11.52	$11.3 \pm 1.1$
$H_0$	67.04	$67.3 \pm 1.2$	67.15	$67.3 \pm 1.2$	67.94	$67.9 \pm 1.0$	67.77	$67.80 \pm 0.77$
Age/Gyr	13.8242	$13.817 \pm 0.048$	13.8170	$13.813 \pm 0.047$	13.7914	$13.794 \pm 0.044$	13.7965	$13.798 \pm 0.037$
$100\theta_s$	1.04136	$1.04147 \pm 0.00062$	1.04146	$1.04148 \pm 0.00062$	1.04161	$1.04159 \pm 0.00060$	1.04163	$1.04162 \pm 0.00056$
$r_{drag}$	147.36	$147.49 \pm 0.59$	147.35	$147.47 \pm 0.59$	147.68	$147.67 \pm 0.50$	147.611	$147.68 \pm 0.45$

**fundamental  
cosmological  
parameters**

**nuisance  
parameters  
(of a  
particular  
cosmological  
probe)**

**derived  
cosmological  
parameters**

# $\Lambda$ CDM problems

**The description of the Universe is far from being finished!**

## **Theoretical development**

- Details of the non-linear structure formation are not well understood
- The LSS is not completely described yet (high-order correlations)
- Relativistic effects not completely studied (larger scales)

## **Observations and interpretation**

- Many observations suffer from systematic effects
- Different observations are well fitted by the model predictions but for different and inconsistent parameter values  $\rightarrow$  the Hubble tension
- Many alternative DE models also fit the data
- Test the assumptions (e.g. cosmological principle)

## **Fundamental concepts**

- Nature of dark matter still unknown
- Nature of dark energy still unknown

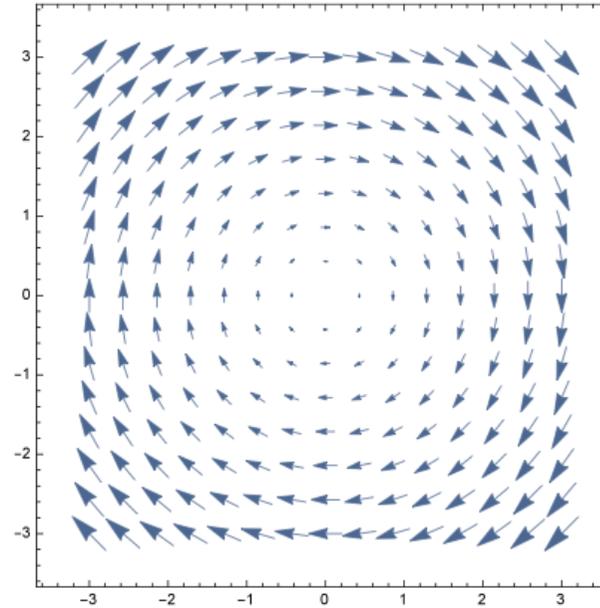
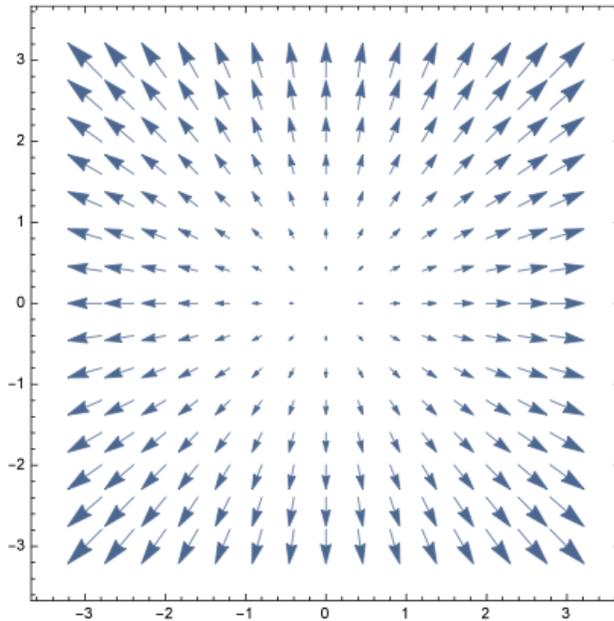
# Cosmological Principle

## the homogeneous universe

### Isotropy

“The Universe observed in any direction (from an observing point) looks the same”

The observed properties are independent of direction (rotational invariance)



isotropic (but not homogeneous)

