Cosmologia Física

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Structure Formation

Cosmic microwave background anisotropies

CMB anisotropies

At the end of the **decoupling process**, baryons and radiation fully decouple, i.e.

the plasma is dissolved, the photons no longer scatter from the baryons

and are free to propagate → forming a radiation background that propagates through all the Universe - known as the **cosmic microwave background radiation** - **CMB**.

Notice that during the plasma epoch, there are **photon perturbations** (which are identical to the baryon perturbations → there are overdensities of photons → they can be clustered, forming structures of light (immediately absorbed and reemitted or scattered).

Those should look like baryonic structures with light (since both are coupled) but the light is not emitted by the baryons (which are not dense enough to form stars), but it is rather made of primordial photons.

Of course, this speculative picture cannot be observed, since the photons cannot travel far in the plasma.

After decoupling, the photons move, and all ‘structures of light’ are destroyed. They become smoothly distributed → **the current photon spatial distribution is homogeneous**. Today there are no photon perturbations.

However, the homogeneous distribution of primordial photons is not isotropic. It is anisotropic because the energy distribution **(the temperature)** of the primordial photons released at z_{dec} is determined by the conditions at their emission from the last scattering surface which is inhomogeneous → it contains information of the plasma inhomogeneities at z_{dec}.

So today we detect a homogeneous distribution of photons, but each direction of detection comes from a specific point of the last scattering surface at z_{dec} and its frequency (temperature) is mostly determined by the inhomogeneities on that point.

The anisotropies are very small, their standard deviation is $\mathbf{10^{-5}}$ around the mean value of **2.7K** (which is in the **microwave band**, hence the name). Given the redshift produced by the expansion, the frequency at emission was **~10 μm** (in the infrared).

These were the only photons existing in the universe until the process of reionization started (at $z_{re} \sim 20$) when the first collapsed baryonic structures formed in the dark matter halos and started emitting new photons.

To be useful as a cosmological probe, the physical properties must be computed with great accuracy (small theoretical uncertainties), to compare with the ones estimated from data.

The baryonic contribution to the total matter power spectrum is not very accurately computed in the Euclidean approximation. It is not good enough to treat the plasma as a fluid. Relativistic particles must be treatet as a system of particles with an energy distribution evolving in the phase space → the conservation equation is not the continuity equation but the (perturbed) **Boltzmann equation**.

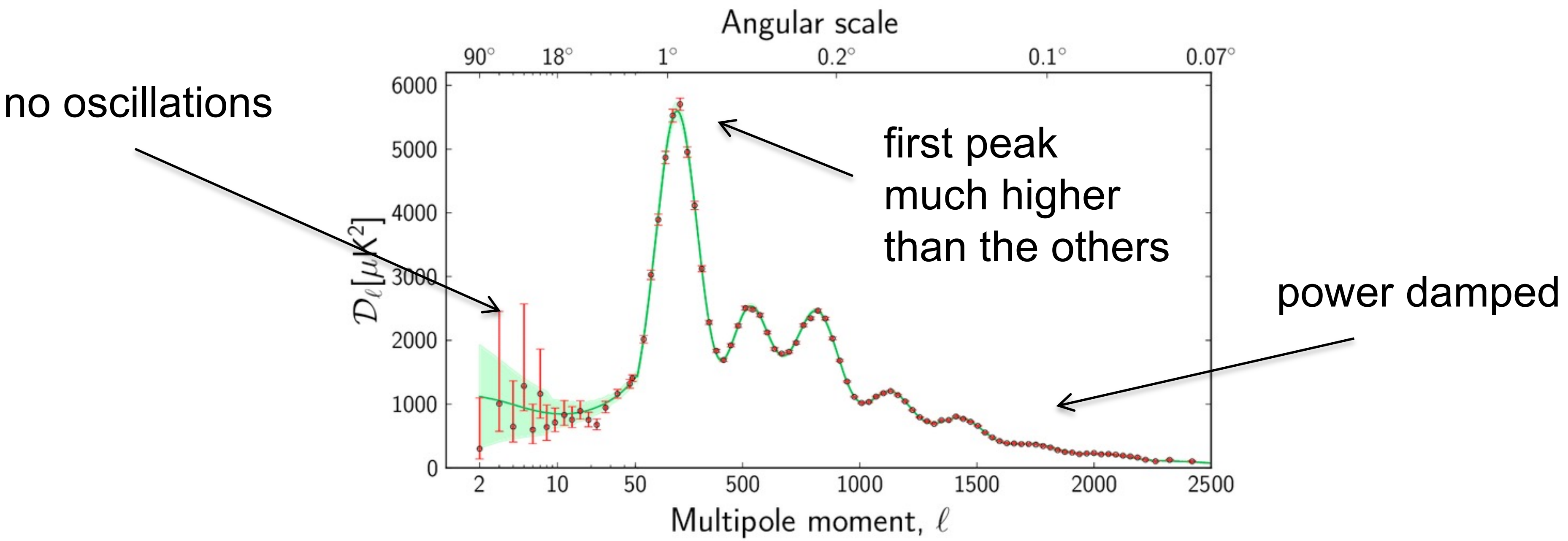
On the other hand, gravity should not be treated as a potential with a Poisson equation, but as a metric property with the **Einstein equations**.

The set of **Einstein-Boltzmann equations** is implemented in various numerical codes (e.g. CLASS, CAMB). They are however slow to compute. Moreover, there are physical effects not captured by the equations that need **additional equations** (e.g. to model the exact features of the decoupling, photon-baryon dragging, etc.)

Let us now describe the main contributions to the calculation of the CMB temperature power spectrum.

Primary temperature anisotropies

The CMB power spectrum is the angular power spectrum (angular because the field is defined on a single redshift) of the temperature contrast of the field of photons temperature.



There are several contributions to the temperature contrast:

$$\left(\frac{\delta T}{T}\right)_{\rm obs} = \frac{1}{4}\delta_\gamma^N - v^N\cdot\hat{n} + \Phi(t_{\rm dec}, x_{\rm ls}) + 2\int \dot{\Phi}\,dt$$

Density contrast

$$\left(\frac{\delta T}{T}\right)_{\rm obs} = \frac{1}{4}\delta_\gamma^N - v^N \cdot \hat{n} + \Phi(t_{\rm dec}, x_{\rm ls}) + 2\int \dot{\Phi} dt$$

The term δ_γ is the most important contribution.

