Planetary boundary layer

Lecture 6

Chap 6.1-6.4

Homework

6.5 – Maria

6.7 – Mariana

6.10 – Sara

6.14 – Cátia

6.16– Diogo

6.17 – Florian

6.27 – Jason

Obukhov length

k – von Karman (~0.4)

$$\frac{\partial \bar{e}}{\partial t} + \bar{U}_j \frac{\partial \bar{e}}{\partial x_j} = + \delta_{i3} \frac{g}{\bar{\theta}_v}\left(\overline{u_i'\theta_v'}\right) - \overline{u_i'u_j'} \frac{\partial \bar{U}_i}{\partial x_j} - \frac{\partial\left(\overline{u_j'e}\right)}{\partial x_j} - \frac{1}{\bar{\rho}} \frac{\partial\left(\overline{u_i'p'}\right)}{\partial x_i} - \varepsilon$$

$\times\, kz/u_*^3$

$$\ldots = -\frac{k\, z\, g\, (\overline{w'\theta_v'})_s}{\bar{\theta}_v\, u_*^3} + \frac{k\, z\, (\overline{u_i'u_j'})_s}{u_*^3} \frac{\partial \bar{U}_i}{\partial x_j} + \ldots - \frac{k\, z\, \varepsilon|_s}{u_*^3}$$

$$\zeta = \frac{z}{L} = \frac{-k\, z\, g\, (\overline{w'\theta_v'})_s}{\bar{\theta}_v\, u_*^3}$$