## Isotropy observed

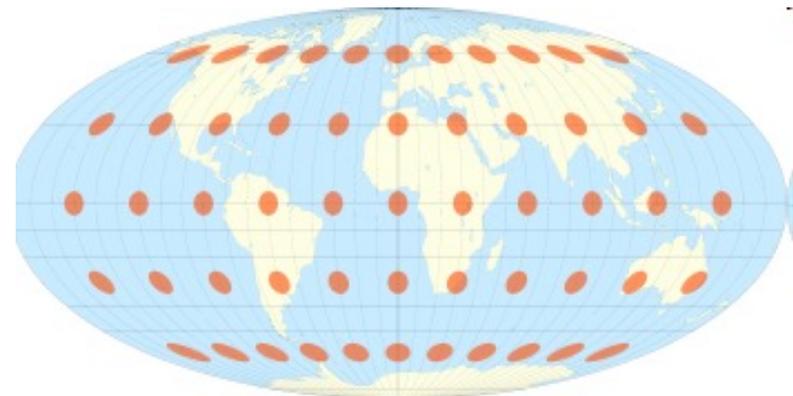
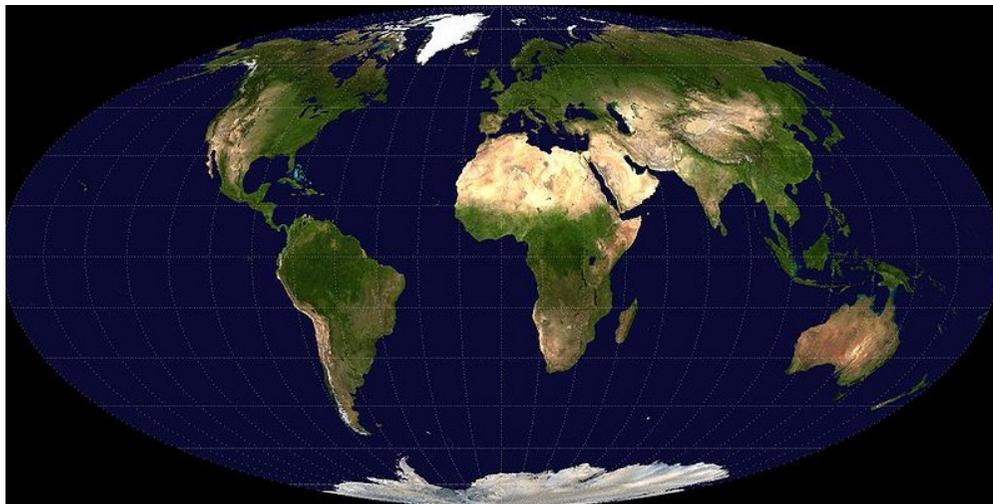
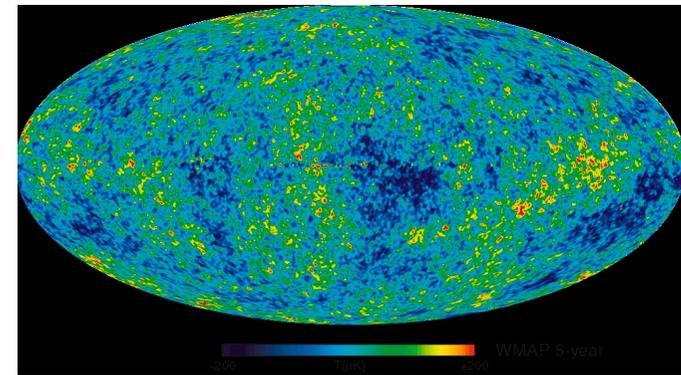
Except for the “nearby” structures, the observed spatial distribution of the Universe looks isotropic.

CMB (Cosmic Microwave Background) is isotropic

$$\Delta T/T \sim 0.00001$$

The sky shown in Mollweide projection  
in galactic coordinates

(preserves areas e distorts shapes)

