Planetary boundary layer

Lecture 5

Homework

5.2 – Jason

5.3 – Maria

5.5 – Mariana

5.7 – Sara

5.12 – Cátia

5.14 – Diogo

5.22 – Florian

5 Turbulence Kinetic Energy, Stability and Scaling

$$\frac{\partial \bar{e}}{\partial t} + \overline{U}_j \frac{\partial \bar{e}}{\partial x_j} = + \delta_{i3} \frac{g}{\overline{\theta}_v}\left(\overline{u_i'\theta_v'}\right) - \overline{u_i'u_j'} \frac{\partial \overline{U}_i}{\partial x_j} - \frac{\partial\left(\overline{u_j'e}\right)}{\partial x_j} - \frac{1}{\overline{\rho}} \frac{\partial\left(\overline{u_i'p'}\right)}{\partial x_i} - \varepsilon \tag{5.1a}$$

I II III IV V VI VII

If we choose a coordinate system aligned with the mean wind, assume horizontal homogeneity, and neglect subsidence, then a special form of the TKE budget equation can be written

$$\frac{\partial \bar{e}}{\partial t} = \frac{g}{\overline{\theta}_v}\left(\overline{w'\theta_v'}\right) - \overline{u'w'} \frac{\partial \overline{U}}{\partial z} - \frac{\partial\left(\overline{w'e}\right)}{\partial z} - \frac{1}{\overline{\rho}} \frac{\partial\left(\overline{w'p'}\right)}{\partial z} - \varepsilon \tag{5.1b}$$

I III IV V VI VII

$$-\varepsilon$$

Turbulence is ***dissipative***. Term VII is a loss term that always exists whenever TKE is nonzero. Physically, this means that turbulence will tend to decrease and disappear with time, unless it can be generated locally or transported in by mean, turbulent, or pressure processes. Thus, TKE is not a conserved quantity. The boundary layer can be turbulent only if there are specific physical processes generating the turbulence. In the next subsections, the role of each of the terms is examined in more detail.

Storage: $\frac{\partial \bar{e}}{\partial t}$

Over a land surface experiencing a strong diurnal cycle, typical order of magnitudes for this term range from about 5×10^{-5} m^2 s^{-3} for surface-layer air over a 6 h interval, to about 5×10^{-3} m^2 s^{-3} for FA air that is spun up over 15 min (i.e., over a time interval corresponding to t_*). Fig 5.2 shows sample observations of TKE made in the surface layer, where TKE varies by about two orders of magnitude.

