Electromagnetismo e análise de circuitos

Resolução serie 3

Prob1

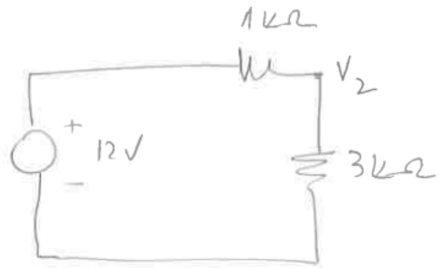
C=ε0 A/d e C = Q/V

a) Q=369 pC
b) C= 80 ε0 A/d = 118 pF; V= Qd/kε0 A = 3.12 V

Prob 2

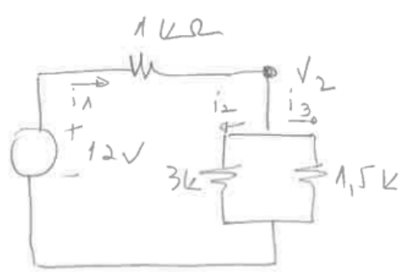
1)

a)



$$V_2 = \frac{3\,k\Omega}{3\,k\Omega + 1\,k\Omega} \times 12\ V = 9\,V$$

b)



$$3k \parallel 1{,}5k = \frac{3 \times 1{,}5}{3 + 1{,}5}\ k\Omega = 1\,k\Omega$$

$$V_2 = \frac{1k}{1k + 1k} \times 12 = 6\ V$$

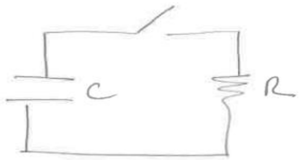
$$i_1 = \frac{(12 - 6)}{1\,k\Omega} = 6\,mA$$

$$i_2 = 6/3k = 2\,mA$$

$$i_3 = i_1 - i_2 = 4\,mA$$

Prob3

1



$R = 1\,M\Omega$
$C = 1\,\mu F$
$V_c = 10 \ (t=0)$
$\Big\} \Rightarrow RC = 10^{6} \times 10^{-6} = \underline{1}$

a) $V_c(t) = V_R(t) = R\,i(t)$

$i(t) = i_0\, e^{-t/RC} = \frac{V_0}{R}\, e^{-t/RC}$

$t = 0 \Rightarrow i = \frac{10\ V}{10^6\ \Omega} = 10\,\mu A$

$t = 1\,s \Rightarrow i = 10\,\mu A \times e^{-1} \simeq 3{,}6\,\mu A$

$t = 2\,s \Rightarrow i = 10\,\mu A \times e^{-2} \simeq 1{,}3\,\mu A$

b) $V_c(t) = V_R(t) = R\,i(t) = \cancel{R}\,\frac{V_0}{\cancel{R}}\, e^{-t/R} = V_0\, e^{-t/RC}$

$\frac{V_c(t)}{V_0} = e^{-t/RC} \Rightarrow -\frac{t}{RC} = \ln\left(\frac{V_c(t)}{V_0}\right)$

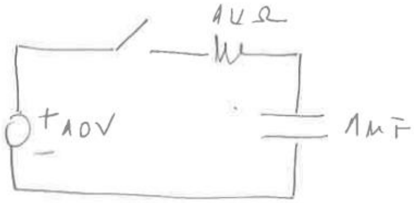
$$\boxed{t = RC \ln\left(\frac{V_0}{V_c(t)}\right)}$$

$RC = 1\,s \Rightarrow t = 1 \times \ln(2) \Rightarrow t \simeq 0{,}69\ s$

c) $\frac{2}{10} = e^{-\left(\frac{1}{R \times 10^{-6}}\right)} \Rightarrow \ln\left(\frac{2}{10}\right) = -\frac{1}{R \times 10^{-6}}$

$\Rightarrow \ln 5 = \frac{1}{R \times 10^{-6}} \Rightarrow R = \frac{1}{\ln(5) \times 10^{-6}} \simeq 0{,}621\,M\Omega$

Prob4



NUMA MALHA RC:

$$\begin{cases} i(t) = \frac{V_0}{R}\, e^{-t/RC} \\ V_R(t) = R \times i(t) = V_0\, e^{-t/RC} \\ V_c(t) = V_0\left(1 - e^{-t/RC}\right) \end{cases}$$

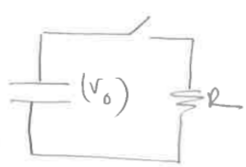
t/τ	$e^{-t/\tau}$	$i/\mu A$	V_R/V	V_C/V
0,5	0,606	6,1	6,1	3,9
1	0,368	3,7	3,7	6,3
2	0,1353	1,3	1,3	8,6
3	0,0498	0,50	0,50	9,5
4	0,0183	0,18	0,18	9,8
5	0,0067	0,07	0,07	9,9

b) $\frac{V_C(t)}{V_0} = 1 - e^{-t/\tau} \Rightarrow \left(1 - \frac{V_C(t)}{V_0}\right) = e^{-t/\tau} \Rightarrow \ln\left(1 - \frac{V_C(t)}{V_0}\right) = -\frac{t}{\tau}$

$-\frac{t}{\tau}(10\%) = \ln(1-0,1) = \ln(0,9) \simeq -0,10$

$-\frac{t}{\tau}(90\%) = \ln(1-0,9) = \ln(0,1) \simeq -2,30$

$\Delta t = (2,3 - 0,1)\tau = 2,2\,(10^{3} \times 10^{-9})\,s \simeq 2,2\,\mu s$

c 

$i(t) = \frac{V_0}{R} e^{-t/RC}$

$P(t) = R i^2 = \cancel{R} \frac{V_0^2}{R^{\cancel{2}}} e^{-2t/RC} = \frac{V_0^2}{R} e^{-2t/RC}$

$E_{TOTAL} = \int_0^{+\infty} P(t)\,dt = \frac{V_0^2}{R} \int_0^{+\infty} e^{-2t/RC}\,dt = \frac{V_0^2}{\cancel{R}}\left(-\frac{\cancel{R}C}{2}\right) \underbrace{\left[e^{-2t/RC}\right]_0^{+\infty}}_{=(0-1)=-1}$

$$\boxed{E_{TOTAL} = \frac{1}{2} C V_0^2}$$

Neste caso:

$E_{TOTAL} = \frac{1}{2}\,10^{-9} \times 5^2 = 12,5\ nJ$