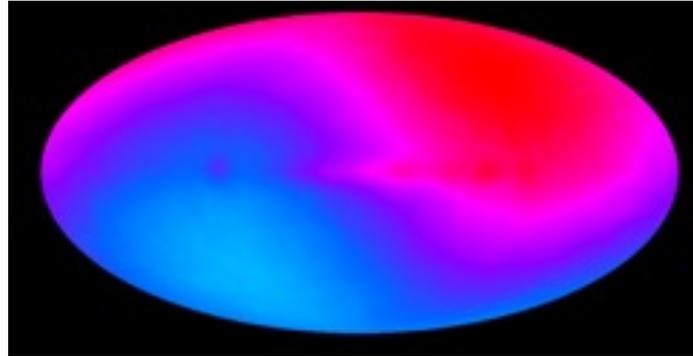


**Anisotropy** is for example

dipole in CMB  $\Delta T/T \sim 0.001$

$\rightarrow \Delta\lambda/\lambda \sim 0.001 \sim v/c$

$\rightarrow v \sim 300 \text{ Km/s}$

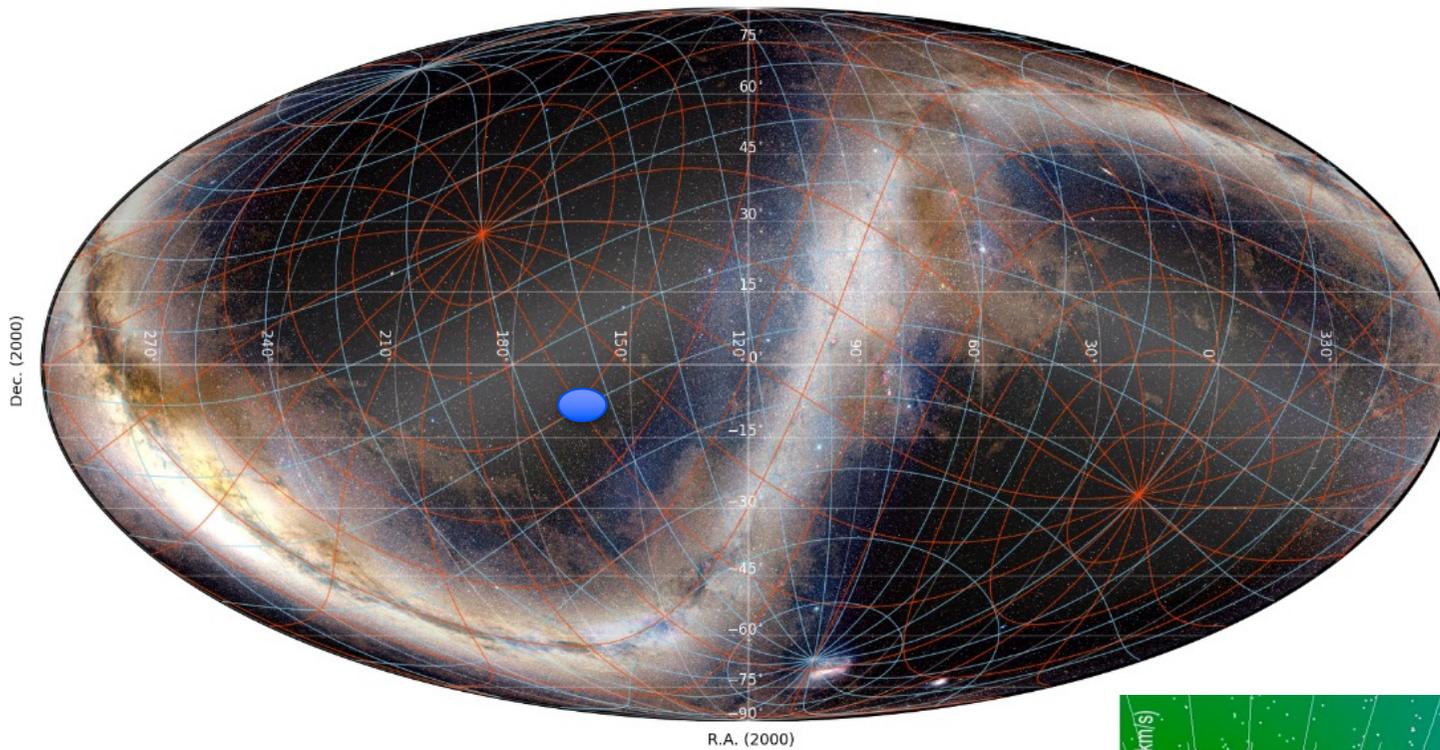


This is the total velocity of the Earth with respect to the CMB frame:

includes Earth's orbital movement + solar system movement in the galaxy + local galaxy movement  $\rightarrow$  **peculiar velocity** of the galaxy  
(it is a perturbation to **Hubble's flow**)

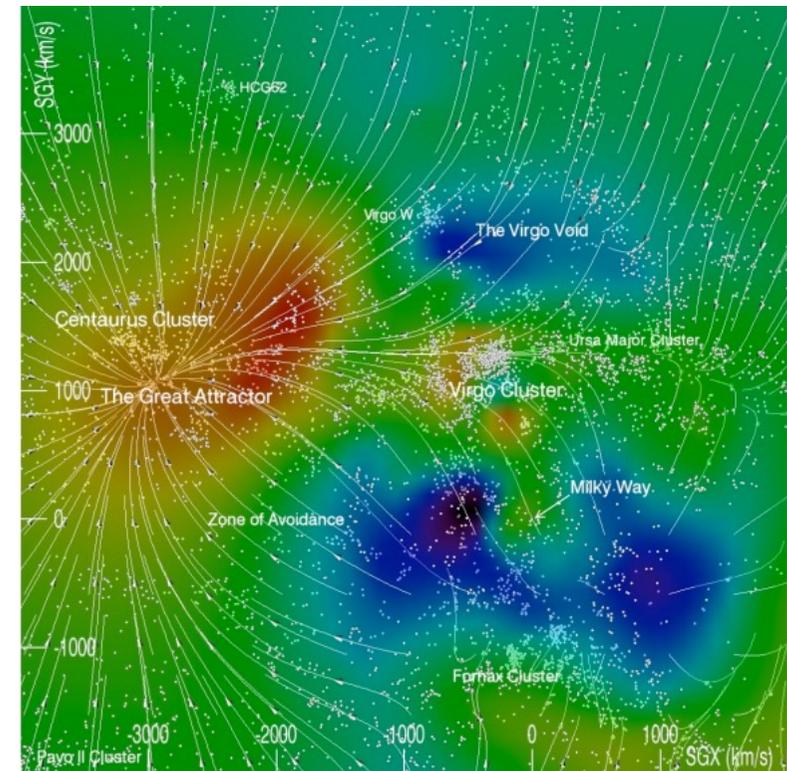
So, there is a "local" anisotropy that can be measured.

The movement is in the direction of the blue pole (ra, dec = 11h11min57s , -7.22°)  
(Leo constellation) towards the **Great Attractor**.