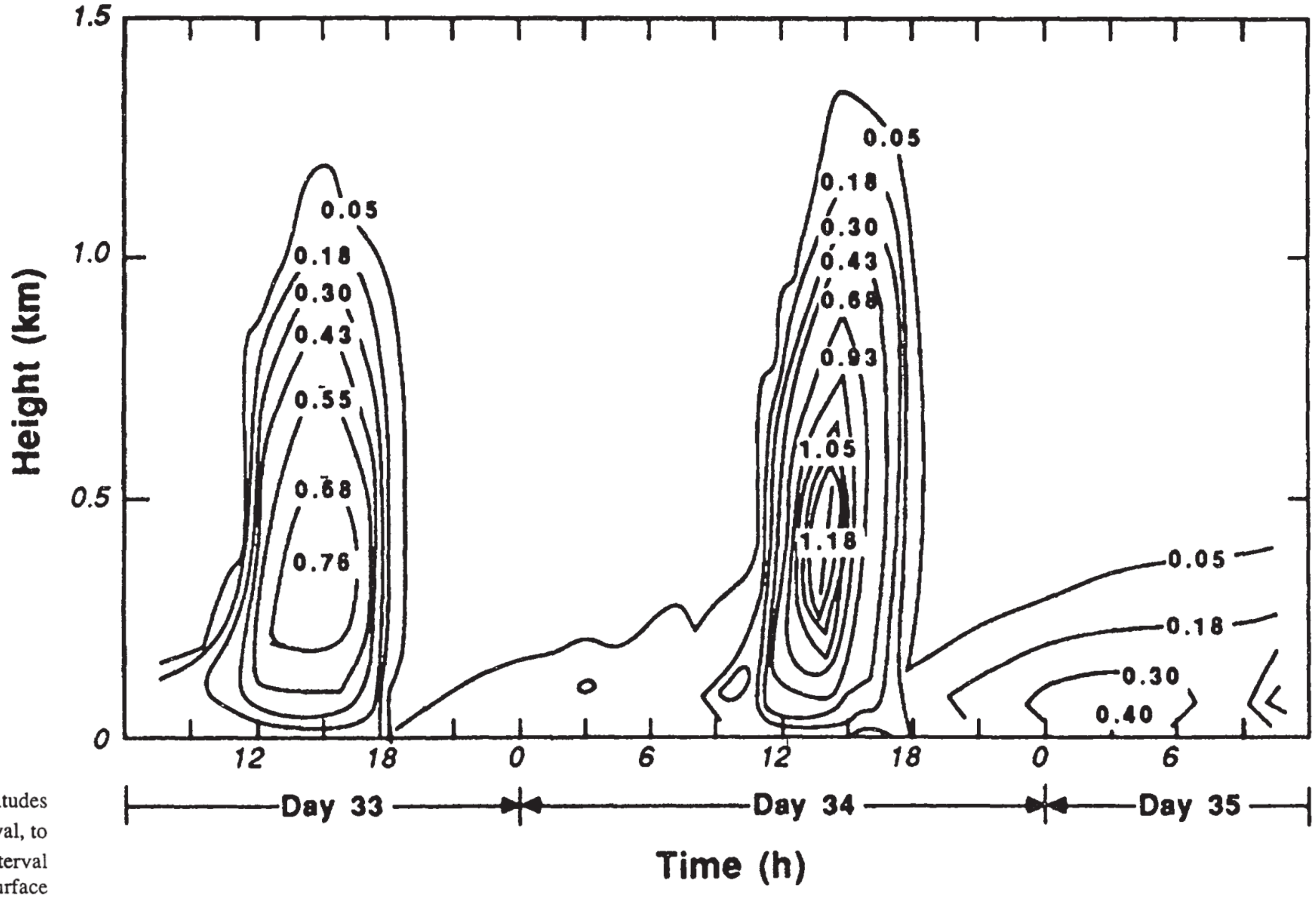
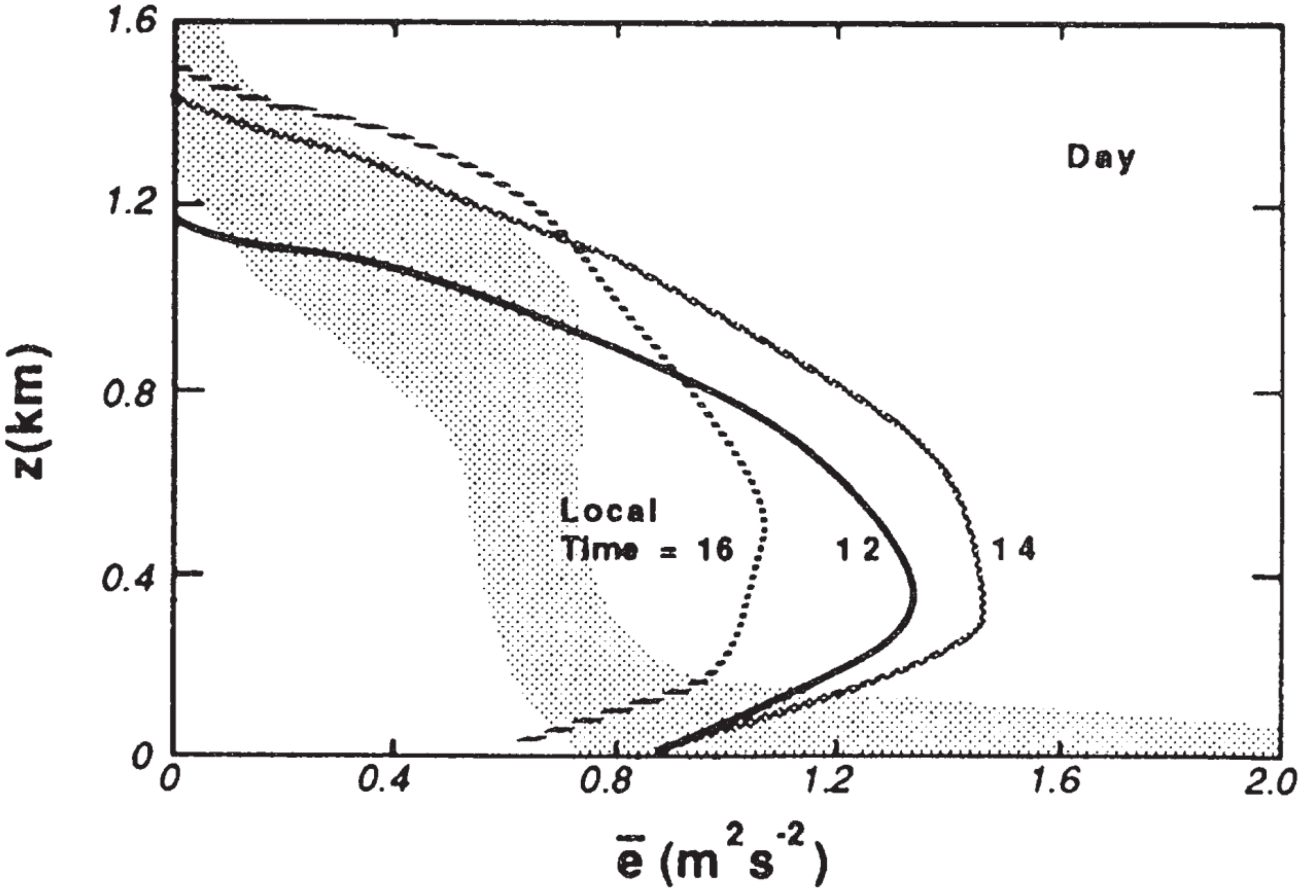


Fig. 5.1 Modeled time and space variation of $\bar{e}$ (turbulence kinetic energy, units m^{2}s^{-2}), for Wangara. From Yamada and Mellor (1975).

Diurnal cycle of TKE/m profile

Fig. 5.3
The lines show modeled vertical profiles of turbulence kinetic energy, $\overline{e}$, during Day 33, Wangara, (after Therry and Lacarrere, 1983). The shaded profile applies when both shears and buoyancy are active (after Hechtel, 1988).