Density contrast in sub-Jeans scales : acoustic oscillations

This is what we saw already, the clustering produced by the Jeans equation:

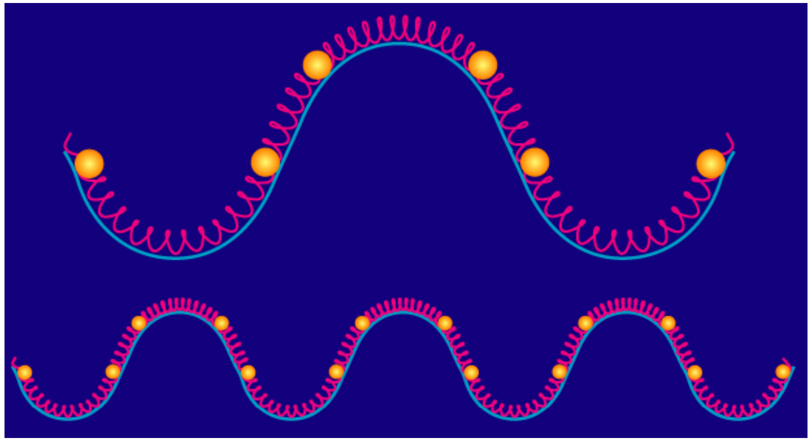
$$\ddot{\delta}_k + 2\frac{\dot{a}}{a}\dot{\delta}_k - \left(4\pi G\bar{\rho} - \frac{c_s^2 k^2}{a^2}\right)\delta_k = 0$$

There is a slight difference in amplitude to be considered, since the density of radiation is not exactly equal to the density of the plasma, but they are related by:

δ_plasma $= \frac{1+R}{1+\frac{4}{3}R}\delta_\gamma$ with $R \equiv \frac{3}{4}\frac{\bar{\rho}_b}{\bar{\rho}_\gamma}$ which explains the factor 1/4

Keep in mind this is an approximation, the detailed temperature fluctuations must be computed using the full formalism of the Einstein-Boltzmann equations.

From density to temperature



springs - photons

spheres - baryons

potential wells - dark matter

The plasma clustering grows up to a maximum δ (compression) and then starts to decrease due to the "pull of the photons".

Larger-scale perturbations reach larger amplitude of clustering and have lower frequency.

At z_{dec} each photon is released from its position, which has a certain density (i.e. from a position in a potential well)

All photons coming from perturbations of equal scale have the same energy (temperature or frequency), because the oscillations are in phase → the oscillation pattern survives in the released CMB power spectrum.
So the anisotropy of temperature detected today shows the inhomogeneities of the baryon density at $z = z_{dec}$

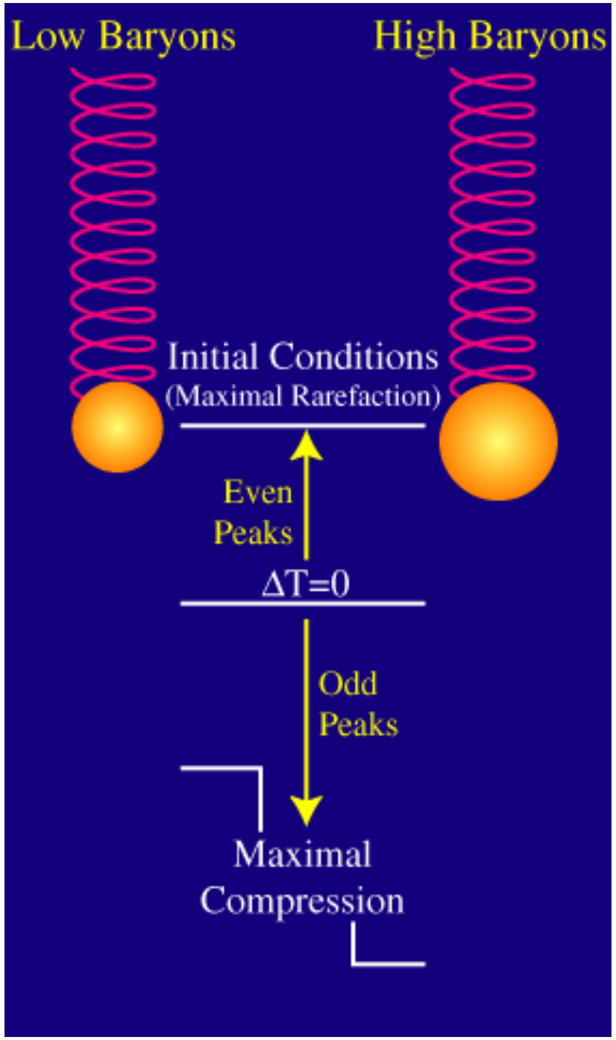
Odd versus even peaks

Now, note that odd peaks in the power spectrum represent scales of maximum compression and even peaks represent scales of maximum rarefaction (because the power spectrum is δ^2 → both peaks and valleys in δ are peaks in δ^2).

In models with **larger values of Ω_b**, the baryons also contribute to the potential → the maximum compression has higher amplitude → **larger amplitude of odd peaks compared to even peaks**

(the 2nd peak of the CMB power spectrum can be smaller than the 3rd peak for some values of the cosmological parameters).

The ratio between the amplitudes of the 2nd and 3rd peaks is a way to measure Ω_b (a completely different method than from the big bang nucleosynthesis).



First peak

The **first peak of the CMB power spectrum** corresponds to the largest scale with oscillations: the scale that reached maximum compression at decoupling → it is given by the maximum distance a perturbation can propagate from $a=0$ to $a=a_{dec}$ → it is the (comoving) sound horizon at decoupling.

This is the same scale responsible for the BAO peak in the matter power spectrum

$$r_s(z_*) = \int_0^{a_*} \frac{da'}{a'^2} \frac{c_s}{H(a')} \quad = \mathbf{105\ Mpc/h}$$

Note: integral from the big bang to the decoupling

Now, the angular scale of the 1st peak is the ratio: $l_a \equiv \pi \frac{r(z_*)}{r_s(z_*)}$

where r (z*) is the (comoving) angular diameter distance to the CMB surface:

$$r(z_*) = \frac{c}{H_0} \int_0^{z_*} \frac{dz'}{E(z')}$$

Note: integral from today to the decoupling

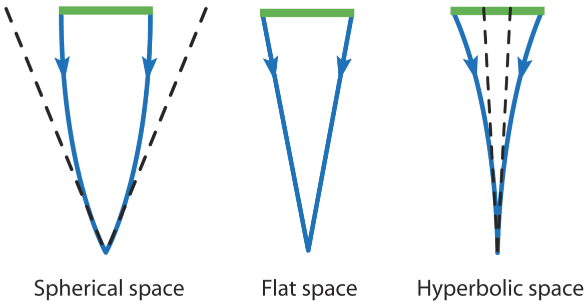
The comoving distance to z=z* is r (z*) ~ **6 Gpc/h** (concordance model)