$$L = \frac{-\bar{\theta}_v\, u_*^3}{k\, g\, (\overline{w'\theta_v'})_s}$$

Obukhov length

$$L = \frac{-\overline{\theta_v}\, u_*^3}{k\, g\, (\overline{w'\theta_v'})_s}$$

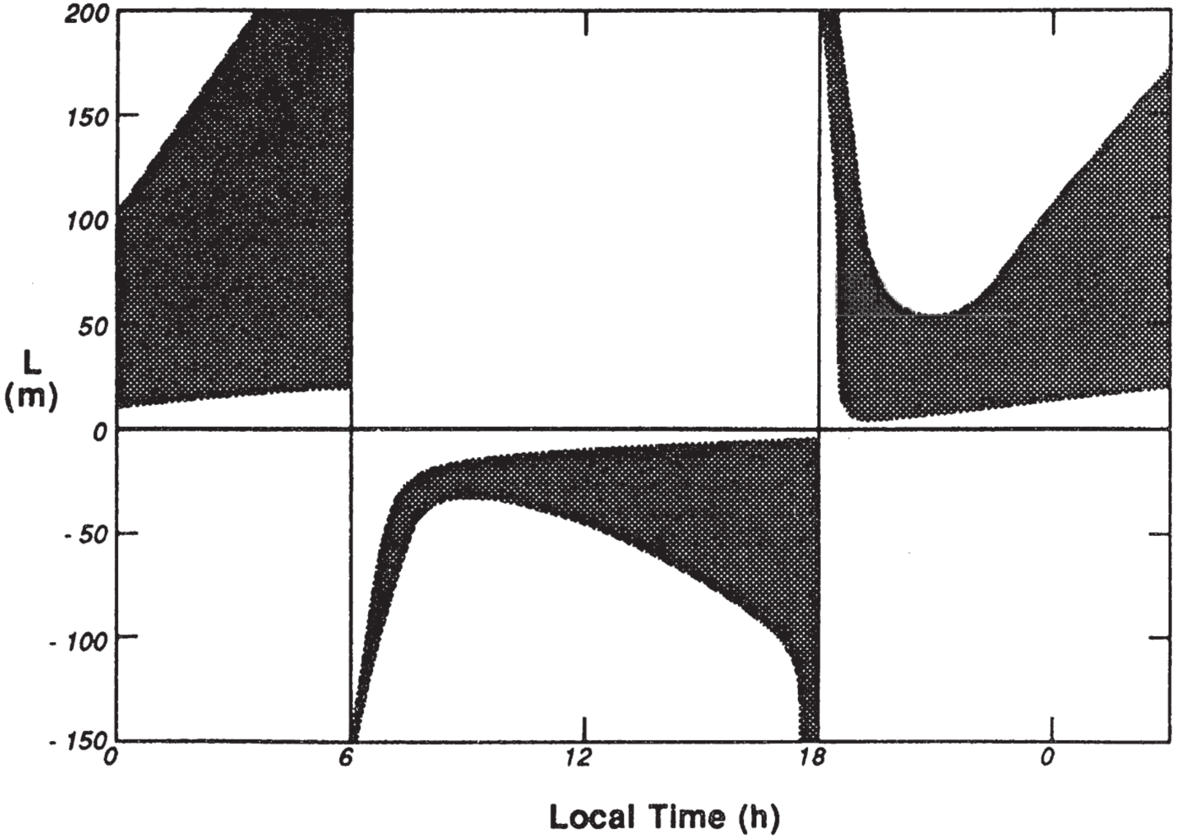

Fig. 5.21 Typical ranges of Obukhov length (L) evolution over a diurnal cycle.

Stability index for the surface layer

k – von Karman ($k \approx 0.4$)

Obukhov length

One physical interpretation of the Obukhov length is that it is proportional to the height above the surface at which buoyant factors first dominate over mechanical (shear) production of turbulence. For convective situations, buoyant and shear production terms are approximately equal at $z = -0.5\,L$. Fig 5.21 shows the typical range of variations of the Obukhov length in fair weather conditions over land.

$$\zeta = z/L$$

Fig. 5.22 shows the variation of TKE budget terms with ζ, as ζ varies between 0 (statically neutral) and -1 (slightly unstable). The decrease in importance of shear and increase of buoyancy as ζ decreases from 0 to -1 is particularly obvious.

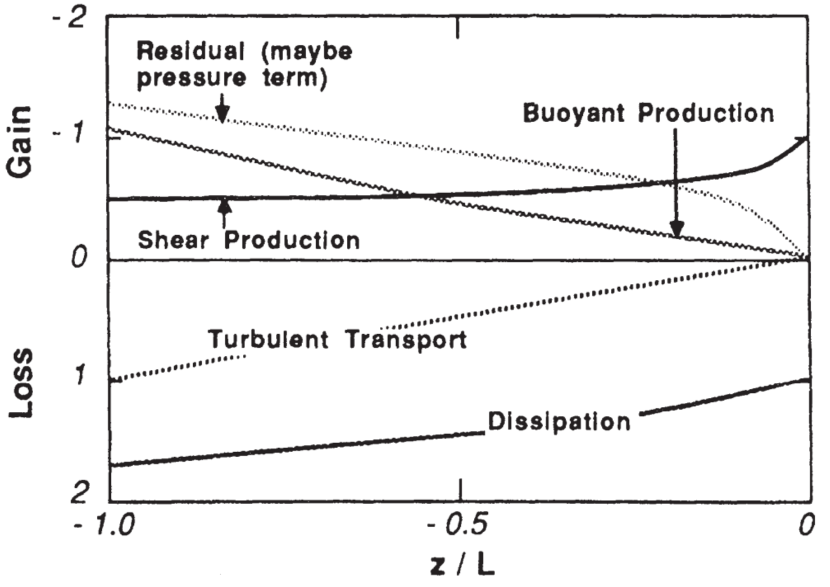

Fig. 5.22
Behavior of the terms (made dimensionless with kz/u_*^3) in the unstable surface-layer turbulence kinetic energy budget. After Wyngaard (1973).