Main terms: vertical profiles

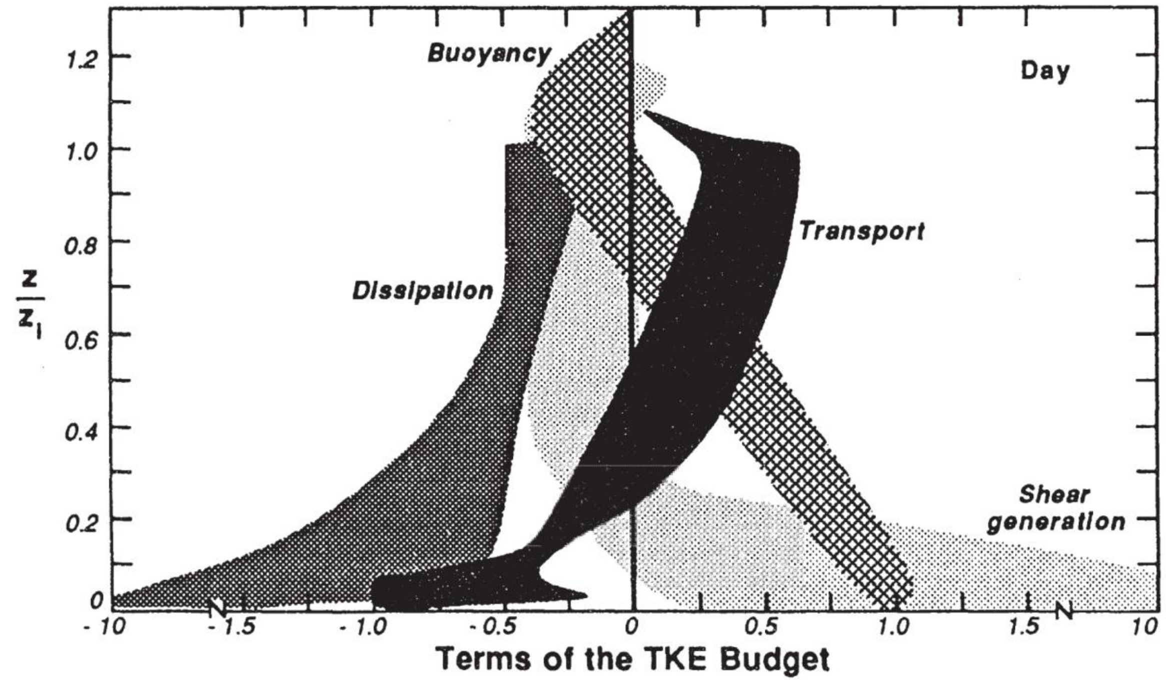


Fig. 5.4 Normalized terms in the turbulence kinetic energy equation. The shaded areas indicate ranges of values. All terms are divided by w_*^3 / z_i, which is on the order of $6x\ 10^{-3} m^2 s^{-3}$. Based on data and models from Deardorff (1974), André et al. (1978), Therry and Lacarrere (1983), Lenschow (1974), Pennell and LeMone (1974), Zhou, et al. (1985) and Chou, et al. (1986).