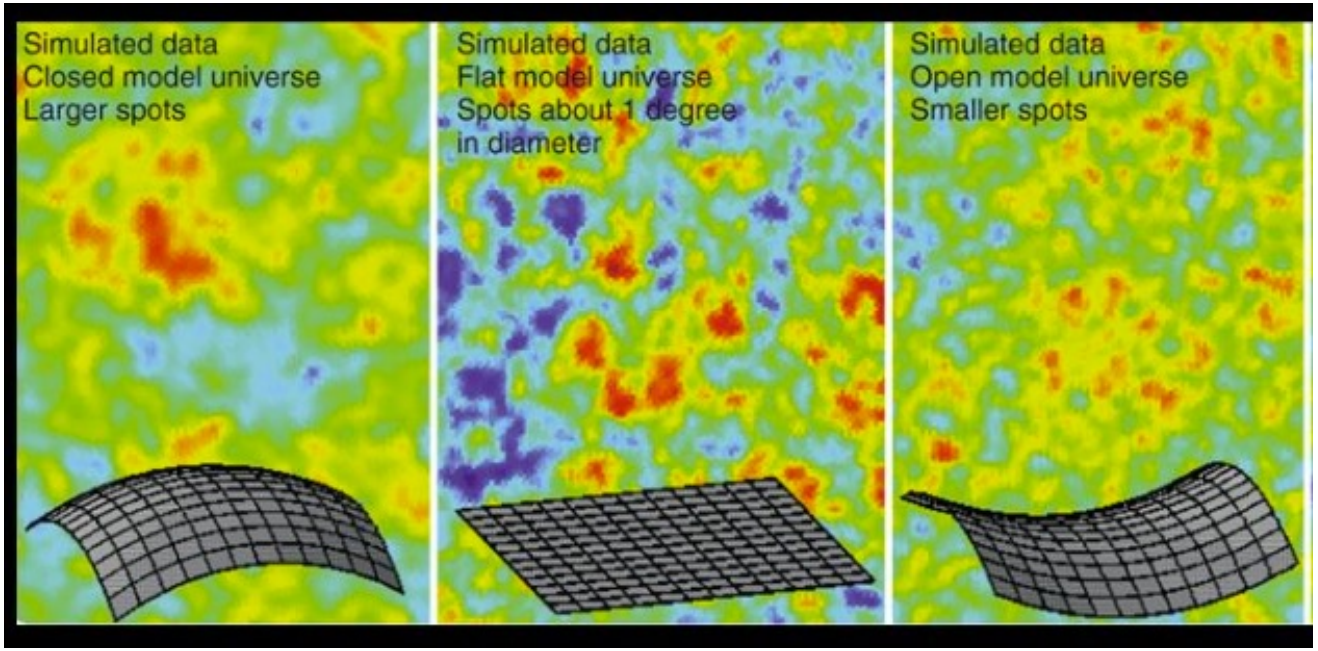
This means that the peak appears at l ~200 → θ ~1 deg

The position of the CMB first peak is a geometric probe :

measuring the peak angular scale constrains the ratio between the size of the sound horizon and the distance to the sound horizon → **the position of the first peak constrains the cosmological parameters** on which this ratio depends (i.e., H_0 and all Ω parameters, i.e., the background cosmological parameters or the parameters of geometry)

In particular, the CMB first peak or the BAO peak are good probes of the **curvature of the Universe**, because the l/θ relation depends explicitly on the curvature:





This was the first indication of the **spatial flatness** of the Universe.

Peculiar velocity **(Doppler effect)**

$$\left(\frac{\delta T}{T}\right)_{\text{obs}} = \frac{1}{4}\delta_\gamma^N - v^N \cdot \hat{n} + \Phi(t_{\text{dec}}, x_{\text{ls}}) + 2\int \dot{\Phi} dt$$

The plasma perturbations have a peculiar velocity, since they are moving on the potential wells.

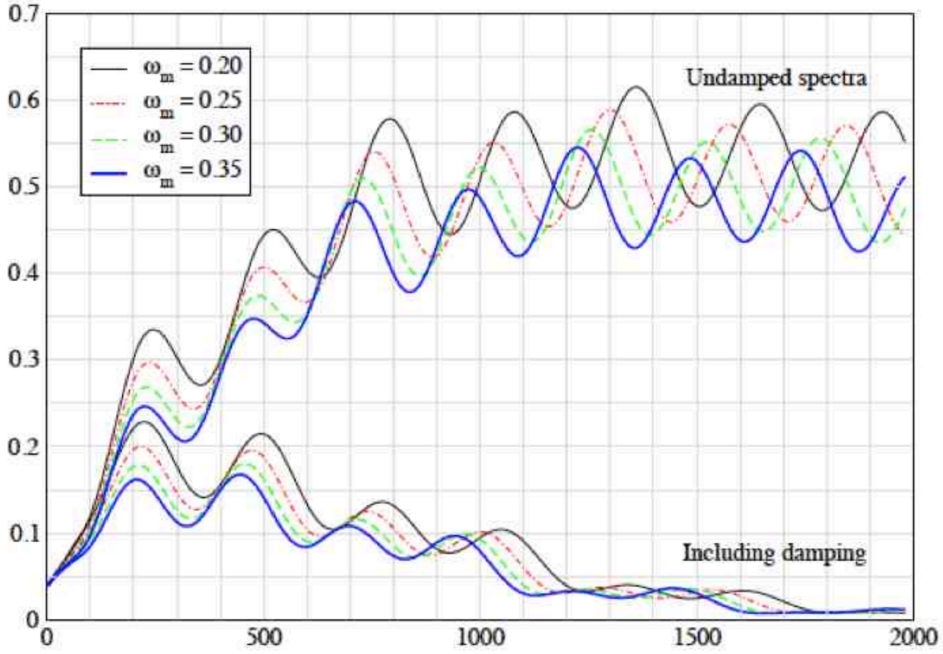
This is zero at the turnaround points (corresponding to maximum compression or rarefaction), i.e., at the positions of the CMB peaks.

But it is not zero at other scales → there is a Doppler effect that is larger for scales in-between peaks → **smoothing of the peak structure of CMB.**

The effect depends on the peculiar velocity, which is function of the potential and thus related with the density contrast.

The **v.n** contribution in the expression for ΔT/T is thus a Doppler effect.

The (dimensionless) CMB temperature power spectrum produced only by the acoustic oscillations (including Doppler shifts) would look like the upper grey curve:



Similar to the dimensionless matter power spectrum, but with peaks of large amplitudes.

This is quite different from what we observe, because there are still other important contributions to consider.

General relativistic effects

$$\left(\frac{\delta T}{T}\right)_{\rm obs} = \frac{1}{4}\delta_\gamma^N - v^N \cdot \hat{n} + \Phi(t_{\rm dec}, x_{\rm ls}) + 2\int \dot{\Phi} dt$$

i) **Density contrast on super-Hubble scales**

On large scales there is no trade-off between pressure and gravity (the Jeans eq. does not apply) → **gravity dominates.**

The growth of δ_γ is related to the growth of the potential (through Poisson equation).