$$\frac{z}{L}\ vs\ Ri$$

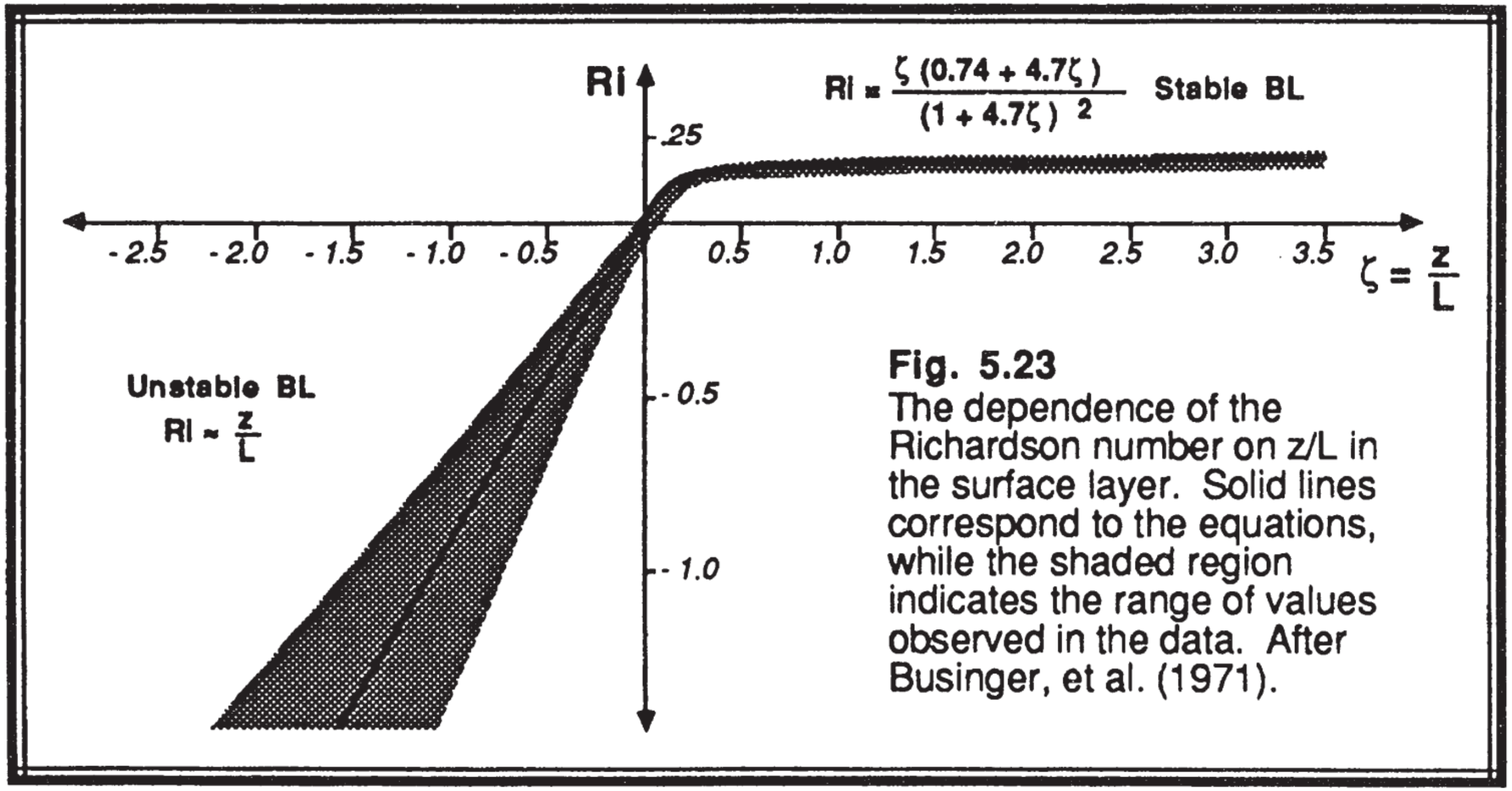

Fig. 5.23
The dependence of the Richardson number on z/L in the surface layer. Solid lines correspond to the equations, while the shaded region indicates the range of values observed in the data. After Businger, et al. (1971).

