# Cosmologia Física

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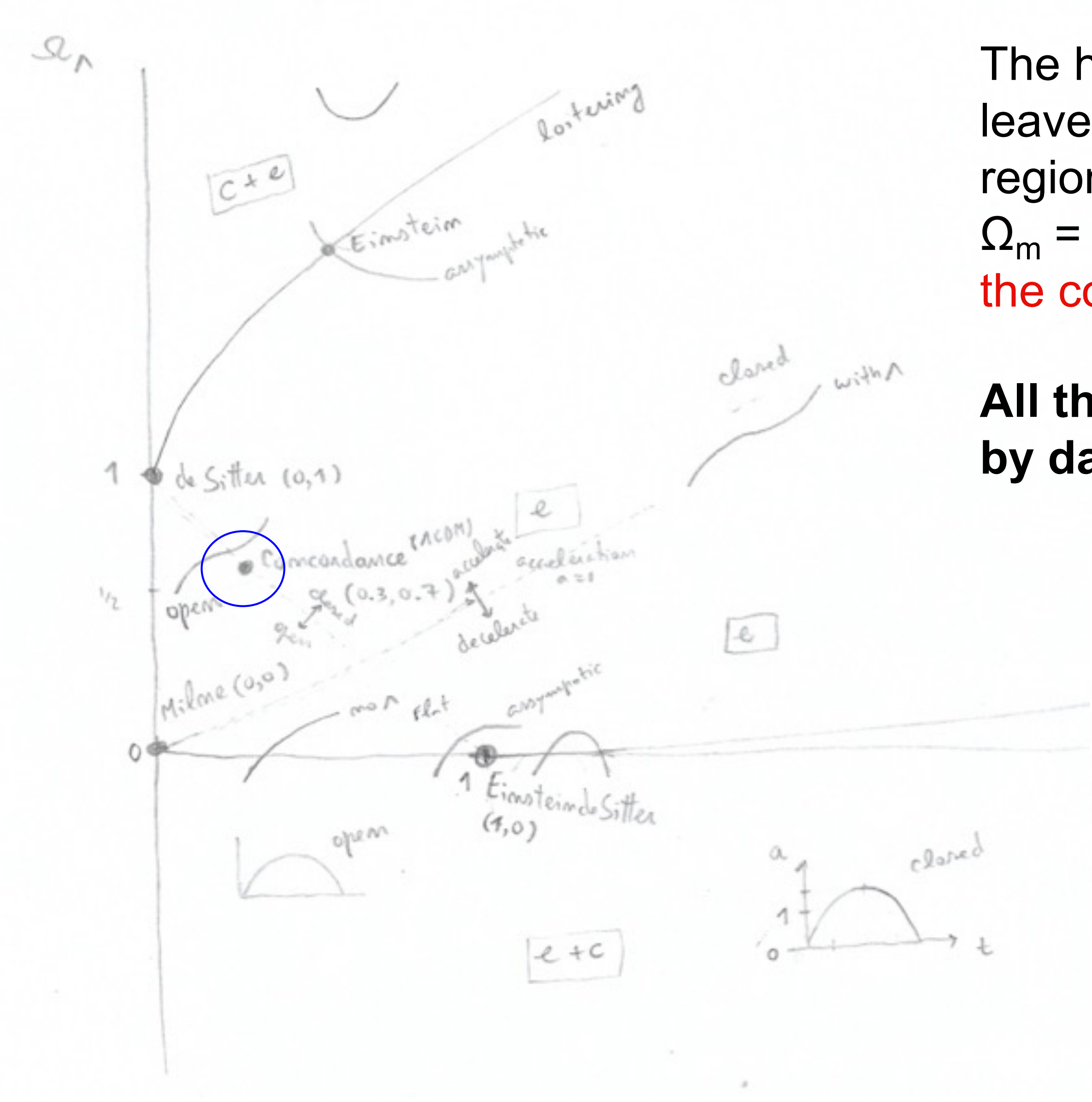
# The Homogeneous Universe

## The concordance model

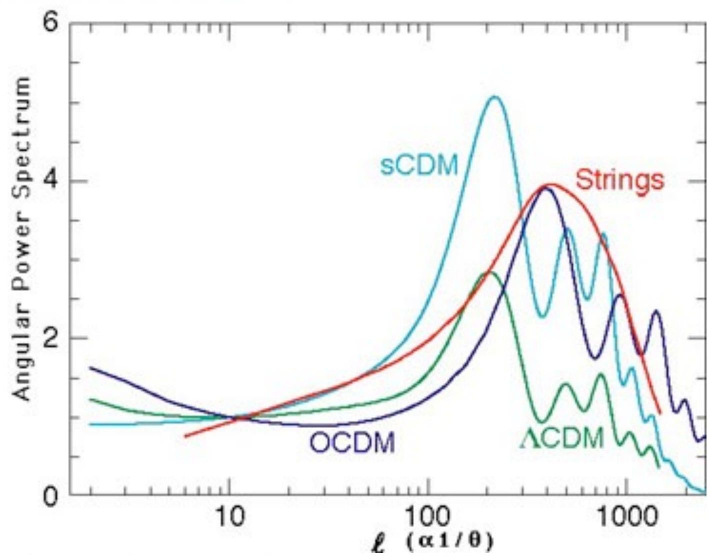
# Concordance model: parameter values

**Which FRW model better corresponds to the reality**? To answer this question, we need to determine the values of its parameters $H_0$, $\Omega_i$, $w_i$