The Einstein equations include equations for the evolution of δ, for the evolution of the potential (a metric perturbation), and a relativistic Poisson equation relating the two quantities. In the plasma epoch, a combination of those equations results in the following relation:

$$\delta = \left[1 + \frac{2}{3}\left(\frac{k}{aH}\right)^2\right]\Phi$$

This shows that on large scales, the perturbations do not relate with the potential only through the Laplacian (k^2 Φ) but have an extra **scale-independent term**.

On very large scales (k small) this term is the dominant one and δ ~ Φ

ii) **Gravitational time-delay**

There is a second GR gravitational effect, which arises because photons coming from denser regions suffer a larger **gravitational redshift** → this translates into a **time-delay** → **they decouple slightly later** → **the temperature of the universe is lower**.

Since in the matter epoch, a(t) ~ $t^{2/3}$ and T ~ 1/a, we can write:

$$\frac{\Delta T}{T} = -\frac{\Delta a}{a} = -\frac{2}{3}\frac{\Delta t}{t} = -\frac{2}{3}\Phi$$

(the potential is responsible for the time-delay).

This means that there is an effect on the emission temperature just because they are emitted later (due to the time-delay created by the potential) and not just because of larger density contrast (also created by the same potential).

The two gravitational effects have opposite impacts on the density contrast:

- through **Poisson equation**: larger potential → larger density contrast → higher CMB δT
- through **time-delay**: larger potential → larger gravitational redshift → lower CMB δT

The sum of the 2 effects is known as the **Sachs-Wolfe effect**:

$\Delta T/T\ (z_{dec}) = 1/3\ \Phi\ (z_{dec})$ → **on large scales, the amplitude of the CMB power spectrum is increased by the value of $\Phi/3$ at z_{dec}**

We need now to consider the **scale-dependence of Φ**, i.e., its power spectrum.

We saw that the primordial dimensionless potential power spectrum is scale-independent:

$$k^3\ P^0_\Phi \propto k^{n_s-1}$$

On large scales, the time-evolution of the potential has no transfer function, i.e., it evolves equally on all scales → at z_{dec}, P_Φ still keeps its original shape → a scale-independent dimensionless power spectrum.

This contribution to the CMB TT power spectrum is known as the Sachs-Wolfe plateau → **the dimensionless CMB power spectrum is constant on large scales**:

$l(l+1)/2\pi\ C_l$ is scale-independent on very large scales

On the scales near the first peak, both the acoustic and the gravitational effects contribute (oscillations and constant Sachs-Wolfe amplitude) → there is an added SW amplitude to the peak → **this is why the first CMB peak is much higher than the others.**

Integrated Sachs-Wolfe effect (ISW)

$$\left(\frac{\delta T}{T}\right)_{\rm obs} = \frac{1}{4}\delta_\gamma^N - v^N \cdot \hat{n} + \Phi(t_{\rm dec}, x_{\rm ls}) + 2\int \dot{\Phi} dt$$

After decoupling, there are extra SW contributions to the CMB temperature power spectrum, if the potential metric perturbation evolves in time.

Indeed, if the potential evolves in time during the time a photon takes to cross it, then the net gravitational frequency shift (redshift entering and blueshift exiting) is not zero → this extra contribution to ΔT/T is the term:

$$2\int \dot{\Phi} dt$$

This does not happen during the matter dominated epoch (where the solution for the evolution of Φ is Φ ~ constant), but **it happens during radiation and dark energy epochs, which originates two different effects:**

i) **Early-time ISW**

During the recombination period, radiation density is still high enough to produce some effect through the potential → the primary CMB (the emitted one) is still affected by this, and the SW contribution needs to include this early-time ISW effect → amplitude of CMB further increases on large scales.

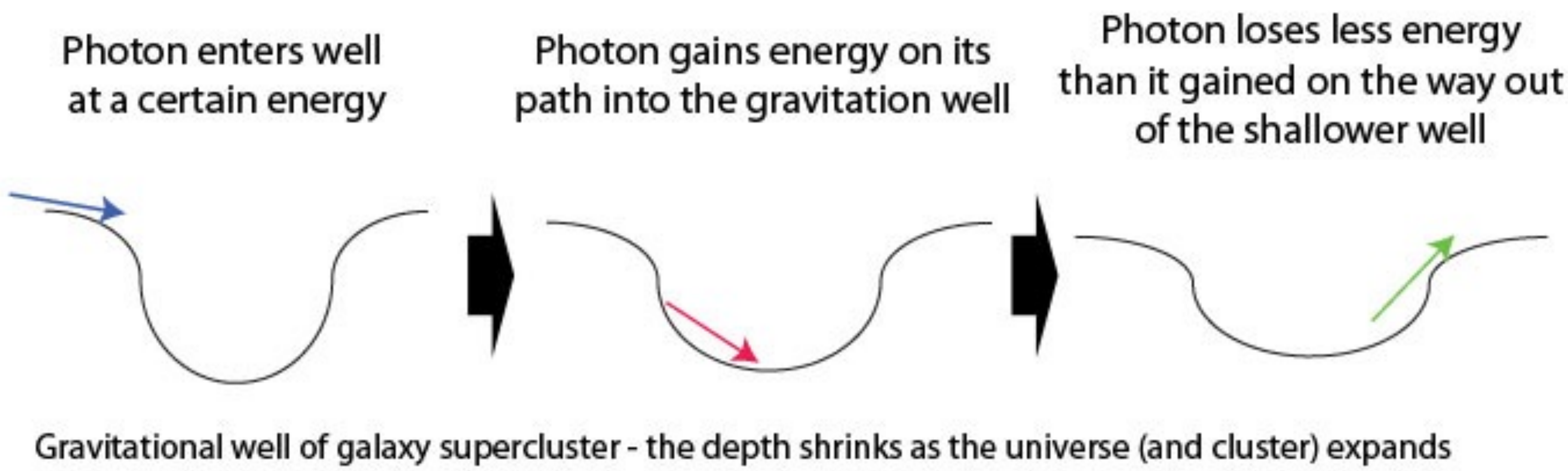
In models with **larger values of Ω_m**, the radiation epoch ends earlier → radiation effects such as the early ISW effect are less important at z_{dec} → no added contribution to the SW plateau → in particular, the **amplitude of the first peak is lower;**

the amplitude of the first peak directly constrains Ω_m

ii) **Late-time ISW**

In the late universe, when **dark energy** starts to be important, **the potential decreases with time** and there is an important contribution from an ISW effect.

This is called late-time ISW effect → amplitude of the secondary anisotropies of the CMB increases on large scales.



When CMB photons cross an evolving LSS potential they are first blueshifted (gain energy when entering) and then redshifted (lose energy when leaving).

The energy balance is not zero, they gain energy if the potentials decay → their temperature increases with respect to their original temperature.