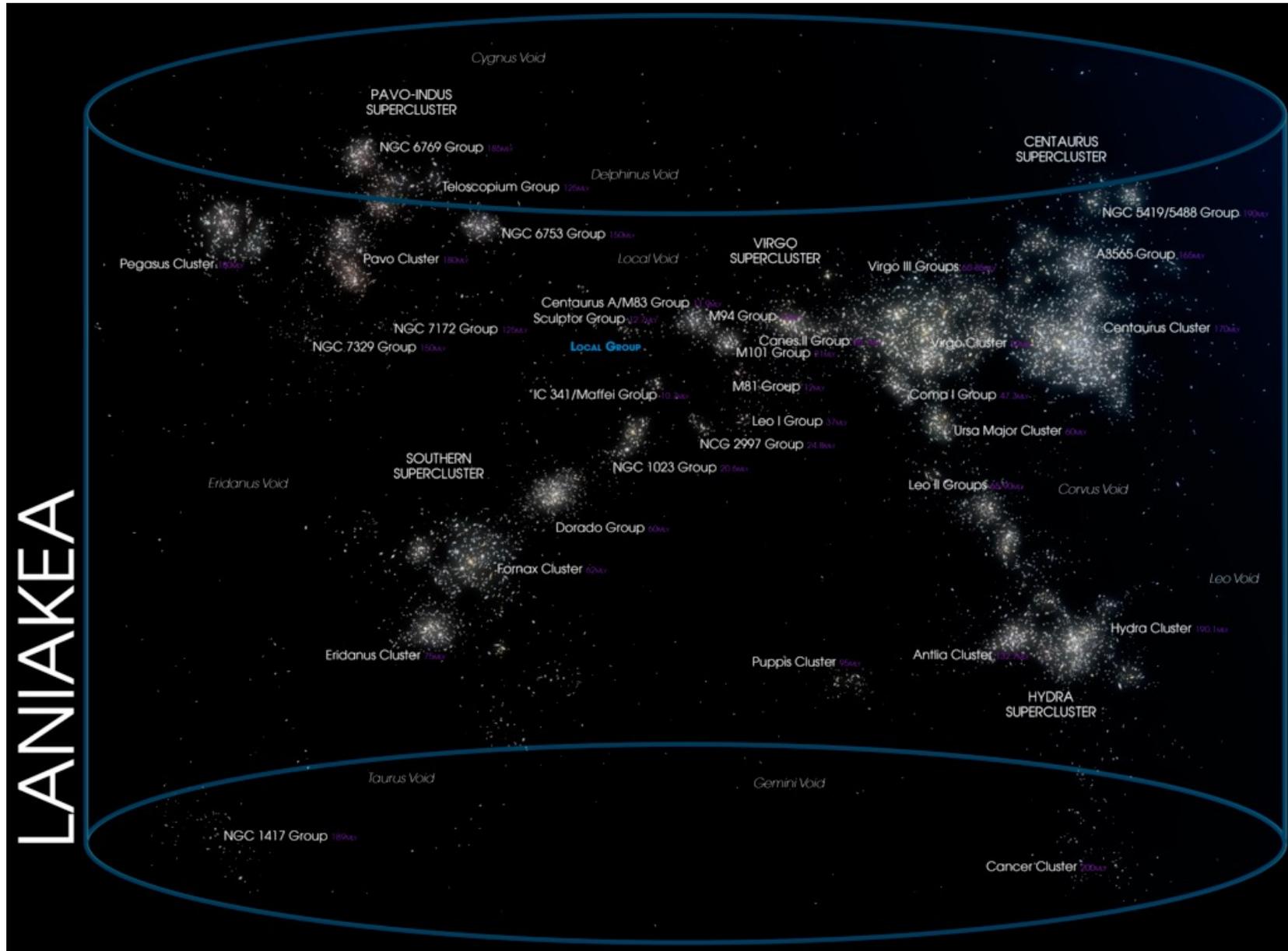
Looking from Earth, the Great Attractor lies on the zodiacal plane and close to the galactic plane → difficult to observe the extra-galactic sky → results are from radio-astronomy (2016)

It is at ~50 Mpc from us  
Parsec is a historical unit of distance. It is the distance to a star that changes its apparent position due to the Earth's orbital movement (parallax) by 1 arcsec. It corresponds to 3.26 lyr.