Cloud-topped tropical PBL

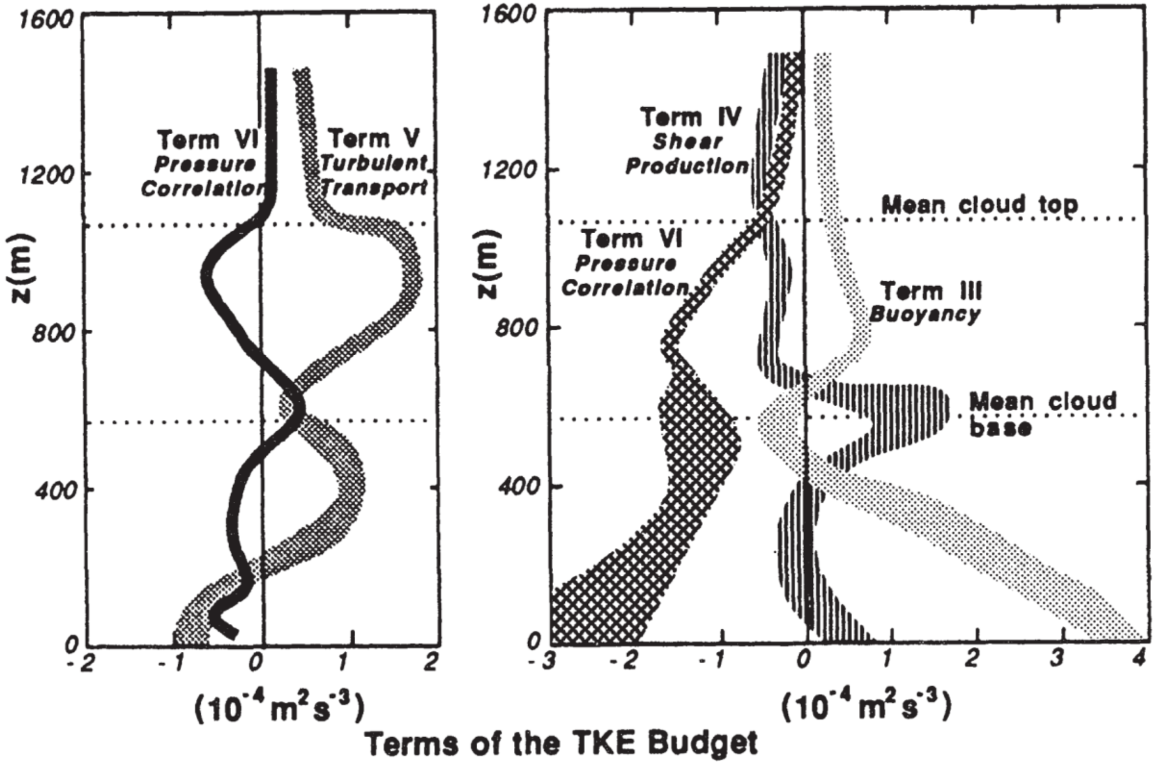


Fig. 5.5 Range of terms in the turbulent kinetic energy budget for a cloud - topped tropical boundary layer. The transport term is split into the pressure correlation (PC) and turbulent transport (T) parts. After Nicholls, et al. (1982).

Normalized for the convective PBL

$$\frac{z_i}{w_*^3}\frac{\partial \bar{e}}{\partial t} = \frac{g\, z_i \left(\overline{w'\theta_v'}\right)}{w_*^3\, \overline{\theta_v}} - \frac{z_i\, \overline{u'w'}}{w_*^3}\frac{\partial \overline{U}}{\partial z} - \frac{z_i}{w_*^3}\frac{\partial\left(\overline{w'e}\right)}{\partial z} - \frac{z_i}{w_*^3\, \bar{\rho}}\frac{\partial\left(\overline{w'p'}\right)}{\partial z} - \frac{z_i\, \varepsilon}{w_*^3} \qquad (5.2.3)$$

O(1)@surface

I III IV V VI VII

Consumption. In statically stable conditions, an air parcel displaced vertically by turbulence would experience a buoyancy force pushing it back towards its starting height. ***Static stability thereby tends to suppress, or consume, TKE***, and is

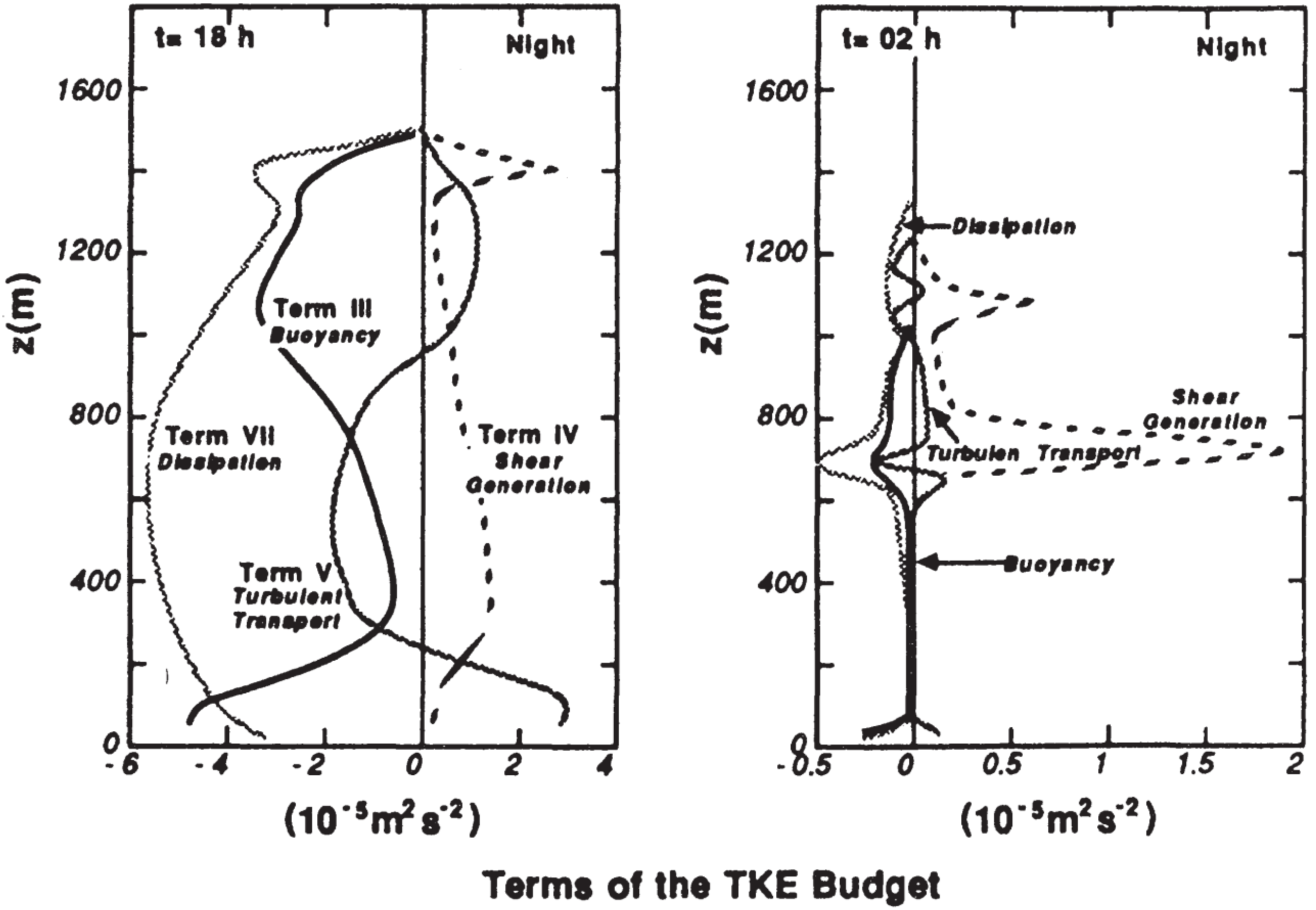


Fig. 5.6 Modeled turbulence kinetic energy budgets at t=18 h and t=02 h during Night 33-34, Wangara. After André et al. (1978).