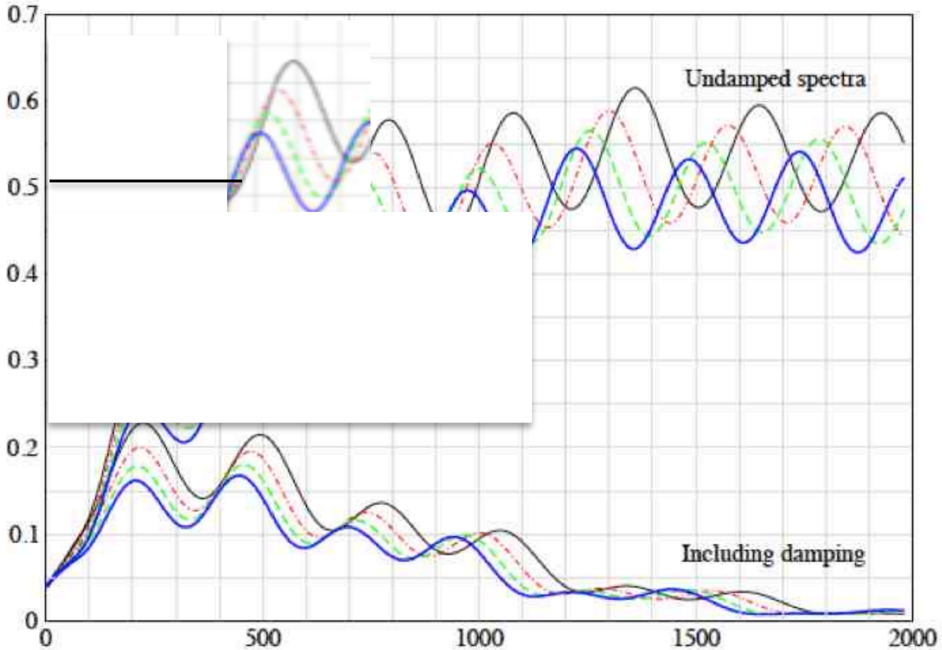
The potential perturbations of different scales can evolve in different ways → different dark energy (or modified gravity) models may have specific signatures for the scale-dependence of this effect.

Because it is a large-scale effect (since the photons take longer to cross the larger potentials it is only relevant on large physical scales) and at low redshift (dark energy dominated epoch), the angular scales are very large → it shows up on the SW plateau and not on the first peak → the observed SW plateau is no longer flat

the amplitude and shape of the SW plateau constrains dark energy and modified gravity models.

Diffusion damping **(Silk damping)**

Putting the four effects together, the CMB should look like a flat line on large scales with an oscillatory pattern on smaller scales, with equal peak amplitudes (except for a higher first peak), as shown in the upper grey curve



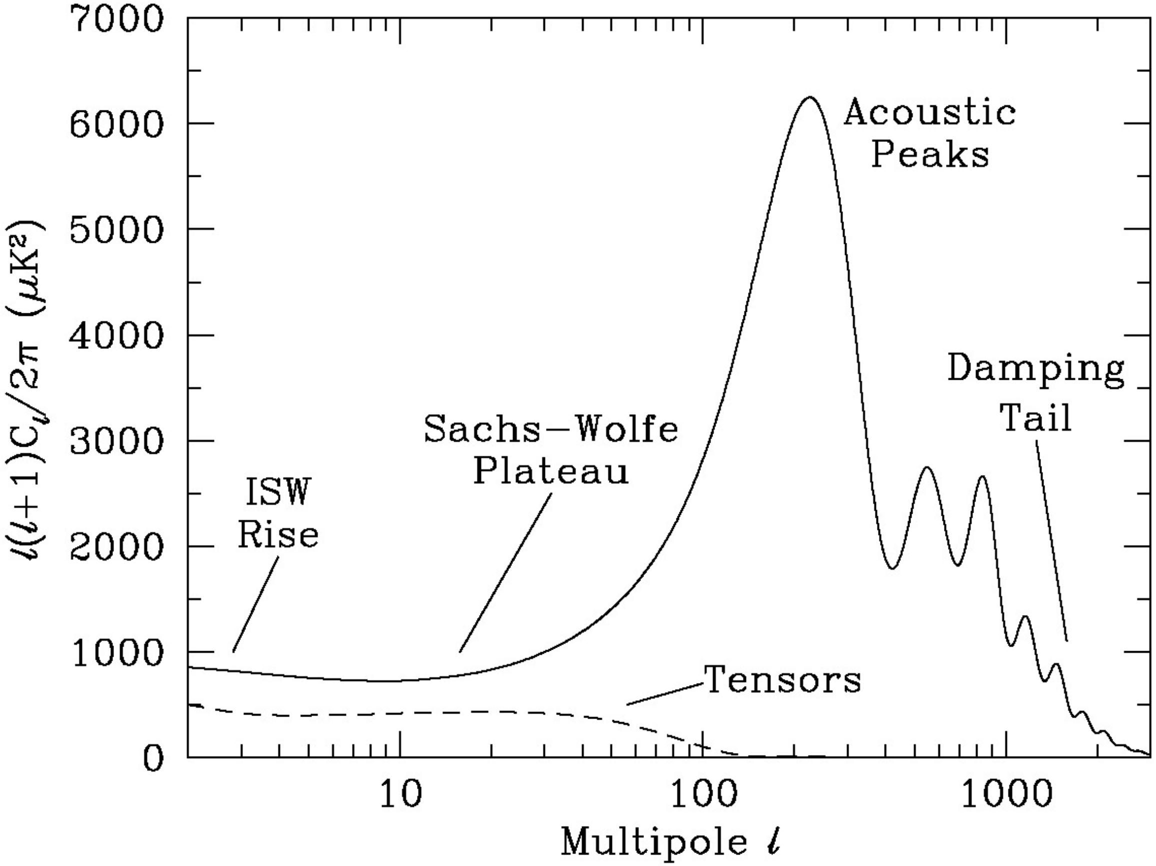
It is no longer similar to a dimensionless matter power spectrum with peaks, because on large scales it is flat (instead of increasing) → on large scales C_{l} is basically a C_{Φ}

However, the observed amplitudes of the peaks decrease with scale.

This is due to the fact that the last scattering surface has a finite thickness, i.e., **decoupling is not instantaneous.** During this delta time, the plasma is "**partially coupled**" → photons start to stream out of the overdensities and drag baryons with them → **destroying the overdensity** on scales below the damping scale.

In summary

Putting all effects together, the dimensionless CMB power spectrum looks like this:



The existence of **various features** – position of the first peak ($\boldsymbol{\Omega}_i$, curvature, H_0), amplitude of the first peak ($\boldsymbol{\Omega}_m$), relative amplitude of odd vs even peaks ($\boldsymbol{\Omega}_b$), general amplitude (A_s or $\boldsymbol{\sigma}_8$, $\boldsymbol{\tau}_{re}$), tilt of the shape (n_s), shape of the SW plateau (DE) – determined by **different physical effects**, explain why the **CMB is the most powerful cosmological probe in constraining the cosmological parameters.**

Secondary temperature anisotropies

In addition to these effects responsible for the CMB primary anisotropies, there are other effects that change the CMB power spectrum during its propagation, the secondary anisotropies → **the (theoretical) observationally estimated CMB is different from the one emitted at z_{dec}.**

Besides the Late-time ISW effect, other secondary effects are:

Rees-Sciama effect - Similar to ISW, but due to peculiar velocities of the potentials (instead of time variations).