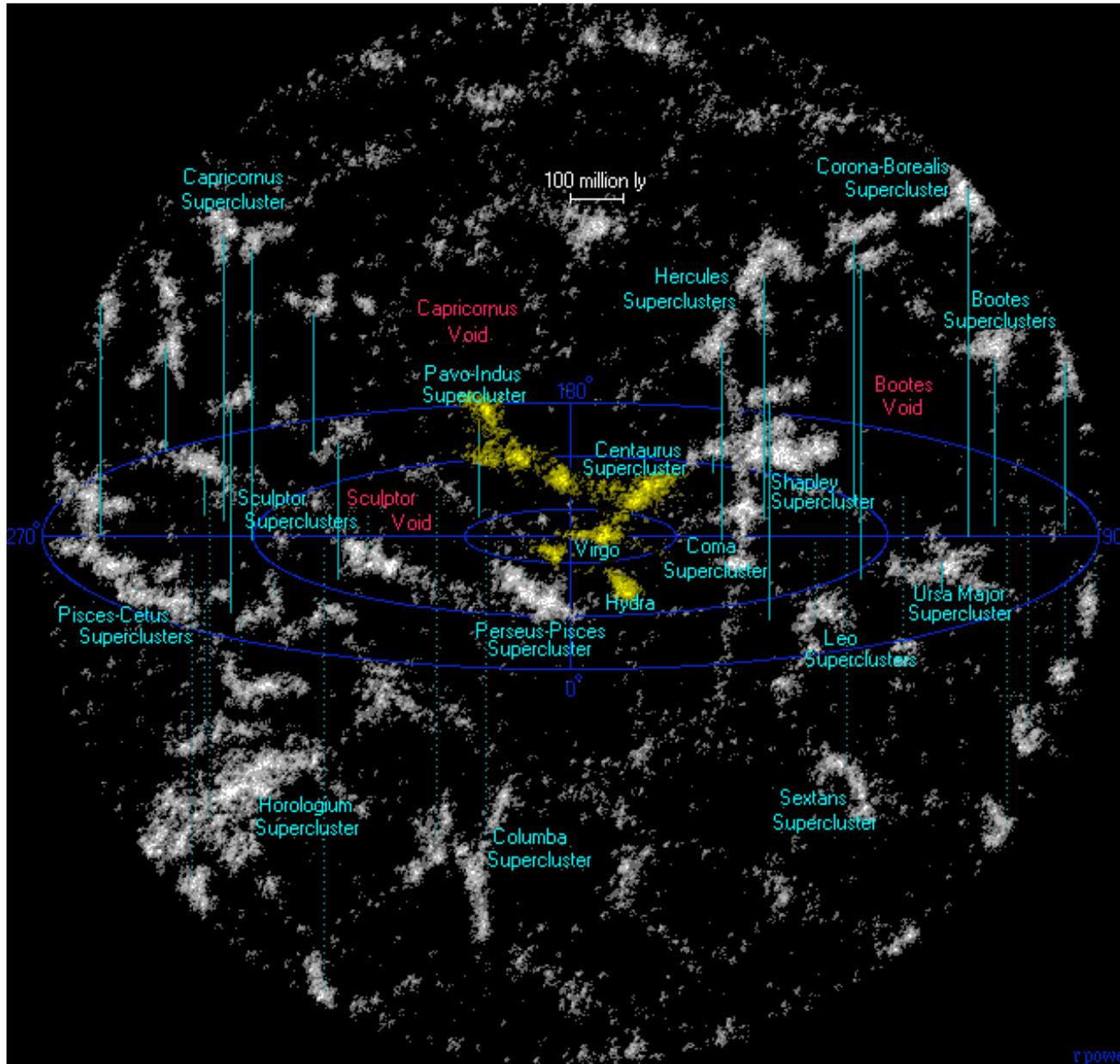


Laniakea: the local super-cluster. Its central gravitational point is the Great Attractor.



Diameter ~170 Mpc

Loosely gravitationally bound. **Linear structure** that contains many non-linear structures.



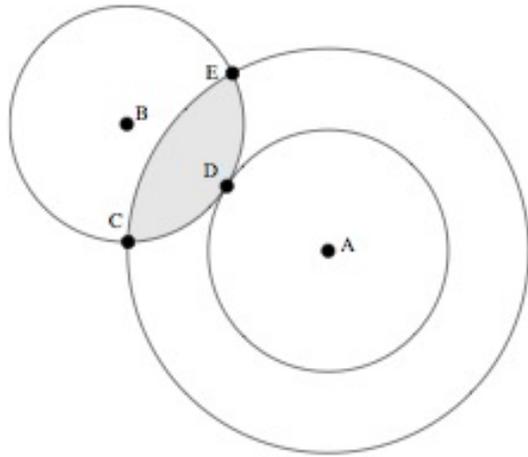
Beyond Laniakea (shown in yellow), the movements with respect to us start to be dominated by the Hubble flow and no longer by peculiar velocities  $\rightarrow$  isotropy

This is roughly redshift  $z \sim 0.1$

**Cosmology starts beyond  $z \sim 0.1$**

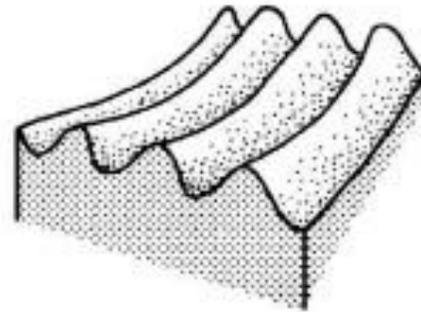
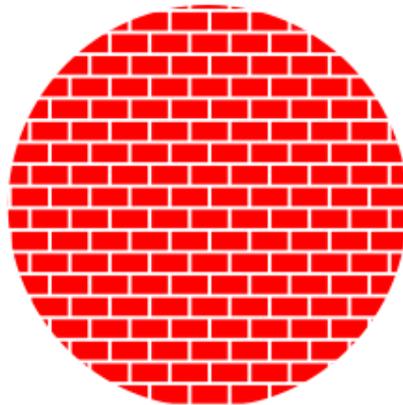
Extrapolation of the Copernican principle  $\rightarrow$  we should not be in a special position. All points should observe isotropy.

Isotropy in all points implies homogeneity.



isotropy around A and around B  
implies that the grey zone is homogeneous.

