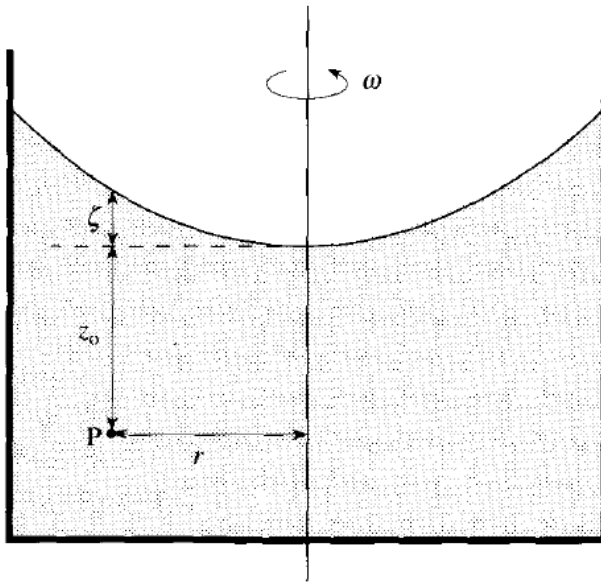


The bucket of liquid

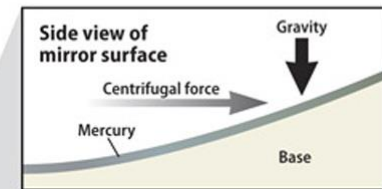
Faber, 2.5



Liquid mirror telescope:
➔ <https://www.youtube.com/watch?v=Q5Cr9P-Q88Y>

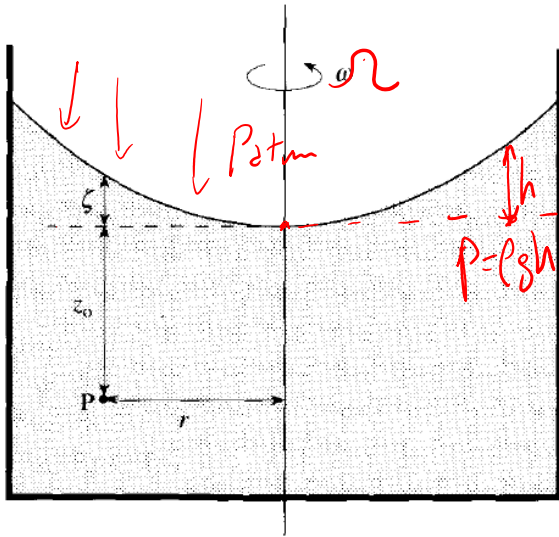


How liquid-mirror telescopes work



A **liquid-mirror telescope** uses a thin layer of mercury within a rotating dish to form a reflective surface to collect light and focus it. As the platform rotates, the combination of gravity and centrifugal force sculpts the liquid mercury into an extremely smooth parabolic surface. The telescope scans a wide swath of the sky directly overhead. Astronomy: Roen Kelly

In the steady state, the velocity profile becomes



$$\vec{u} = \Omega r \hat{\varphi} = -\Omega y \hat{x} + \Omega x \hat{y}$$

$$\hat{\varphi} = \frac{-y \hat{x} + x \hat{y}}{\sqrt{x^2 + y^2}}$$

Vorticity

$$\vec{\omega} = \nabla \times \vec{u} = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ -\Omega y & \Omega x & 0 \end{vmatrix} = \hat{z} \left(\frac{\partial}{\partial x} (\Omega x) + \frac{\partial}{\partial y} (\Omega y) \right) = 2\Omega \hat{z}$$

Why Euler? The velocity field above leads to zero off-diagonal components of the stress tensor (check). Thus, the viscous effects are not relevant and we can use the ideal fluid approximation.

$$\sigma_{ij} = -P\delta_{ij} + \eta \left(\frac{\partial u_j}{\partial x_i} + \frac{\partial u_i}{\partial x_j} \right)$$

Euler's equation

$$\frac{D\mathbf{u}}{Dt} = -\frac{1}{\rho} \nabla p + \mathbf{g},$$

or

$$\cancel{\frac{\partial \mathbf{u}}{\partial t}} + \underbrace{(\nabla \wedge \mathbf{u})}_{\vec{\omega}} \wedge \mathbf{u} = -\nabla \left(\frac{p}{\rho} + \frac{1}{2} \mathbf{u}^2 + \cancel{\chi} \right) \quad \text{gh}$$

$$\vec{\omega} \times \vec{u} = -\nabla H$$

$$\Rightarrow \vec{\omega} \times \vec{u} = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ 0 & 0 & 2\Omega \\ -\Omega y & \Omega x & 0 \end{vmatrix} = -\hat{x} (2\Omega^2 x) + \hat{y} (-2\Omega^2 y) \\ = \boxed{-2\Omega^2 (x \hat{x} + y \hat{y})}$$

$$\Rightarrow \vec{u}^2 = \Omega^2 y^2 + \Omega^2 x^2$$

$$\begin{aligned} \nabla \vec{u}^2 &= \left(\hat{x} \frac{\partial}{\partial x} + \hat{y} \frac{\partial}{\partial y} + \hat{z} \frac{\partial}{\partial z} \right) (\Omega^2 x^2 + \Omega^2 y^2) \\ &= 2\Omega^2 x \hat{x} + 2y \Omega^2 \hat{y} \end{aligned}$$

$$\Rightarrow \nabla p = 0 \quad (\text{Superficie d'atm})$$

$$\Rightarrow \nabla(\rho h) = \rho \nabla h$$

Composante x

$$-2\Omega^2 x = -\frac{1}{x} \chi \Omega^2 x - \rho g \frac{\partial h}{\partial x} \Rightarrow \frac{\partial h}{\partial x} = \frac{\Omega^2 x}{g}$$

$$h(x) = \frac{\Omega^2 x^2}{2g} + C_1(y, z)$$

Componente y

$$-2\Omega^2 y = -\frac{1}{r} \cancel{r} y \Omega^2 - g \frac{\partial h}{\partial y} \Rightarrow \frac{\partial h}{\partial y} = \frac{\Omega^2 y}{g}$$

$$h(y) = \frac{\Omega^2 y^2}{2g} + C_2(x, z)$$

$$\Rightarrow \boxed{h = \frac{\Omega^2}{2g} (x^2 + y^2) + C_{1c}} = \frac{\Omega^2}{2g} r^2 + C_{1c}$$

Calcular "p" para $h = cte$

Componente x

$$\Rightarrow \nabla h = 0, \quad \nabla p \neq 0$$

$$-2\Omega^2 x = -\frac{1}{\rho} \frac{\partial p}{\partial x} - \frac{1}{\cancel{r}} 2\Omega^2 x \Rightarrow \frac{\partial p}{\partial x} = \rho \Omega^2 x$$

$$p = \frac{\rho \Omega^2 x^2}{2} + C_1(y, z)$$

Componente y

$$-2\Omega^2 y = -\frac{1}{\rho} \frac{\partial p}{\partial y} - \frac{1}{2} 2\Omega^2 y \Rightarrow p = \frac{\rho \Omega^2 y^2}{2} + C_2(x, z)$$

$$p = \frac{\rho \Omega^2}{2} (x^2 + y^2) + Gz = \frac{\rho \Omega^2}{2} r^2 + p_0$$

$$\rightarrow h = \frac{\Omega^2 r^2}{2g} \Rightarrow \Omega^2 r^2 = 2gh$$

$$p = \frac{\rho}{2} 2gh + p_0 = \boxed{\rho gh + p_0}$$