Gravitational waves - They may also change the potentials and create another ISW effect.

Gravitational lensing - Deflects each photon to another direction in a coherent way. It mixes the directions → smearing out the anisotropies **(smoothing the peaks)**. In addition, it also changes the original polarization of the CMB.

Sunyaev-Zeldovich effect - Scattering of CMB photons if they pass in the hot gas of galaxy clusters

Reionization - New free electrons start to be available from first stars (galaxy formation) → they scatter CMB photons, which become more isotropically distributed → this reduces the amplitude of the anisotropies on all scales by a factor exp($-\tau^2$) → If the effect was too strong it would completely destroy the CMB anisotropies.

The optical depth of this scattering is **another fundamental parameter of the ΛCDM cosmological model**. It is defined as:

$$\tau = \int_0^{z_{\rm rec}} dz\, n_e\, \sigma_T \qquad\qquad n_e = \Omega_{\rm gas} \frac{3H_0^2}{8\pi G} \frac{1}{\mu m_p} (1+z)^3$$

it depends on the electron density in the stars, and the cross section for the interaction

In alternative to τ, the effect can also be parameterized by the redshift of reionization, $z_{re;}$

the amplitude of the CMB power spectrum constrains the redshift of the first stars.

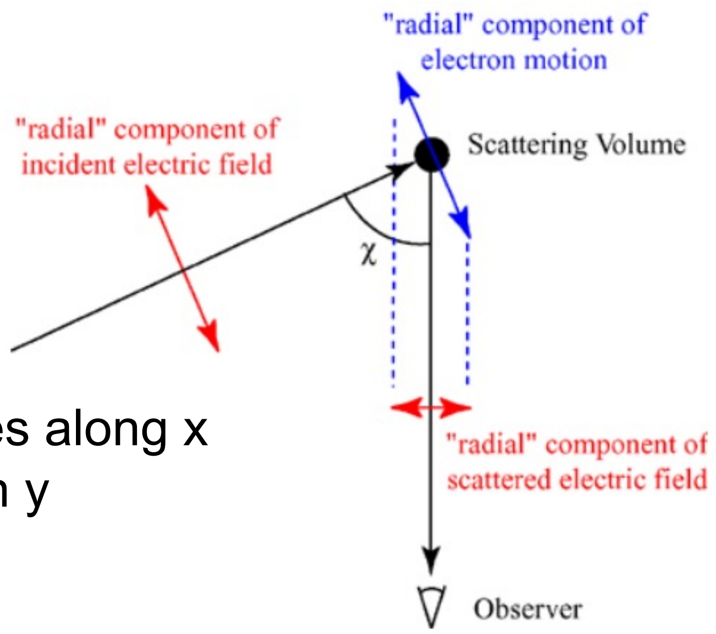
CMB polarization

At the end of the plasma epoch, as the temperature decreases, the interaction between baryons and photons starts to be dominated by **Thomson scattering.**

This is the low-energy limit of Compton scattering. It occurs for lower energy photons and the interaction does not produce a change in the frequency of the photon, or a change in the kinetic energy of the baryon.

So, photons no longer lose energy but can get polarized → this is the origin of the **CMB polarization**

In general:



photon moves along x with E field in y

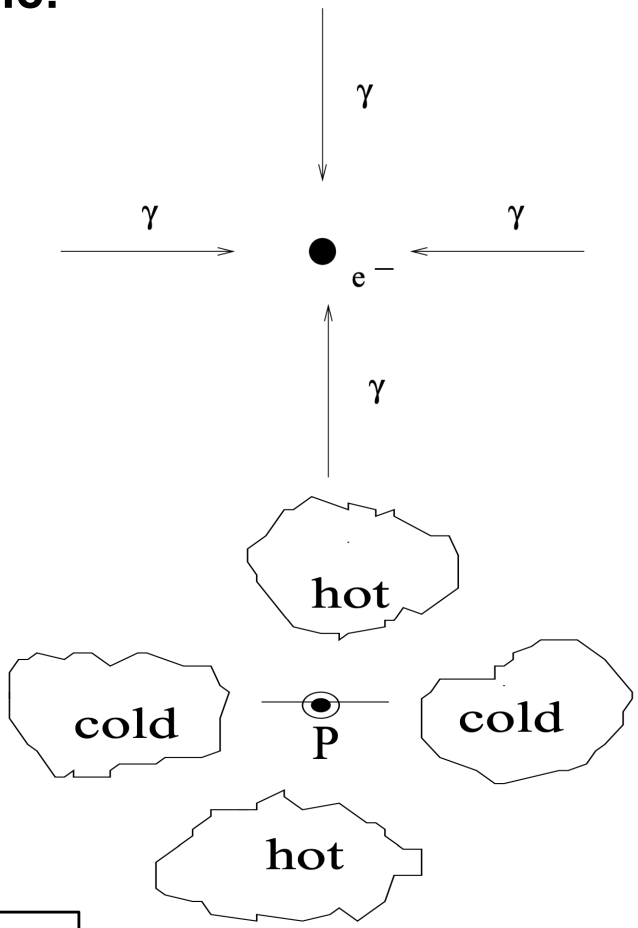
photon scatter off the baryon and makes it to oscillate along y → new photon propagates along z with E field in y

If the incoming radiation is unpolarized but anisotropic:

Incoming photons are unpolarized → they contain all polarization directions perpendicular to the propagation.

- photons moving along x have E field in y,z plane
- photons moving along y have E field in x,z plane

Anisotropy → example: y photons have slightly more energy than x photons → they induce baryon movement along x → polarized emission along z (to the observer) with slight polarization along x.



CMB polarization can only be produced by Thomson scattering at the **last scattering surface** → it is the "purest" probe of inhomogeneities at the LSS.

CMB temperature anisotropies have contributions from secondary anisotropies along the way → they are not a pure image of the LSS.