The high-precision of current data only leaves a small uncertainty around the region defined by $\Omega_m = 0.3$ , $\Omega_\Lambda = 0.7$ (and so $\Omega_K = 0$): the concordance model

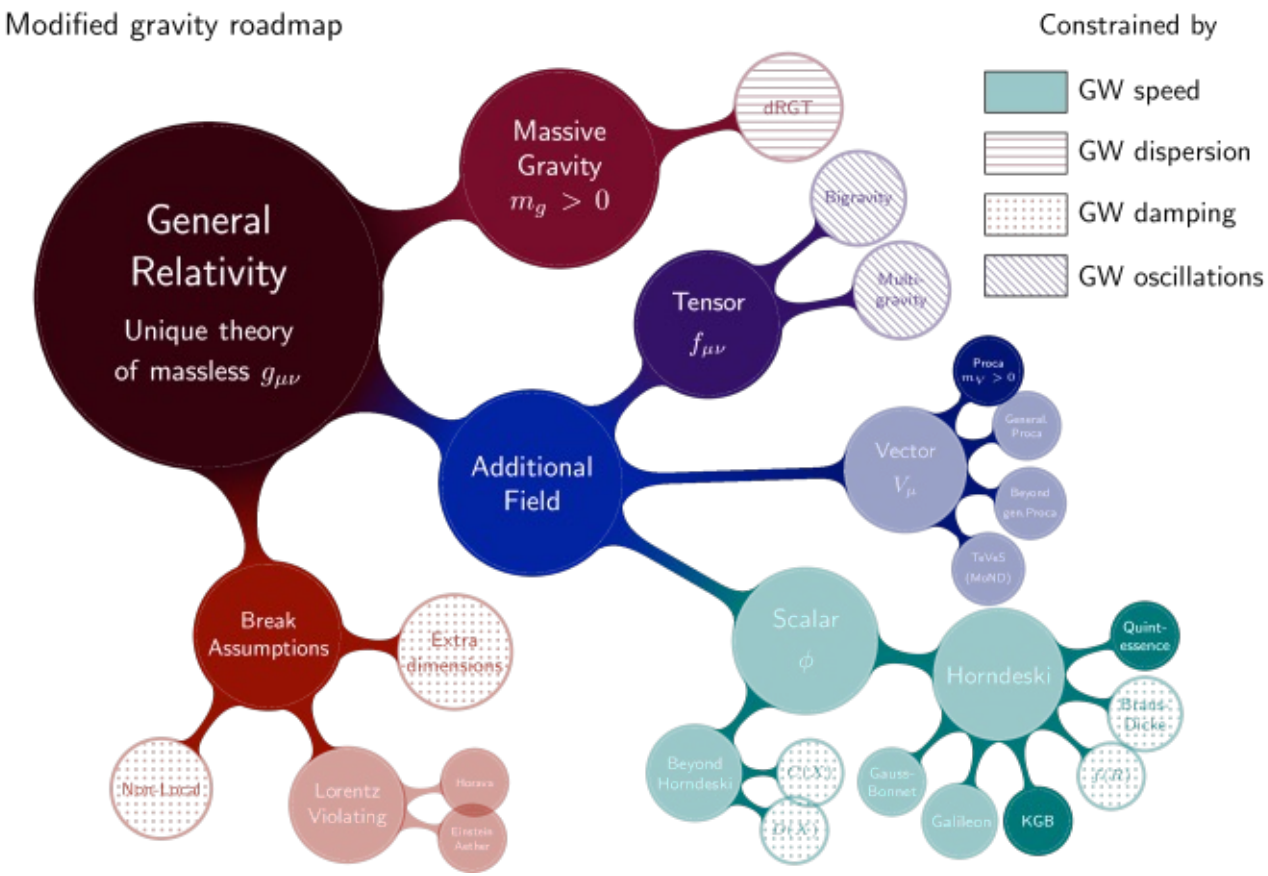
**All these other models are ruled out by data.**



**Note that even though these models are ruled out, there is still room for models other than the concordance model.**

Viable cosmological models do not spread out in this plane of parameters, since they need to be close to the concordance model.