Turbulence regimes: shear *vs* buoyancy

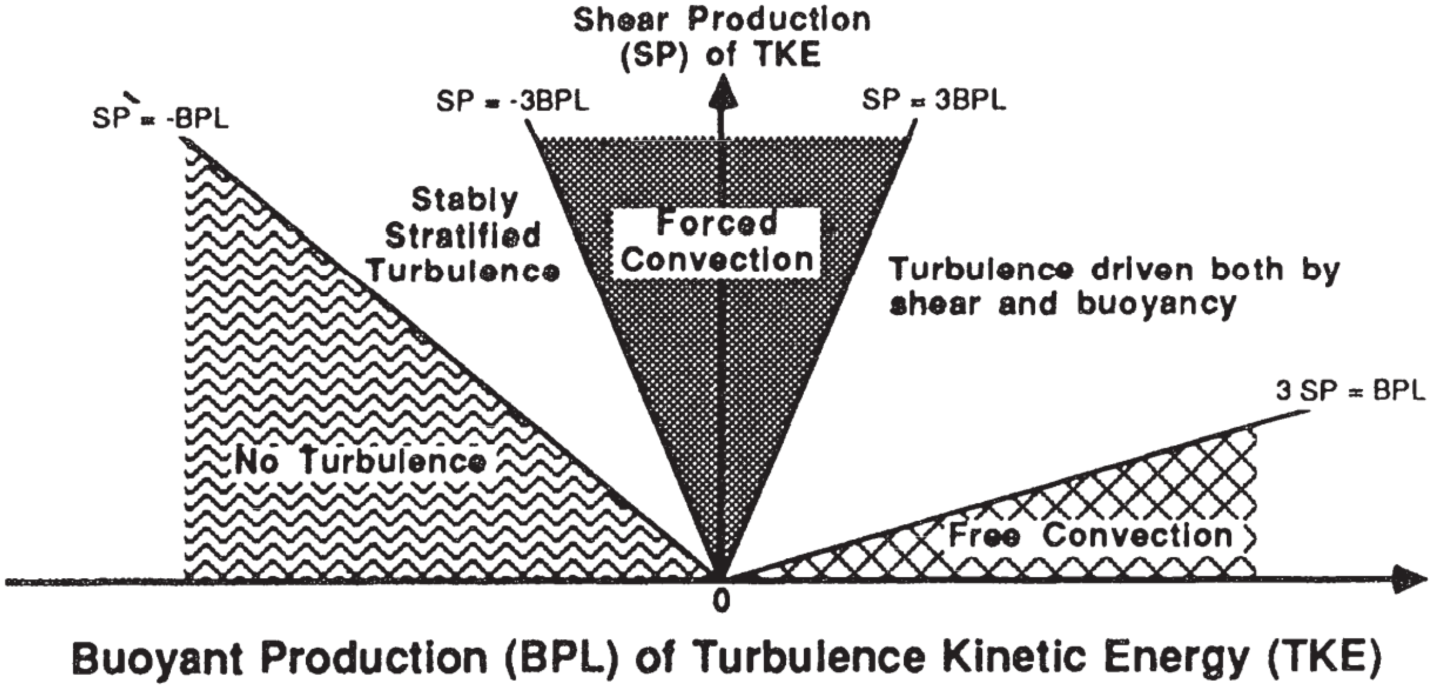


Fig. 5.7 Approximate regimes of free and forced convection.

dissipation

Note different scales

Fig. 5.13
Range of normalized dissipation rate(ε) profiles during the daytime, where z_i is ML depth and w_* is the convective velocity scale. (After Caughey et al. (1979) and Kitchen et al. (1983)).