The anisotropic **quadrupole** configuration (C_2) is the dominating source of polarization

The polarization signal is a measure of the electric field:

$$E_x = a_x \cos(\omega t - \xi_x); \qquad E_y = a_y \cos(\omega t - \xi_y).$$

It is usually decomposed in the **Stokes parameters:**

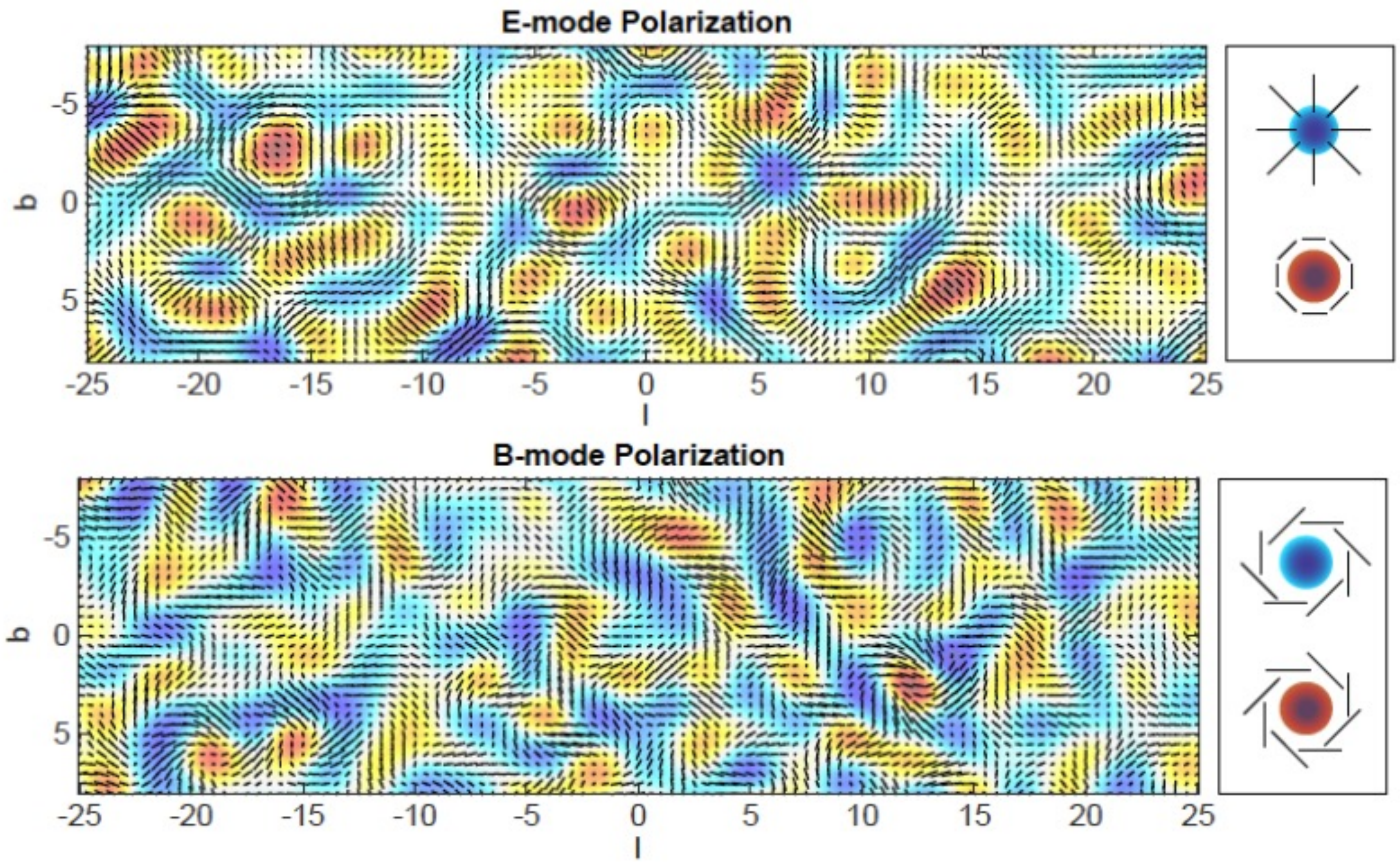
Intensity	$I = a_x^2 + a_y^2,$
linear polarization +	$Q = a_x^2 - a_y^2,$
linear polarization x	$U = 2a_x a_y \cos(\xi_x - \xi_y),$

They depend on the scattering cross-section, which depends on the density of baryons, etc → contain physical information on the cosmological model.

$$I = \frac{3\sigma_T}{16\pi} I' \left(1 + \cos^2\theta\right) \qquad Q = \frac{3\sigma_T}{16\pi} I' \sin^2\theta \qquad U = 0$$

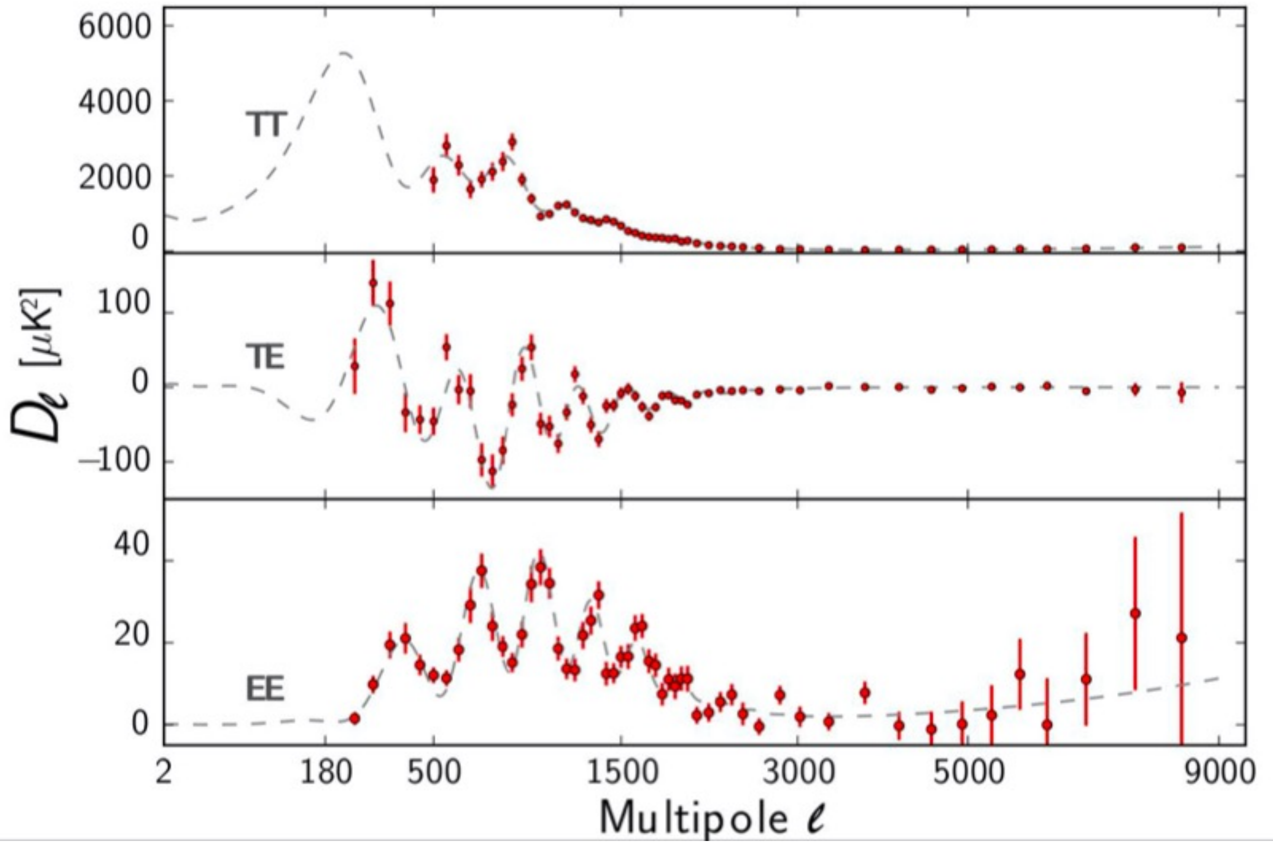
The intensity of the signal is very small, because only a fraction of the initial intensity scatters in the direction of the observer.