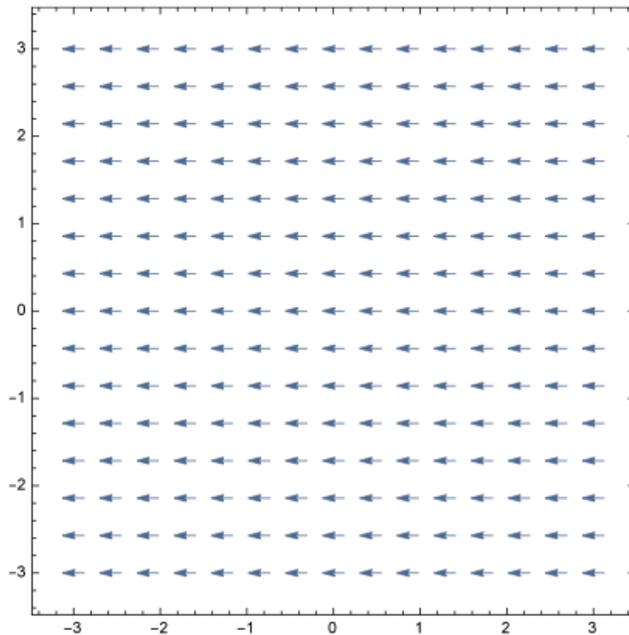
On the contrary, homogeneity does not imply isotropy



## Homogeneity

“The Universe is identical in all points, at each instant”

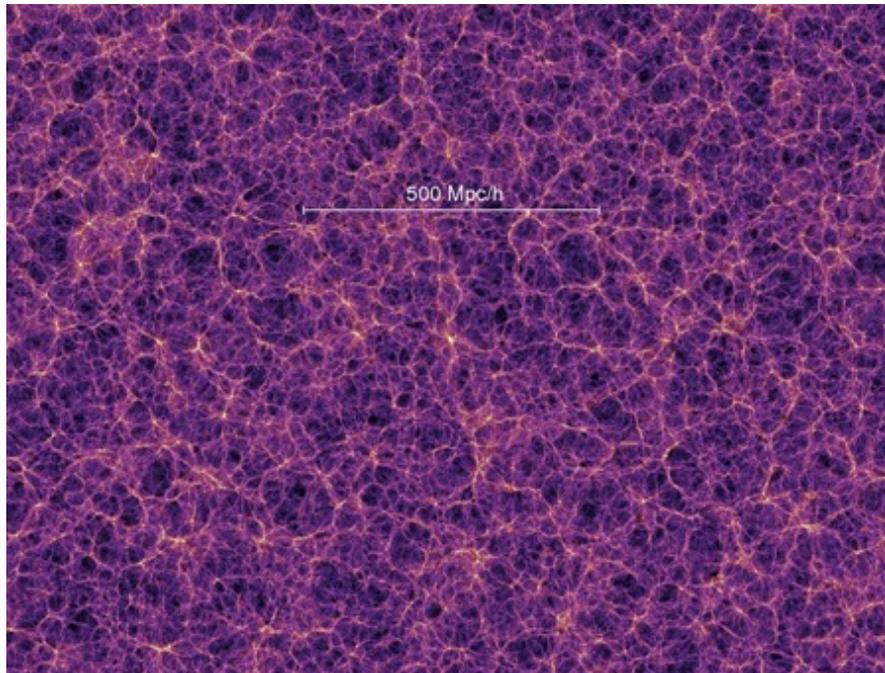
The observed properties are independent of location (translational invariance)



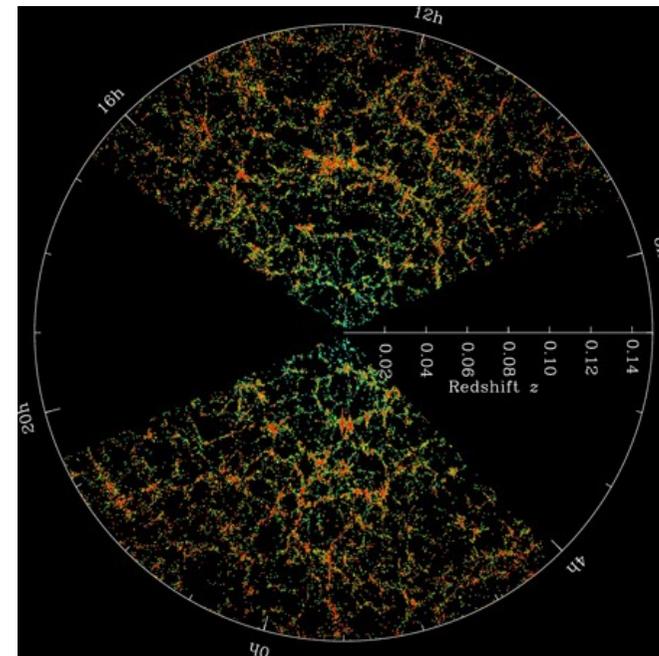
homogeneous (but not isotropic)

## Homogeneity observed

- Galaxy counts as function of volume
- The absence of structures on “very large scales” – the average matter density contrast on very large scales is very low.



(dark matter N-body simulation)



(observations of galaxies)

Homogeneity scale  $> 100$  Mpc

## Cosmological Principle

The Universe is **homogeneous** and **isotropic** (on “large-enough scales”)

This implies that there is a set of observers that have the same history of the Universe and to which all observables are independent of direction. This defines a fundamental reference frame where the physical properties are the same on all points. This is the **comoving frame** - that follows Hubble’s flow

Physical fields (matter density or CMB temperature) have the same values for all comoving observers.

The time rate is also identical, which allows to define an universal time and separate space and time coordinates.

In practice:  $\rho(t,x) \rightarrow \rho(t)$  where  $\mathbf{t}$  is universal

To be in agreement with the cosmological principle, let us consider the Universe as a **homogeneous** sphere that expands **isotropically**.