Stability Re and Ri

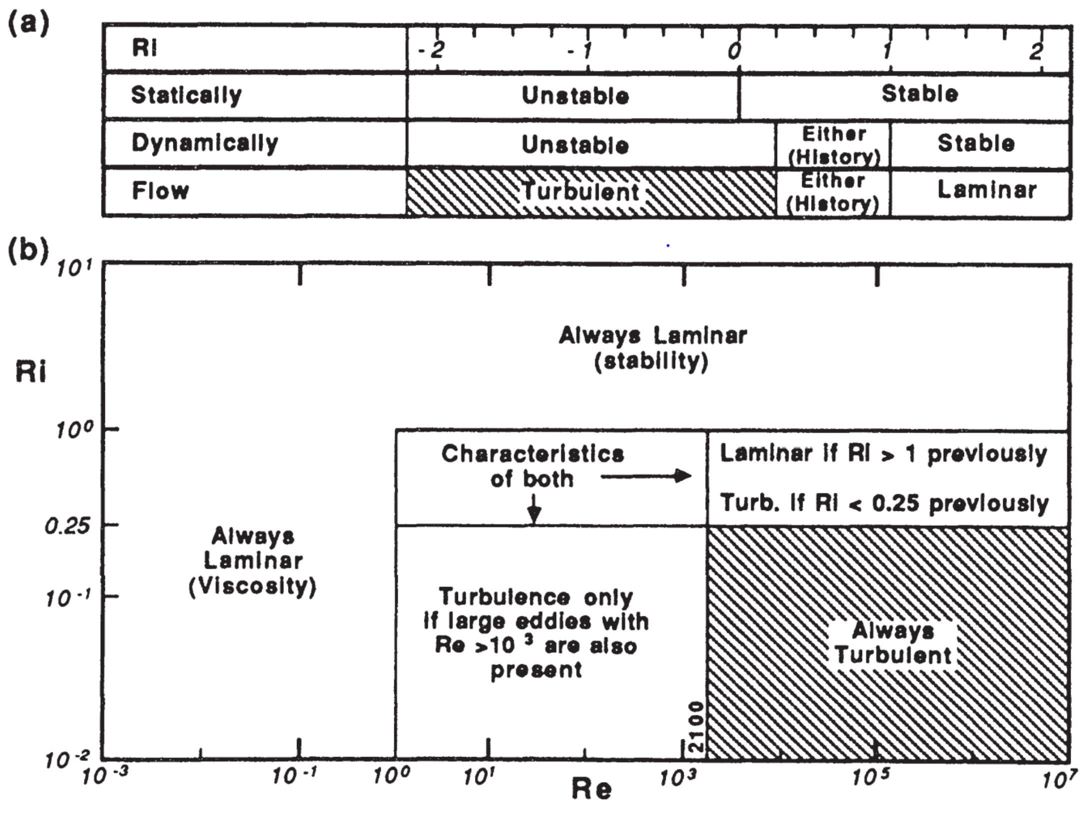

Fig. 5.24 Stability parameter relationships (see text). (After Woods, 1969).

Turbulence closure (Chap. 6)

Table 6-1. Simplified example showing a tally of equations and unknowns for various statistical moments of momentum, demonstrating the closure problem for turbulent flow. The full set of equations includes even more unknowns.

Prognostic Eq. for:	Moment	Equation	Number of Eqs.	Number of Unknowns
$\overline{U_i}$	First	$\frac{\partial \overline{U_i}}{\partial t} = \ldots - \frac{\partial \overline{u_i' u_j'}}{\partial x_j}$	3	6
$\overline{u_i' u_j'}$	Second	$\frac{\partial \overline{u_i' u_j'}}{\partial t} = \ldots - \frac{\partial \overline{u_i' u_j' u_k'}}{\partial x_k}$	6	10
$\overline{u_i' u_j' u_k'}$	Third	$\frac{\partial \overline{u_i' u_j' u_k'}}{\partial t} = \ldots - \frac{\partial \overline{u_i' u_j' u_k' u_m'}}{\partial x_m}$	10	15

Unknowns in each order of closure

Order of Closure	Correlation Triangle of Unknowns
Zero	$\overline{U}$
	$\overline{V}$ $\quad$ $\overline{W}$
First	$\overline{u'^2}$
	$\overline{u'v'}$ $\quad$ $\overline{u'w'}$
	$\overline{v'^2}$ $\quad$ $\overline{v'w'}$ $\quad$ $\overline{w'^2}$
Second	$\overline{u'^3}$
	$\overline{u'^2v'}$ $\quad$ $\overline{u'^2w'}$
	$\overline{u'v'^2}$ $\quad$ $\overline{u'v'w'}$ $\quad$ $\overline{u'w'^2}$
	$\overline{v'^3}$ $\quad$ $\overline{v'^2w'}$ $\quad$ $\overline{v'w'^2}$ $\quad$ $\overline{w'^3}$

Rules to follow

Most importantly,the parameterization for an unknown quantity should be physically reasonable. In addition, the parameterization must:

- have the same dimensions as the unknown.
- have the same tensor properties.
- have the same symmetries.
- be invariant under an arbitrary transformation of coordinate systems.
- be invariant under a Galilean (i.e., inertial or Newtonian) transformation.
- satisfy the same budget equations and constraints.

