# Planetary boundary layer

Lecture 2

# Homework

1-5 – Cátia

2-2 – Diogo

2-3 – Florian

2-5 – Jason

2-8 – Maria

2-9 – Mariana

2-11 – Sara

# The equations in a continuum (10 eq, 10 unknows)

$$\frac{\partial \vec{v}}{\partial t} = -\vec{v}.\nabla\vec{v} - \frac{1}{\rho}\nabla P + \vec{g} - 2\overrightarrow{\Omega} \times \vec{v} + \nu\nabla^2\vec{v}$$

Navier-Stokes

$$\frac{\partial \theta}{\partial t} = -\vec{v}.\nabla\theta + \dot{Q}_{rad} + \dot{Q}_{lat} + \kappa\nabla^2\theta$$

Thermodynamics

$$\frac{\partial \rho}{\partial t} = -\nabla.(\rho\vec{v}) = -\vec{v}.\nabla\rho - \rho\nabla.\vec{v}$$

Continuity

$$\frac{\partial q_{v,l,s,\dots}}{\partial t} = -\vec{\mathrm{v}}.\nabla q_{v,l,s,\dots} + G_{q_{v,l,s,\dots}} + \kappa_D\nabla^2 q_{v,l,s,\dots}$$

Water budget

$$p = R_d\rho T(1 + 0.61\, q_v)$$

State equation

$$\theta = T\left(\frac{P}{P_{00}}\right)^{-\frac{R_d}{c_p}}$$

Potential temperature

# The Navier-Stokes equation is an application of Newton's second law

$$\vec{F} = m\vec{a} \Longrightarrow \vec{a} = \frac{d\vec{v}}{dt} = \frac{\vec{F}}{m}$$

it applies to a moving body of mass $m$. In a fluid we can not follow a set of fluid molecules. Euler proposed to look instead at fixed (not moving) elements of volume. The time evolution of a variable at a fixed position is its tendency, given by the partial derivative $\partial/\partial t$. Euler showed that:

$$\frac{d\vec{v}}{dt} = \frac{\partial\vec{v}}{\partial t} + u\frac{\partial\vec{v}}{\partial t} + \mathrm{v}\frac{\partial\vec{v}}{\partial y} + \mathrm{w}\frac{\partial\vec{v}}{\partial z}$$

Where $\vec{v} \equiv (u, v, w)$.

This leads to the Euler equation:

$$-\vec{v}.\nabla\vec{v}$$

$$\frac{\partial\vec{v}}{\partial t} = -u\frac{\partial\vec{v}}{\partial t} - \mathrm{v}\frac{\partial\vec{v}}{\partial y} - \mathrm{w}\frac{\partial\vec{v}}{\partial z} + \frac{\vec{F}}{m}$$

tendency advection

$$\frac{\partial \vec{v}}{\partial t} = -\vec{v}.\nabla\vec{v} - \frac{1}{\rho}\nabla P + \vec{g} - 2\vec{\Omega} \times \vec{v} + \nu\nabla^2\vec{v}$$

Navier and Stokes introduced viscosity. Viscosity changes the order of the equation and allows the no-slip condition at the boundary.

Within the fluid, viscosity promotes diffusion: the transport of properties leading to homogeneization. It appears in different equations:

$$\nu\nabla^2\vec{v};\ \kappa\nabla^2\theta;\ \kappa_D\nabla^2 q_{v,l,s,\dots}$$

If a given variable is spatially homogenous, $\nabla^2\theta = 0$ and this terms are zero.

# The equations can only be solved numerically

The equations are non-linear. In the Navier-Stokes equation, non-linearity comes from 2 terms:

$$\frac{\partial \vec{v}}{\partial t} = -\vec{v}.\nabla\vec{v} - \frac{1}{\rho}\nabla P + \vec{g} - 2\overrightarrow{\Omega} \times \vec{v} + \nu\nabla^2\vec{v}$$

To solve the equations (by analytical or numerical methods) we need initial and boundary conditions: we need observations of the initial state, and other equations for the Earth and Ocean surface.

# We cannot observe the continuum

Reynolds noted that to relate the equations with real world states, one needs to apply spatial/time averaging: that is what sensors do.

Reynolds averaging

# Equations

Approximations by scaling: compare the relevance of different terms

Perturbation methods: $u = U + u'$

Linearization: compute $\frac{\partial u'}{\partial t}$ $(with\, u' \ll U)$

Turbulence: compute $\frac{\partial \overline{u}}{\partial t}$