Each point of the sky has a certain value of polarization Q,U (also called E,B). There are thus two **CMB polarization maps** in addition to the CMB temperature map.



B-type polarization is not produced by interaction with baryons. It is produced by interaction with tensor-type metric perturbations (i.e. it is produced by gravitational waves) → U is a signature of **gravitational waves.**

The **power spectra** of these maps is measured and is also computed theoretical



The amplitudes of the polarization power spectra are much lower than the TT power spectrum.

Cross-correlation with the temperature map is useful to confirm that the TT signal comes primarily from the LSS.

ACTPol data

LSS probes: Sensitivity to the cosmological parameters

We described the features of **the two central cosmological functions** of the inhomogeneous universe:

Matter power spectrum → structure in the dark matter density field + effect of baryons (oscillations).

It is a function of redshift.

It is not directly observable, but determines the power spectra of several observable cosmological fields that are derived from it (galaxy clustering, weak gravitational lensing, peculiar velocity, cluster abundance, Lyman-alpha forest, etc.)

CMB Temperature power spectrum → anisotropy in the cosmological photons' temperature field → structure in the radiation/baryon plasma density field.

At a *fixed redshift* (decoupling).

It is *directly observable*.

Our information on the properties of the inhomogeneous universe comes mostly from measurements of these two power spectra.

The CMB power spectrum is the most powerful one to constrain the cosmological parameters because:

- it has a rich structure (much more features than the matter power spectrum);

- it depends on many cosmological parameters in different ways;

- it contains information from both the early universe and the recent universe (through the secondary anisotropies);

- it can be measured with high precision and on the whole sky;

- it is less affected by theoretical biases than other probes because its perturbations are linear;

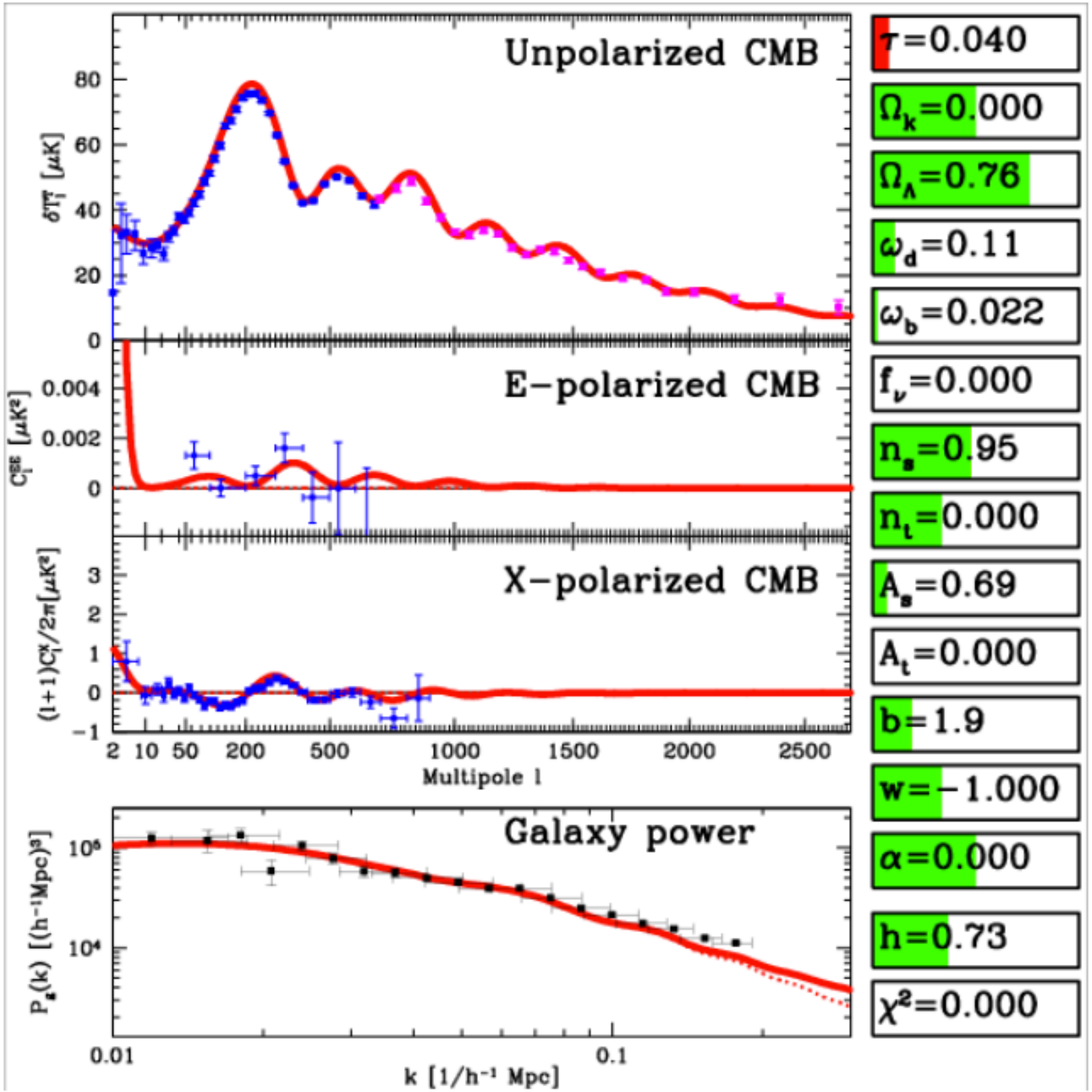
- it is less affected by astrophysical biases than other probes (the primary anisotropies).

The various LSS power spectra can be computed for **input values of the cosmological parameters.**

There are several **codes** that implement the set of equations (Newtonian, Einstein, Boltzmann, transfer function, non-linear fits):

CLASS
http://class-code.net/

CAMB



These **interactive CMB and matter power spectra movies** made from the outputs of those codes can be found at: ***https://space.mit.edu/home/tegmark/parmovies***