Example

As an example, Donaldson (1973) has proposed that the ***unknown*** $\overline{u_i'u_j'u_k'}$ be parameterized by:

$$-\Lambda \bar{e}^{1/2}\left[\frac{\partial(\overline{u_j'u_k'})}{\partial x_i}+\frac{\partial(\overline{u_i'u_k'})}{\partial x_j}+\frac{\partial(\overline{u_i'u_j'})}{\partial x_k}\right]$$

where Λ is a ***parameter*** having the dimension of length (m), and the ***knowns*** are $\bar{e}$ (turbulence kinetic energy per unit mass, m^2s^{-2}) and $\overline{u_i'u_j'}$ (momentum flux, m^2s^{-2}).

This parameterization has the same dimensions (m^3s^{-3}) and the same tensor properties (unsummed i, j & k) as the original unknown. The symmetry of the original unknown is such that the order of the indices i, j, k is not significant. The same symmetry is achieved in the parameterization by having the sum of the three terms in square brackets. If only one term had been used instead of the sum, then a change in the order of the indices would have produced a different numerical result (because $\partial \overline{u'v'}/\partial z$ is not necessarily equal to $\partial \overline{u'w'}/\partial y$). Since the gradient of the momentum flux is taken in all three Cartesian directions in the square brackets, any rotation or displacement of the coordinate system will not change the result. Also, movement of the coordinate system at constant velocity c_i (a Galilean transformation) does not change the parameterization, as can be seen by setting $x_i = X_i + c_i t$.

First order closure

$$\frac{\partial \overline{U}}{\partial t} = f_c(\overline{V} - \overline{V_g}) - \frac{\partial(\overline{u'w'})}{\partial z}$$

$$\frac{\partial \overline{V}}{\partial t} = -f_c(\overline{U} - \overline{U_g}) - \frac{\partial(\overline{v'w'})}{\partial z}$$

$$\frac{\partial \overline{\theta}}{\partial t} = - \frac{\partial(\overline{w'\theta'})}{\partial z}$$

$$\overline{u_j'\xi'} = -K \frac{\partial \overline{\xi}}{\partial x_j}$$

$$\overline{u_j'\xi'} = -K \frac{\partial \overline{\xi}}{\partial x_j}$$

K is known by a variety of names:

- eddy viscosity
- eddy diffusivity
- eddy-transfer coefficient
- turbulent-transfer coefficient
- gradient-transfer coefficient

