Cosmologia Física

Homework 1

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Exercise 1: Expansion

1.1) The detected frequency of an electromagnetic wave emitted by a cosmological source is redshifted and it is thus different from the emitted frequency.

a) Derive the relation between this cosmological redshift and the scale factor of the Universe at emission time.

Exercise 2: Distances and Volume

2.1) The comoving volume element is given by

$$dV_C = D_A^2 (1+z)^2 \, d\Omega \, dD_C,$$

where $d\Omega$ is the solid angle, D_A the angular diameter distance and dD_C the comoving distance element.

a) Derive a similar expression for dV_C/dt.

b) Derive a similar expression for dV_C/da.

c) Derive a similar expression for dV_C/dz.

2.2) Consider two identical galaxy clusters (meaning they have similar intrinsic sizes and luminosities) in the Einstein-de Sitter universe. The clusters are at different redshifts, z_1 and z_2, with $z_1 < z_2$.

a) Show that the comoving distance $D_c(z)$ in this universe is

$$D_c = 2 \frac{c}{H_0} \left[1 - (1+z)^{-1/2} \right].$$

b) Compute which of the 2 clusters appears to be the brightest to the observer.

c) Compute which of the 2 clusters appears to be the largest to the observer.

d) Which one of distances, luminosity distance or angular diameter distance, could be used as a time indicator in the evolution of the universe?