Radial expansion is the only expansion model that keeps the homogeneity (note however that it is not the only possibility to ensure isotropy).

Consider the following:



Let us write  $\vec{r} = a(t) \vec{x}$  → homogeneity allows to separate time and space.  
position of matter element

$\vec{x}$  = comoving coordinate is constant in time  
 it is also the physical distance at  $t = a(t) = 1$

(this is  $t_0$  by definition, i.e. today)  $0 < a < 1$

$a(t)$  cosmic scale factor

↓  
 $1 < x$

Some quantities appear naturally with the expansion

$x$  - **comoving coordinate** - the absolute reference frame → a particle that is comoving with the expansion keeps a constant value of its comoving coordinate.

$a(t)$  - **scale factor**

$r(t)$  - **proper coordinate**

convention:  $0 < a < 1 \rightarrow r < x$

We can also consider the derivatives of the expansion  $r = ax$ .

$$r = ax \rightarrow v = da/dt x + a dx/dt = \dot{a}/a ax + v_{\text{peculiar}} = H(t) r + v_{\text{peculiar}}$$

(note:  $\dot{a} = da/dt$  )

This defines the **Hubble flow** (a feature of the zeroth-order Universe;  $H(t)r$  is also known as the **recession velocity**) and the **peculiar velocity** (a feature of the first-order Universe).

In the local Universe the **Hubble function**  $H(t)$  is constant , and is known as the **Hubble constant**  $H_0 = \dot{a}/a|_{a=1}$

which is a very important cosmological parameter, that sets the size and age of the Universe.

Its value is usually written as

$$H_0 = 100 h \text{ Km/s/Mpc}$$

**$h \sim 0.7$**   $\rightarrow H_0 \sim 70 \text{ Km/s/Mpc} \rightarrow$  a galaxy that is 1 Mpc more distant than another one closer to the observer, recedes with a velocity 70 Km/s faster than the one that is closer.

# Perturbations

## the inhomogeneous universe

The cosmological principle is a first approximation to study the Universe. It is not verified on smaller scales where “local” structures differ from point to point, defining local gravitational potentials.

**Is it a good approximation?** In other words, *what is the amplitude of the gravitational potentials associated to the astrophysical structures?*

To address this question let us consider the **Theorem of the Virial** (for the dynamics of the gravitational collapse of a local system of  $N$  particles of masses  $m$  at positions  $x$ )

### Tensor of Inertia

is the matrix of the second-order moments of the mass distribution in the system

second-order derivative of the inertia tensor

Sistema de  $N$  partículas com massas  $m_\alpha$  em posição  $\vec{x}_\alpha$

Momento de inércia: 
$$I_{jk} = \sum_{\alpha=1}^N m_\alpha x_j^\alpha x_k^\alpha \quad \begin{matrix} \text{3D} \\ j,k=1,2,3 \end{matrix}$$

Calculamos a 2ª derivada em relação ao tempo:

$$\frac{\partial^2 I_{jk}}{\partial t^2} = \sum_{\alpha=1}^N m_\alpha (\ddot{x}_j^\alpha x_k^\alpha + 2 \dot{x}_j^\alpha \dot{x}_k^\alpha + x_j^\alpha \ddot{x}_k^\alpha)$$

Consideremos a aceleração (newtoniana) de uma partícula:

acceleration  
of each particle:

$$\ddot{x}_j^\alpha = \sum_{\substack{\beta=1 \\ (\beta \neq \alpha)}}^N \frac{G m_\beta (x_j^\beta - x_j^\alpha)}{|\vec{x}^\beta - \vec{x}^\alpha|^3}$$

$$\Rightarrow \frac{\partial^2 I_{jk}}{\partial t^2} = \underbrace{2 \sum_{\alpha=1}^N m_\alpha \dot{x}_j^\alpha \dot{x}_k^\alpha}_{\text{Tensor de Energia Cinética}} + \underbrace{\sum_{\alpha=1}^N \sum_{\substack{\beta=1 \\ (\alpha \neq \beta)}}^N \frac{G m_\alpha m_\beta}{|\vec{x}^\alpha - \vec{x}^\beta|^3} \left[ (x_j^\beta - x_j^\alpha) x_k^\alpha + (x_k^\beta - x_k^\alpha) x_j^\alpha \right]}_{\text{Tensor de Energia Potencial}}$$

$4 K_{jk}$   
Tensor de  
Energia Cinética

Kinetic energy tensor

Tensor de Energia Potencial

$$V_{jk} = \sum_{\alpha=1}^N \sum_{\substack{\beta=1 \\ (\alpha \neq \beta)}}^N \frac{G m_\alpha m_\beta x_j^\alpha (x_k^\beta - x_k^\alpha)}{|\vec{x}^\alpha - \vec{x}^\beta|^3}$$

Ou seja,

$$\frac{1}{2} \frac{\partial^2 I_{jk}}{\partial t^2} = 2 K_{jk} + V_{jk}$$

This means that the evolution of the tensor of inertia of the set of gravitationally interacting particles is subject to this constraint (by definition).