Mixing length

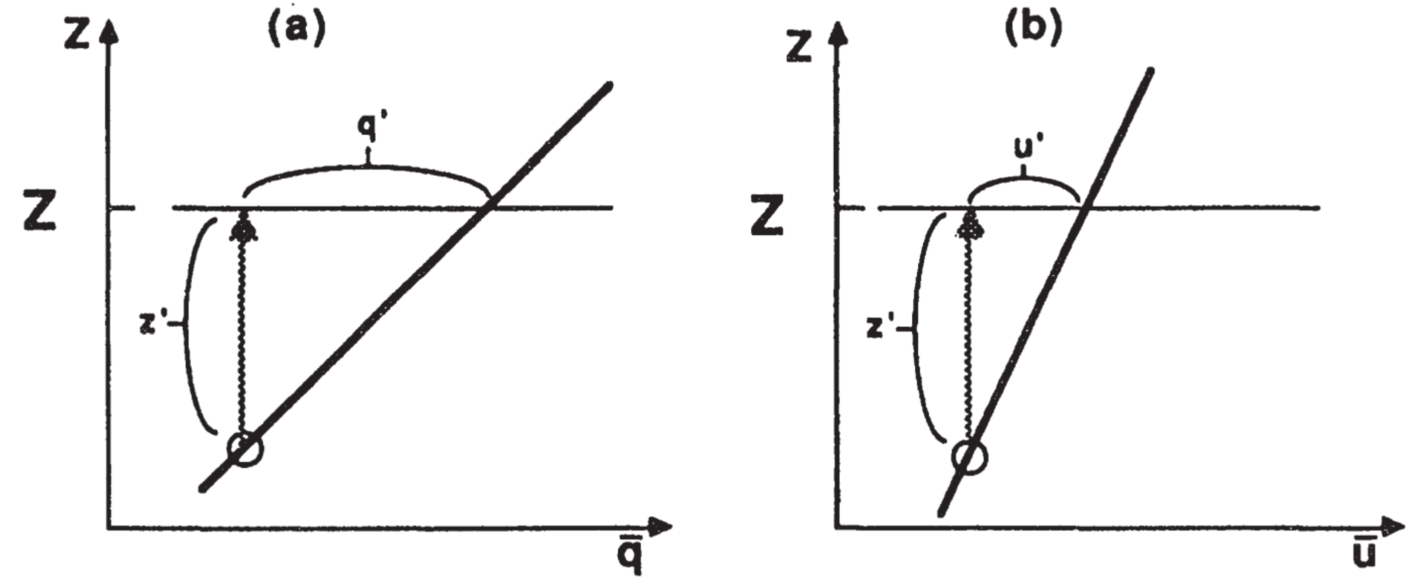

Fig. 6.1 Movement of an air parcel (shaded line) within a background having linear moisture and wind gradients (heavy line). The superposition of many such parcels, starting at different levels but all ending at level Z, forms the conceptual basis for "mixing length theory."

$$q' = -\left(\frac{\partial \overline{q}}{\partial z}\right) z'$$

$$u' = -\left(\frac{\partial \overline{U}}{\partial z}\right) z'$$

$$w' = c \left| \frac{\partial \overline{U}}{\partial z} \right| z'$$

$$R = \overline{w'q'}$$

$$q' = -\left(\frac{\partial \bar{q}}{\partial z}\right) z'$$

$$u' = -\left(\frac{\partial \bar{U}}{\partial z}\right) z'$$

$$w' = c \left| \frac{\partial \bar{U}}{\partial z} \right| z'$$

$$R = -c\,\overline{(z')^2} \left| \frac{\partial \bar{U}}{\partial z} \right| \cdot \left(\frac{\partial \bar{q}}{\partial z} \right) \qquad (6.4.4d)$$

We recognize $\overline{z'^2}$ as the variance of parcel displacement distance. The square root of it is a measure of the average distance a parcel moves in the mixing process that generated flux R. In this way, we can define a ***mixing length***, l, by $l^2 = c\,\overline{z'^2}$. Thus, the final expression for moisture flux is

$$R = -l^2 \left| \frac{\partial \bar{U}}{\partial z} \right| \cdot \left(\frac{\partial \bar{q}}{\partial z} \right) \qquad (6.4.4e)$$

This is directly analogous to K-theory if

$$K_E = l^2 \left| \frac{\partial \bar{U}}{\partial z} \right| \qquad (6.4.4f)$$

In the surface layer, the size of the turbulent eddies is limited by the presence of the earth's surface. Thus, it is sometimes assumed that $l^2 = k^2z^2$, where k is the von Karman constant. The resulting expression for eddy viscosity in the surface layer is

$$K_E = k^2 z^2 \left| \frac{\partial \bar{U}}{\partial z} \right| \tag{6.4.4g}$$

For SBLs, Delage (1974) proposed the following parameterization for mixing length that has since been used as a starting point for other parameterizations (Estournel and Guedalia, 1987; and Lasser and Arya, 1986):

$$\frac{1}{l} = \frac{1}{k\,z} + \frac{1}{0.0004\,G\,f_c^{-1}} + \frac{\beta}{k\,L_L} \tag{6.4.4h}$$

6.4.5 Sample Parameterizations of K

The eddy viscosity is best not kept constant, as mentioned earlier, but should be parameterized as a function of the flow. The parameterizations for K should satisfy the following constraints:

- $K = 0$ where there is no turbulence.
- $K = 0$ at the ground ($z=0$).
- K increases as TKE increases.
- K varies with static stability (in fact, one might expect that a different value of K should be used in each of the coordinate directions for anisotropic turbulence).
- K is non-negative (if one uses the analogy with viscosity).

K(z)

