UNIVERSO PRIMITIVO

Mestrado em Física Astronomia 2020-2021

Exercise Sheet 3

1. Show that the entropy of the primordial fluid can be defined as $S = V\,(\rho + P)/T$.
   Use the continuity equation to show that the entropy, S, is constant in time. [Hint: Use thermodynamics and the Maxwell relations for the first part of the problem.]

2. According to the standard model of particle physics, photons, electrons, neutrinos and their anti-particles are the main relativistic thermal species in the temperature plateau $1 \leq T/\mathrm{MeV} \leq 30$.

   2.1. Compute the number and energy densities of primordial photons at $T = 1\mathrm{MeV}$, in cgs units. Compare your findings with present CMB observations of these quantities: $n_{\gamma,0} \simeq 410\ \mathrm{cm}^{-3}$; $\rho_{\gamma,0} \simeq 4 \times 10^{-34}\ \mathrm{gcm}^{-3}$.

   2.2. Compute the effective number of relativistic species in energy and entropy (g_* and g_{*S}) during the temperature period from 1 to 30 MeV. What would be the value of g_* and g_{*S} in a particle physics model with 4 families of massless neutrinos, all with just one helicity state?

   2.3. Derive exact expressions for the plasma temperature, energy density and entropy density, $s = S/V$, as a function of redshift, z, assuming that $T = 1\ \mathrm{MeV}$ at $z = 6 \times 10^9$.

3. Consider the Friedmann equation written as in exercise 3.2 of problems sheet 1.

   3.1. Explain why the energy density of relativistic particles (the radiation term in this equation) should be modified to:

$$\Delta_R(a)\ \Omega_{r0}\left(\frac{a_0}{a}\right)^4, \qquad \text{where:} \qquad \Delta_R(a) = \frac{g_*(a)}{g_*(a_0)}\left(\frac{g_{*S}(a_0)}{g_{*S}(a)}\right)^{4/3}.$$

   3.2. Compute the age of the universe by the end of the Big Bang Nucleosynthesis, $T = 0.1$ MeV, assuming the following approximation for g_* and g_{*S}:

$$g_* \simeq g_{*S} \simeq \begin{cases} 100 & T > 300\ \mathrm{MeV} \\ 10 & 300\ \mathrm{MeV} > T > 1\ \mathrm{MeV} \\ 3 & T <\ \mathrm{MeV} \end{cases}$$

   Use the present-day values $H_0 \simeq 1.44 \times 10^{-42}$ GeV, $\Omega_{r0} \simeq 9.2 \times 10^{-5}$. Consider that $T = 0.1\mathrm{MeV}$ at $z = 4 \times 10^8$, for the normalisation of the temperature – z relation.

   3.3. Repeat the calculation now using the tabulated values of g_* and g_{*S} in Ref. astro-ph/1609.04979 (Table A1). Compare with your findings in 3.2.

4. Read sections 3.1 and 3.2 in Ref. astro-ph/1808.08968 where the authors discuss the effect of extra-degrees of freedom in the spectrum of gravitational waves (GW) from a network of cosmic strings. Explain by your own words their findings in Fig. 6 and say if the LISA space mission would be able to discriminate between the models they investigate.

5. In the extra-dimensional model of Randal-Sundrum, the Friedmann equation gains a quadratic correction in the energy density,

$$H^2 = \frac{8\pi G}{3}\rho\left(1 + \frac{\rho}{2\lambda}\right),$$

   Assuming that the quadratic term is dominant and that the universe is radiation dominated up until nucleosynthesis, at what time does nucleosynthesis happens? Write your result in the form below. How does $f(\lambda)$ varies with λ?

$$t_{nuc} = 132\,\mathrm{sec}\left(\frac{0.1\mathrm{MeV}}{T_{nuc}}\right)^4\ f(\lambda).$$