Costumamos calcular o traço para trabalhar com quantidades escalares

$$I = \sum_{j=1}^3 I_{jk} \quad (j=k)$$

Introduce the trace, just to work with scalar quantities

$$K = T_n (K_{jk}) = \frac{1}{2} \sum_{\alpha=1}^N m_{\alpha} v_{\alpha}^2$$

$$V = T_n (V_{jk}) = -\frac{1}{2} \sum \sum \frac{G m_{\alpha} m_{\beta}}{|\vec{x}^{\alpha} - \vec{x}^{\beta}|}$$

T. Virial:

$$\frac{1}{2} \frac{d^2 I}{dt^2} = 2K + V$$

Theorem of the Virial

Sistema Virializado (ex: estruturas em equilíbrio após colapso)

$$V = -2K$$

When the system is virialized - the collapse has ended, the inertial tensor does not change anymore and the

$$(E = K + V) \rightarrow E = -K ; E = \frac{V}{2}$$

system remains with this energy condition  $E = V/2$

## Potential of an astrophysical structure

$$\Phi = \frac{V}{m} = \frac{Gm}{r}$$

(note it has dimensions of velocity square:  $[G M / r] = v^2$  )

Theorem of the virial  $\rightarrow \Phi = -\frac{2K}{m} = -v^2$

i.e. the amplitude of the gravitational potential of a virialized structure is given by the **velocity dispersion**

Exemple of structures in the Universe : Clusters :  $v = 1000$  km/s

Galaxies :  $v = 200$  km/s

We need to compare these values with the amplitude of the “**gravitational potential**” of the **homogeneous Universe**.

But what is the potential of the Universe?

From the **equivalence principle** :

Gravitational field (gravitational mass)  $\leftrightarrow$  Acceleration of the reference system (inertial mass)

In the well-known “gedanken” experiment in special relativity:

Photon travel time from ceiling to floor  $t = h/c$

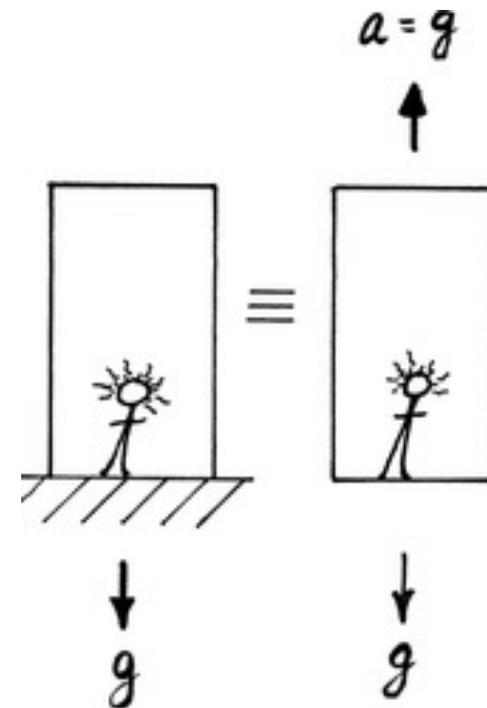
Floor's velocity increased by  $a h/c$

Frequency shift  $\Delta v/v = \Delta v/c = a h/c^2$

**Time dilation**  $\Delta t/t = a h/c^2$

Equivalence principle:  $a=g \rightarrow ah/c^2 = gh/c^2 = \Delta\Phi /c^2$

$\rightarrow$  time dilation  $= \Delta\Phi /c^2$



This implies that the Minkowski metric  $ds^2 = -c^2dt^2 + dx^2$  that in an accelerated frame is written as

$$ds^2 = -(1-v^2/c^2)^2 dt^2 + dx^2$$

can also be written as  $ds^2 = - \left(1 - \frac{\Phi}{c^2}\right)^2 dt^2 + dx^2$  (which shows explicitly the **gravitational redshift** effect)

**So the potential impacts the  $g_{00}$  term of the metric.**

An astrophysical structure contributes with  $\Phi / c^2$

**and the potential of the homogeneous Universe is 1**  $\rightarrow$  the kinetic velocity of the Universe (which is equivalent to a potential) is  $v^2 = c^2$

We saw that galaxies and clusters have “small” dispersion velocities  $v \ll c \rightarrow$  their gravitational potential is much smaller than the global potential of the homogeneous Universe  $\Phi \ll c^2 \rightarrow$  The astrophysical structures in the Universe are only a **perturbation** to the homogeneous (Robertson-Walker) metric.

**Note:** The Robertson-Walker metric of a Universe with perturbations will be modified to include a term

$$\left(1 - \frac{\Phi}{c^2}\right)^2 \approx 1 - \frac{2\Phi}{c^2}$$

**Note:**

*Astrophysical structures are **scalar** perturbations to the homogeneous metric of the Universe. There may be other types of perturbations to the metric. For example, **gravitational waves** are tensor perturbations to the spatial part of the homogeneous metric.*

## **We conclude that the structures in the Universe can be considered perturbations to the cosmological principle**

**Homogeneous Universe** - is in expansion - its gravitational dynamics are described by the homogeneous metric (Robertson-Walker).

**Inhomogeneous Universe** - consisting on global expansion + local linear clustering - its gravitational dynamics is described by the homogeneous metric with perturbation terms.

**Collapsing structures** - regions of space that are locally not expanding, they follow a non-linear collapse or are already collapsed - its gravitational dynamics is not described by the homogeneous metric with perturbation terms (they are weak gravitational fields, GR is not the best way to describe them. An exception are the black holes, which are strong gravitational fields and are described by a GR metric - but not the RW metric).