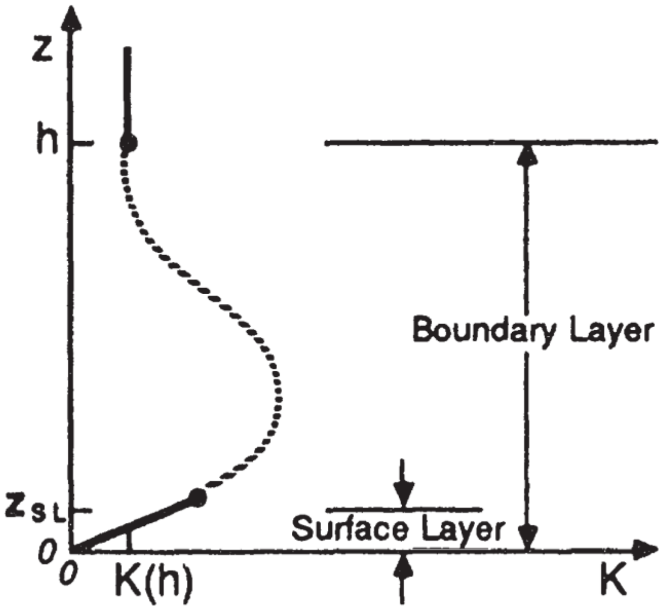

Fig. 6.2 Typical variation of eddy viscosity, K, with height in the boundary layer. After O'Brien (1970).

Ekman spiral

Even with first-order closure, the equations of motion (3.5.3) are often too difficult to solve analytically. However, for the special case of a steady state $[\partial\overline{(\ )}/\partial t=0]$, horizontally homogeneous $[\partial\overline{(\ )}/\partial x = 0,\ \partial\overline{(\ )}/\partial y = 0]$, statically neutral $[\partial\overline{\theta_v}/\partial z = 0]$, barotropic atmosphere $[\overline{U_g}$ & $\overline{V_g}$ constant with height] with no subsidence $[\overline{W}=0]$, the equations of motion can be reduced to:

$$\begin{cases} 0 = -f_c\left(\overline{V_g} - \overline{V}\right) - \dfrac{\partial\left(\overline{u'w'}\right)}{\partial z} \\ 0 = +f_c\left(\overline{U_g} - \overline{U}\right) - \dfrac{\partial\left(\overline{v'w'}\right)}{\partial z} \end{cases}$$

K=const, align with geostrophic wind

Atmosphere: The following derivations are based on the approach of Businger (1982). Define the magnitude of the geostrophic wind, $\overline{G}$, by $\overline{G} = [\,\overline{U_g}^2 + \overline{V_g}^2\,]^{1/2}$. Pick an x-axis aligned with the geostrophic wind; thus, $\overline{V_g} = 0$ and $\overline{U_g} = \overline{G}$. Use first-order local closure K-theory, with constant K_m. Hence, $\overline{u'w'} = -K_m\, \partial\overline{U}/\partial z$ and $\overline{v'w'} = -K_m\, \partial\overline{V}/\partial z$. Inserting these into (6.4.6a) leaves the following set of coupled second-order differential equations:

$$\begin{cases} f_c \overline{V} = -K_m \dfrac{\partial^2 \overline{U}}{\partial z^2} \\[2ex] f_c (\overline{U} - \overline{G}) = +K_m \dfrac{\partial^2 \overline{V}}{\partial z^2} \end{cases} \qquad (6.4.6b)$$