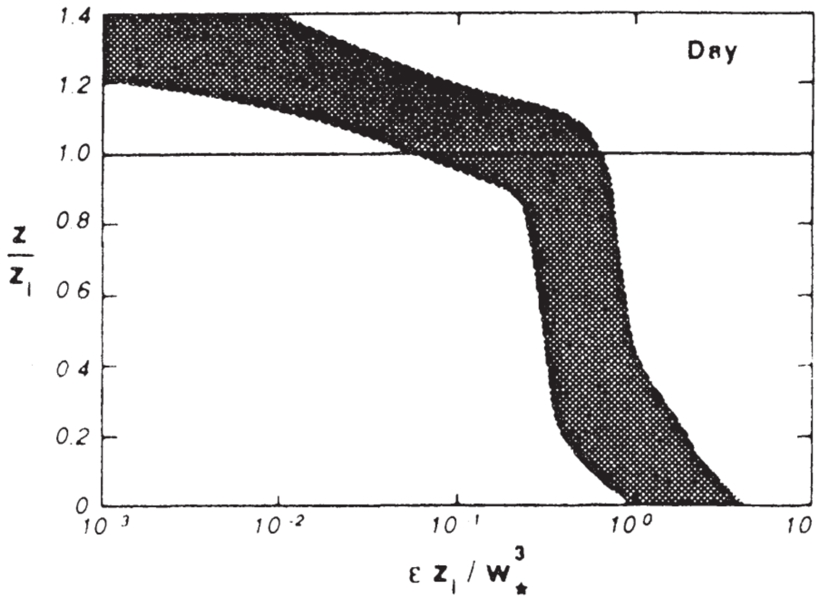
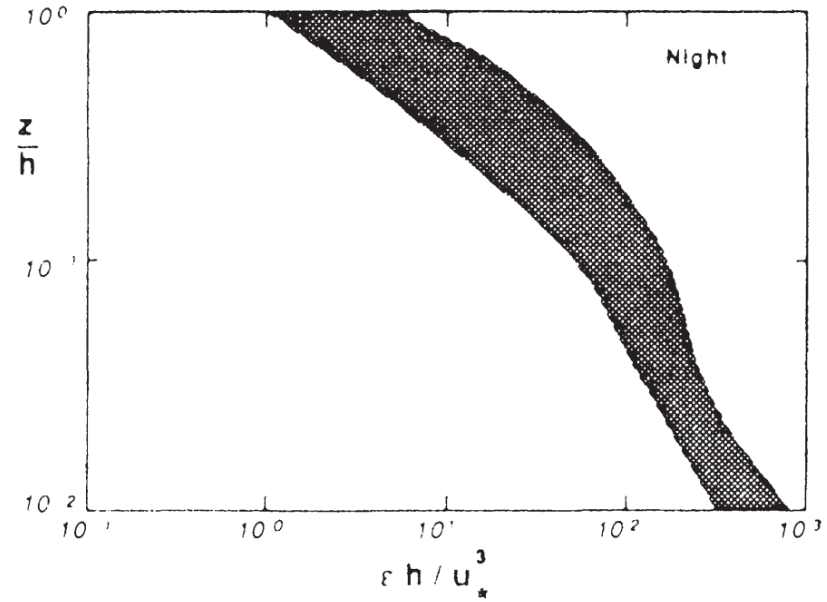


Fig. 5.14
Range of normalized dissipation rate(ε) profiles at night, where h is boundary layer depth and u_* is the friction velocity. After Caughey, et al. (1979).



Eddy size

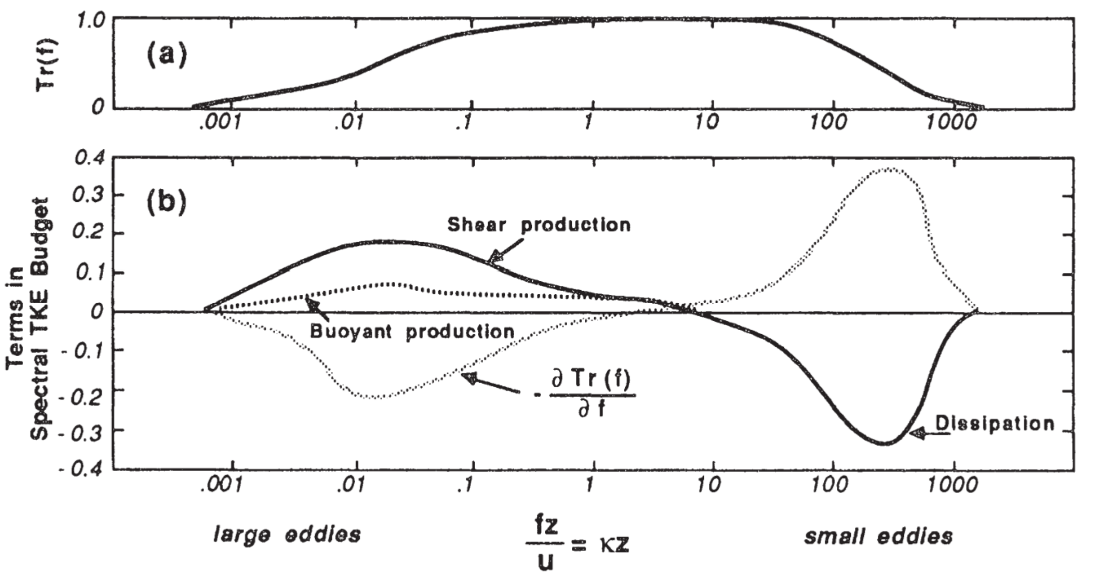


Fig. 5.16 Example of spectral energy budget terms for z/L = -0.29. Shown in (b) are the shear and buoyant production and the dissipation terms as functions of frequency f. The shaded curve (labelled $\partial Tr(f) / \partial f$) is equal to minus the sum of the shear, buoyant production and dissipation terms. The Tr(f) curve (a) was obtained by integrating $\partial Tr(f) / \partial f$. Here Tr(f) is the transfer of energy in f space required to balance the production and dissipation. The symbol f is frequency and κ is wave number. After McBean and Elliott (1975).

One measure of the smallest scales of turbulence is the Kolmogorov microscale, η, given by: $\eta = (\nu^3/\varepsilon)^{1/4}$. This scaling assumes that the smallest eddies see only turbulent energy cascading down the spectrum at rate ε, and feel only the viscous damping of ν. For the example of Fig 5.16, $\eta \cong 1$ mm, which occurs at a normalized frequency of about 3000.

