Cosmologia Física

Homework 2

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Exercise 1: Olbers' Paradox

1.1) Consider a sample of N galaxies, with identical luminosity and constant comoving number density n_0, contained in a comoving volume V_C.

a) Show that the number of galaxies per redshift and solid angle is given by,

$$\frac{dN}{d\Omega dz} = n_0 D_A^2 (1+z)^2 \frac{c}{H(z)}\,.$$

b) Inserting the luminosity distance, show that the flux function is given by

$$\frac{dN}{d\Omega dF} = -\frac{n_0}{2}\left(\frac{L}{4\pi}\right)^{3/2} F^{-5/2} \frac{c}{H(z)} \frac{1}{(1+z)^2} \frac{dz}{dD_L},$$

where F is the flux.

c) Show that in the local universe ($z \ll 1$) this expression reduces to

$$\frac{dN}{d\Omega dF} = -\frac{n_0}{2}\left(\frac{L}{4\pi}\right)^{3/2} F^{-5/2}\,.$$

Hint: Consider the local Hubble function and a flat model.

Note: This expression is called the Euclidean limit of the flux function, while the remaining factor is a cosmological correction:

$$\frac{c}{H(z)} \frac{1}{(1+z)^2} \frac{dz}{dD_L}\,.$$

d) Using the Euclidean flux function compute the total flux in the volume, including sources with fluxes down to a minimum F_0, and recover Olbers' paradox.

e) Consider now Milne's Universe, which is an open (i.e. with negative curvature) cosmological model where the Hubble function is $H(z) = H_0\,(1+z)$. Show that in this case the flux cosmological correction is given by $(1+z)^{-4}$.

Hint: Use $D_L = (1+z)\,D_M$.

f) Compute the total flux in Milne's Universe and solve Olbers' paradox!

Hint: It will be useful to write the cosmological correction as function of flux.

Exercise 2: Friedmann's equations

2.1) Consider a 2-fluid flat universe with dark matter and cosmological constant.

a) Compute the redshift of the transition to the dark energy epoch.

b) Derive the expression of the line of no acceleration in a $(\Omega_m, \Omega_\Lambda)$ plane.

2.2) Consider the concordance universe of the ΛCDM 3-fluid model, i.e. a flat cosmology with $h = 0.7$, $\Omega_m = 0.3$, $\Omega_\Lambda = 0.7$ and $\Omega_r = 8 \times 10^{-5}$.

a) Even though the redshift range extends from 0 to ∞, the redshift at half of the age of the universe is a surprisingly low value. Compute it for the concordance universe.
Hint: solve the integral numerically, for example using Wolfram Alpha online.

Exercise 3: The rabbit and the turtle

3.1)

a) In terms of the dynamics of the expansion, think about what is the physical parameter that defines the moment when a universe reaches $a = 1$. Is it a fixed moment in time? Is it a fixed size of the universe? Is it some other physical parameter (and in this case say which parameter do you think it is)?

b) Show that a decelerating universe is younger than the Hubble time.

c) The result in b) may seem unexpected because it leads to the conclusion that a slower universe (the decelerating one) is younger than a fast-growing accelerating universe. The fact that the slowly expanding Universe is able to reach the current state $a = 1$ sooner than the rapidly expanding universe may seem a paradox. Explain that this result is not a paradox, but it is in fact the expected behaviour.

Hint: Consider two universes with two different power law expansions $a \propto t^n$ and compare the evolution of their Hubble functions, H(a). From this comparison and taken into account the answer to a) it should become clear that there is no paradox.