The spiral

$$\overline{U} = \overline{G}\left[1 - e^{-\gamma_E z}\cos(\gamma_E z)\right]$$

$$\overline{V} = \overline{G}\left[e^{-\gamma_E z}\ \sin(\gamma_E z)\right]$$

$$\gamma_E = [f_c/(2K_m)]^{1/2}\ .$$

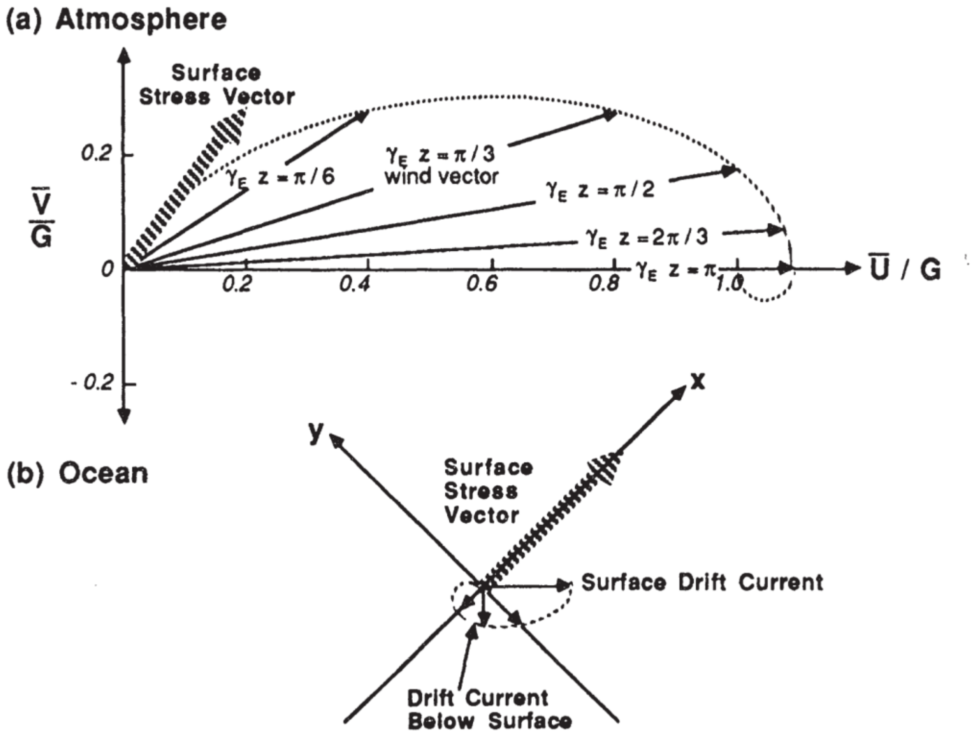

Fig. 6.3 Ekman spiral hodographs for wind and current vectors. After Businger (1982).

According to this solution, the surface wind vector is 45° to the left of the geostrophic wind vector in the Northern Hemisphere. Hence, the surface stress is also in this direction, because it is caused by the drag of the surface wind against the surface. Use u_*^2 as a measure of the surface stress magnitude, where $u_*^2 = [(\overline{u'w'})^2 + (\overline{v'w'})^2]^{1/2} = [(K_m \partial \overline{U}/\partial z)^2 + (K_m \partial \overline{V}/\partial z)^2]^{1/2}$ evaluated at z=0. Inserting (6.4.6c) into this expression yields:

$$u_*^2 = \overline{G}\left(K_m f_c\right)^{1/2} \qquad (6.4.6d)$$

Ekman spiral in the ocean

Ocean. The ocean drift current is driven by the wind stress at the surface, neglecting pressure gradients in the ocean. Hence, the equations of motion reduce to:

$$f_c \overline{V} = - K_m \frac{\partial^2 \overline{U}}{\partial z^2}$$

$$f_c \overline{U} = + K_m \frac{\partial^2 \overline{V}}{\partial z^2} \qquad (6.4.6f)$$

This time, let us choose a coordinate system with the x-axis aligned with the surface stress, and z positive up. Thus, the four boundary conditions become: $K_m \partial\overline{U}/\partial z = u_*^2$ at $z = 0$, $\partial\overline{V}/\partial z = 0$ at $z = 0$, $\overline{U} \to 0$ as $z \to -\infty$, and $\overline{V} \to 0$ as $z \to -\infty$. Thus, the current is assumed to go to zero deep in the ocean. In the equations above, K_m and u_* refer to their ocean values, where $\rho_{water} \cdot u_{*water}^2$ = surface stress = $\rho_{air} \cdot u_{*air}^2$.

The solution is:

The solution is:

$$\overline{U} = \left[\frac{u_*^2}{\left(K_m f_c\right)^{1/2}} \right] \left[e^{\gamma_E z} \cos\left(\gamma_E z - \frac{\pi}{4}\right) \right]$$

$$\overline{V} = \left[\frac{u_*^2}{\left(K_m f_c\right)^{1/2}} \right] \left[e^{\gamma_E z} \sin\left(\gamma_E z - \frac{\pi}{4}\right) \right] \qquad (6.4.6g)$$

where K_m and γ_E now apply to ocean values. This solution gives a surface current that is 45° to the right of the surface stress, making it parallel to the geostrophic wind in the atmosphere. Based on typical values of eddy viscosity in the air and ocean, the magnitude of the surface drift current is roughly G/30. Deeper in the ocean the current reduces in speed, and turns to the right as shown in Fig 6.3b. This causes horizontal convergence in the ocean under atmospheric regions of horizontal divergence, and vice versa. Hence, we expect ***downwelling*** water movement under synoptic high pressure systems, and ***upwelling*** under lows.

In chapter 6

We won’t study 6.5 and after sections (higher order closures)

Past Homework

5.2 – Jason

5.3 – Maria

5.5 – Mariana

5.7 – Sara

5.12 – Cátia

5.14 – Diogo

5.22 – Florian