The nature of the atmospheric turbulence spectrum is directly related to the fact that production and dissipation are not happening at the same scales. Production is feeding only the larger size eddies (anisotropically, as we learned earlier), but dissipation is acting only on the smaller sizes. Thus, the rate of transport across the middle part of the spectrum is equal to the rate of dissipation, ε, at the small-eddy end. Such transfer can be thought of as happening inertially — larger eddies creating or bumping into smaller ones, and transferring some of their inertia in the process. This middle portion of the spectrum is called the ***inertial subrange***.

Mean kinetic energy

$$\frac{\partial\left(0.5\overline{U}_i^{\,2}\right)}{\partial t} + \overline{U}_j \frac{\partial\left(0.5\,\overline{U}_i^{\,2}\right)}{\partial x_j} = -g\delta_{i3}\overline{U}_i + f_c\varepsilon_{ij3}\overline{U}_i\,\overline{U}_j - \frac{\overline{U}_i}{\overline{\rho}}\frac{\partial\overline{P}}{\partial x_i} + \nu\overline{U}_i\frac{\partial^2\overline{U}_i}{\partial x_j^2} - \overline{U}_i\frac{\partial\left(\overline{u_i'u_j'}\right)}{\partial x_j} \qquad (5.4a)$$

I II III IV V VI X

Term I	represents storage of MKE.
Term II	describes the advection of MKE by the mean wind.
Term III	indicates that gravitational acceleration of vertical motions alter the MKE.
Term IV	shows the effects of the Coriolis force.
Term V	represents the production of MKE when pressure gradients accelerate the mean flow.
Term VI	represents the molecular dissipation of mean motions.
Term X	indicates the interaction between the mean flow and turbulence.

Shear production is a transfer from MKE

$$\frac{\partial\left(0.5\overline{U}_i^{\,2}\right)}{\partial t} + \overline{U}_j \frac{\partial\left(0.5\overline{U}_i^{\,2}\right)}{\partial x_j} = -g\overline{W} - \frac{\overline{U}_i}{\overline{\rho}}\frac{\partial \overline{P}}{\partial x_j} + \nu\,\overline{U}_i \frac{\partial^2 \overline{U}_i}{\partial x_j^2} + \overline{u_i'u_j'}\frac{\partial \overline{U}_i}{\partial x_j} - \frac{\partial\left(\overline{u_i'u_j'}\,\overline{U}_i\right)}{\partial x_j} \tag{5.4b}$$

$$\frac{\partial(\mathrm{TKE/m})}{\partial t} = \ldots - \overline{u_i'u_j'}\frac{\partial \overline{U}_i}{\partial x_j}$$

$$\frac{\partial(\mathrm{MKE/m})}{\partial t} = \ldots + \overline{u_i'u_j'}\frac{\partial \overline{U}_i}{\partial x_j}$$

Static stability

Parcels conserve θ_v

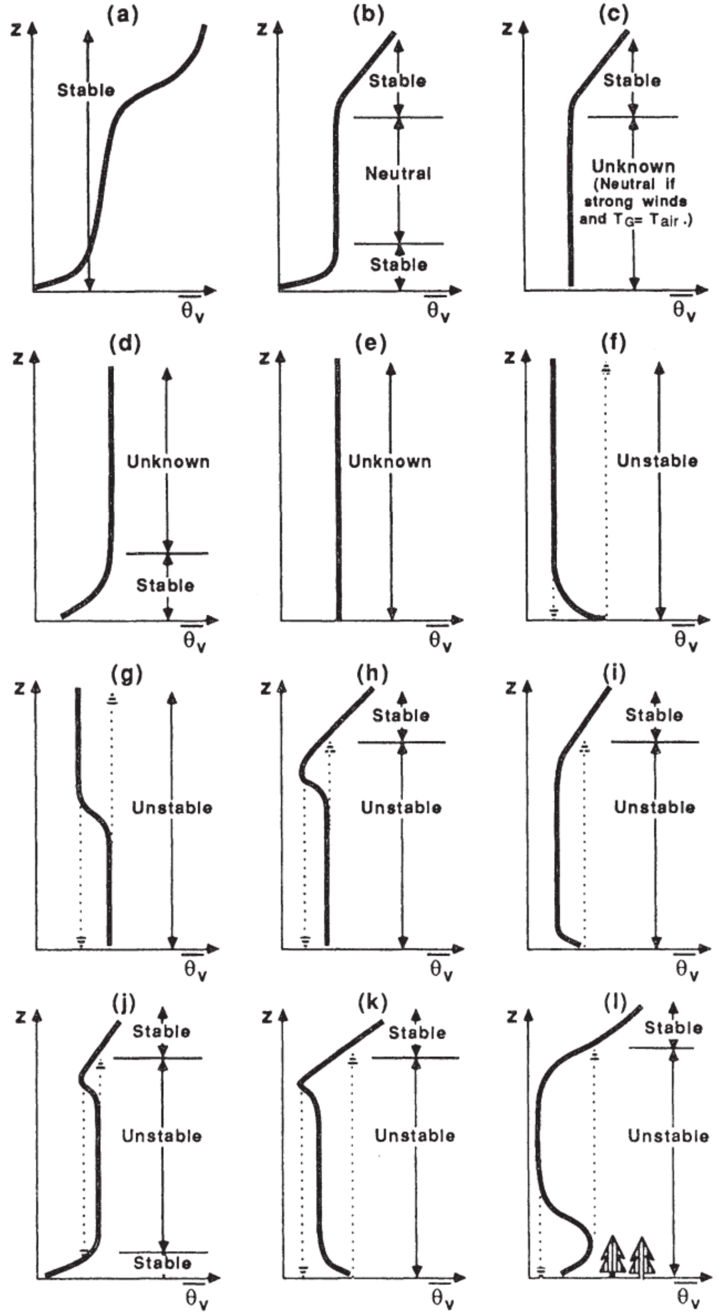


Fig. 5.17 Static stabilitiy as a function of the $\overline{\theta_v}$ profile. Dotted lines denote parcel movement.

