Cosmologia Física

Homework 5 due 14 May 2026 (iatereno@fc.ul.pt)

Exercise 1: Dark matter linear perturbations

1.1) Consider a flat Universe with some amount of non-relativistic pressureless massive neutrinos and no baryonic matter, and no dark energy. In this Universe, the mean matter density Ω_m has two contributions: cold dark matter $\Omega_{\rm cdm}$ and neutrinos Ω_ν, i.e., $\Omega_m = \Omega_{\rm cdm} + \Omega_\nu$. We assume that only the cold dark matter component clusters and forms structure.

a) Compute the evolution of the scale factor $a(t)$ in the matter-dominated epoch for this universe.

b) Compute the evolution of the cold dark matter density contrast for this universe in the matter-dominated epoch.

c) Assume the neutrino fraction is 10% of the total matter density. In this case, using the result found in b), what is the growth rate (n) of the dark matter overdensities: $\delta_{\rm cdm} \propto a^n$?

d) Consider now a ΛCDM universe (Ω_ν is now 0, but there is a non-zero Ω_Λ) where the dark matter growth rate is exactly the same you found in c) (i.e. n=0.94). What is the value of Ω_Λ? Hint: Remember the definitions of the growth parameters f and γ.

e) Consider now a dynamical wCDM universe (i.e. w=w(z)) with the same growth rate as in the universes of c) and d). Assume this model is indistinguishable from the concordance ΛCDM at z=0. Show that w(z) changed by roughly an order of magnitude since $z = 1$.

1.2) Consider a flat Universe with dark matter and a cosmological constant, such that the transition from matter to dark energy epochs occurs only today, at $a = 1$. In the matter epoch, the matter density contrast grows as $\delta \propto a$. Assume that $\delta(t)$ keeps that rate all the way until the transition (i.e., dark energy does not make the rate to decrease during the matter epoch). This means that we can write $\delta(a) = a\,\delta_0$, where δ_0 is the clustering amplitude today. After the transition, the universe is dominated by the cosmological constant. Assume that the mean dark matter density can be neglected immediately after the transition: $\Omega_m(a > 1) = 0$.

a) Solve the equation of motion for the dark matter density constrast δ in the dark energy dominated epoch $a > 1$.

b) In a) you must have found that the solution for $\delta(t)$ is the sum of a decaying solution plus a constant term (an integration constant). Show that the constant term in the solution can be written in the form $n\delta_0$. Find out the value of n.

c) Assume that collapsed structures (i.e. non-linear) did not yet form in this universe at $a = 1$, i.e., $\delta_0 < 1$. What is the minimum value that δ_0 must have for collapsed structure to be able to form in this universe in the future?

Exercise 2: Dependence on the cosmological parameters

2.1) Consider the matter power spectrum and the CMB power spectrum. Say if the following sentences are true or false, justifying your answer (no calculations are needed).

With respect to the concordance model:

a) increasing the value of Ω_m, the main peak of the matter power spectrum moves to the right.

b) including an early dark energy model that increases the speed of sound in the baryon-photon plasma, the first peak of the CMB moves to the right.

c) increasing the value of the Hubble constant, there is an effect on the amplitude of the first peak of the CMB.

d) increasing the value of the Hubble constant, the first peak of the CMB moves to the left.

e) decreasing the value of Ω_b, the amplitudes of the odd peaks of the CMB increase with respect to the amplitudes of the even peaks.

f) increasing the value of Ω_m, the amplitude of the first peak of the CMB increases due to the early ISW effect.

g) increasing the value of n_s, the amplitude of the first peak of the CMB increases.

h) increasing the value of n_s, the amplitude of the fifth peak of the CMB increases.