Kelvin-Helmoltz

Shear instability (with static stability)

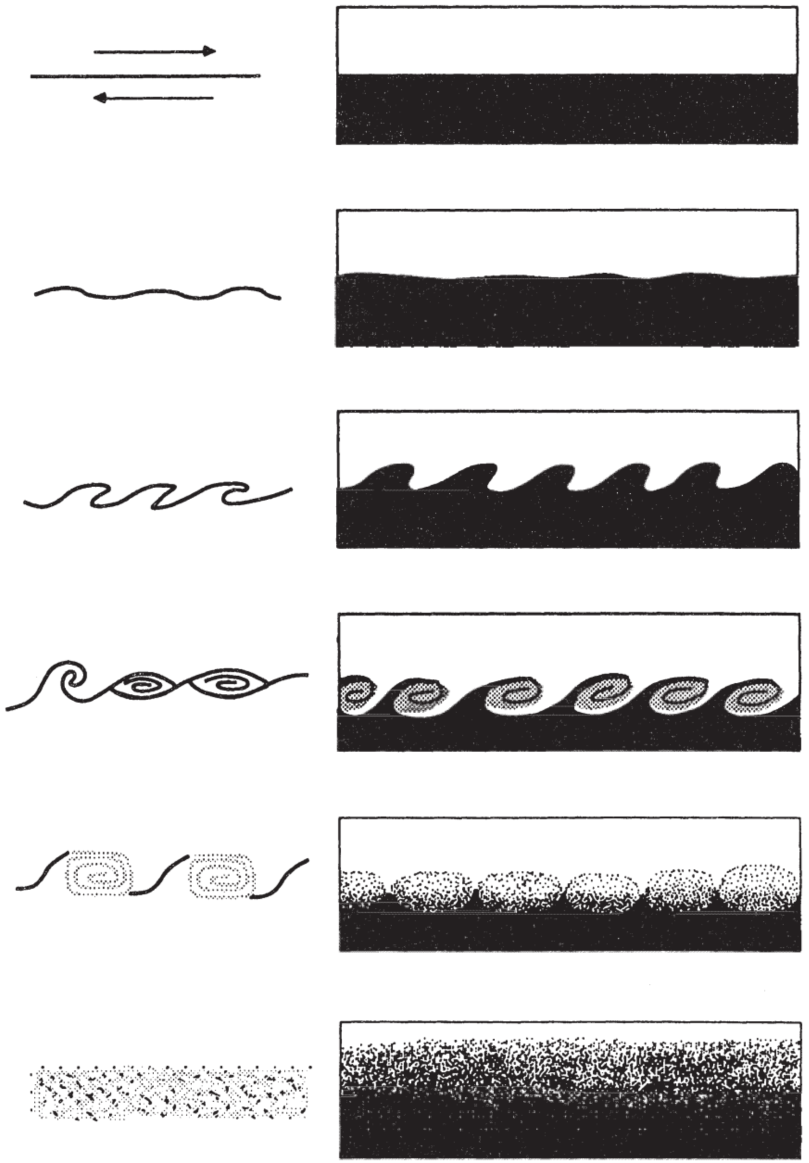


Fig. 5.18 Schematic diagram of Kelvin-Helmholtz instability in a laboratory experiment where shear flow has been generated. The upper layer, water, flows to the right, and the lower more dense fluid, dyed brine, flows to the left. The figures are about half a second apart. After Thorpe (1969,1973) and Woods (1969).

Flux Richardson number

$$R_f = \frac{\left(\frac{g}{\overline{\theta_v}}\right)\left(\overline{w'\theta_v'}\right)}{\left(\overline{u_i'u_j'}\right)\frac{\partial \overline{U_i}}{\partial x_j}}$$

$$R_f = \frac{\left(\frac{g}{\overline{\theta_v}}\right)\left(\overline{w'\theta_v'}\right)}{\left(\overline{u'w'}\right)\frac{\partial \overline{U}}{\partial z} + \left(\overline{v'w'}\right)\frac{\partial \overline{V}}{\partial z}}$$

Gradient Richardson number

1st order closure

$$Ri = \frac{\frac{g}{\overline{\theta_v}} \frac{\partial \overline{\theta_v}}{\partial z}}{\left[\left(\frac{\partial \overline{U}}{\partial z} \right)^2 + \left(\frac{\partial \overline{V}}{\partial z} \right)^2 \right]}$$

Bulk Richardson number

$$R_B = \frac{g \ \Delta\overline{\theta_v} \ \Delta z}{\overline{\theta_v} \left[(\Delta\overline{U})^2 + (\Delta\overline{V})^2\right]}$$

Richardson

Ri<0: static instability

0<Ri<0.25 KH instability

1<Ri: turbulence dies