They consist mainly of different evolutions for $\rho_{DE}$ (a) and $w_{DE}$(a) (instead of being constant), but that lead to the same $\Omega_{DE}$ ~ 0.7 and w(a=1) ~ -1




*Ezquiaga & Zumalacarregui 2018, https://arxiv.org/abs/1807.09241v1*

The best way to find the values of the cosmological parameters is to estimate them from measurements of "cosmological functions", i.e., quantities like distances or power spectra that depend on the cosmological model. However, it is also possible to make "**direct (or astrophysical) measurements**" of some astrophysical properties that may provide good approximations to the values of the cosmological parameters.

## Radiation

$\Omega_r = \Omega_\gamma + \Omega_\nu$ (relativistic)

The main contribution to the cosmological radiation are the **CMB photons.**

The **energy density of the CMB photons** is found by summing up the energy of all photons. The CMB has a blackbody spectrum and so the energy distribution of the photons is well-known and is determined by the temperature.

The energy density is then the integral of hν with a window function (the Bose-Einstein distribution):

$$\rho_{CMB} = \frac{2}{(2\pi)^3} \int d^3q \, \hbar\nu \, \frac{1}{e^{\hbar\nu/kT} - 1} \qquad (c=1)$$

2 d.o.f. — Bose-Einstein distribution

(here h is the Planck constant)

$$\rho_{CMB} = \frac{1}{c^2} \frac{\pi^2}{15} \frac{(k_B T_{CMB})^4}{(\hbar c)^3} \simeq 4.5 \times 10^{-34} \, g/cm^3$$

using T_CMB = 2.725 K

Dividing by the critical density, $\rho_c = 3\,H_0^2 / 8\pi G = 1.88 \times 10^{-29}\, h^2$ g cm$^{-3}$,

the dimensionless radiation density is **$\Omega_\gamma = 4.8 \times 10^{-5}$** (with h=0.7)

The **massless neutrinos** also give an important contribution to the radiation of the Universe. The **energy density of massless neutrinos** is computed in the same way, but using the Fermi-Dirac distribution instead and a different number of degrees-of-freedom:

Add massless neutrinos

Fermi-Dirac; Fermion factor

$$\rho_\nu = \frac{6}{(2\pi)^3} \int d^3E \; E \; \frac{1}{e^{E/kT_\nu}+1} = 6\;\frac{7}{8}\;\frac{\pi^2}{30}\;T_\nu^4 = 3 \cdot \frac{7}{8} \left(\frac{T_\nu}{T_\gamma}\right)^4 \rho_{CMB}$$

6 d.o.f.

$\nu_e, \bar{\nu}_e, \nu_\mu, \bar{\nu}_\mu, \nu_\tau, \bar{\nu}_\tau$

From the thermal history of the Universe, we know that neutrinos decouple before the CMB, when the temperature was higher, such that:

$$\frac{T_\nu}{T_{CMB}} = \left(\frac{4}{11}\right)^{1/3}$$

So, their density is $\rho_\nu = 0.68\ \rho_\gamma$

$$\rho_{r_{total}} = \left(1 + 3 \cdot \frac{7}{8}\left(\frac{4}{11}\right)^{4/3}\right)\rho_{CMB} = 1.68\ \rho_{CMB}$$

and in terms of the dimensionless density parameter: $\mathbf{\Omega_r = \Omega_\gamma + \Omega_\nu \sim 8.0 \times 10^{-5}}$ (with h=0.7). It is a negligible contribution to the density of the Universe today.

Note however, that **neutrinos are massive**, and the massless neutrinos scenario is only a good approximation when the temperature of the Universe is $T >> M_\nu$ .

Later in the Universe, neutrinos become non-relativistic fermionic particles and the **density of massive neutrinos** is computed as:

$$\rho_\nu = N_\nu m_\nu = M_\nu \frac{6}{(2\pi)^3}\int d^3E \frac{1}{e^{E/kT}+1}$$

using again:

$$\frac{T_\nu}{T_{CMB}} = \left(\frac{4}{11}\right)^{1/3}$$

The result is:

$$\Omega_\nu = M_\nu \frac{1}{94\ eV} h^{-2}$$

The density depends on the neutrino mass (here **$M_\nu$ is the sum of the masses of the 3 neutrinos**) and is no longer fully determined by the temperature.

For example, a neutrino mass of 0.1 eV would give a small but non-negligible contribution to the total energy density of $\Omega \sim 0.001$

Neutrinos then contribute both to the radiation density and to the matter density.

An additional cosmological parameter N_eff (effective number of relativistic species) was introduced to model what fraction of neutrino density is considered relativistic and contributes to the radiation density, and what fraction is non-relativistic and contributes to the matter density affecting structure formation on small scales.

# Baryonic matter $\Omega_b$

Its total density can be determined by nucleosynthesis.

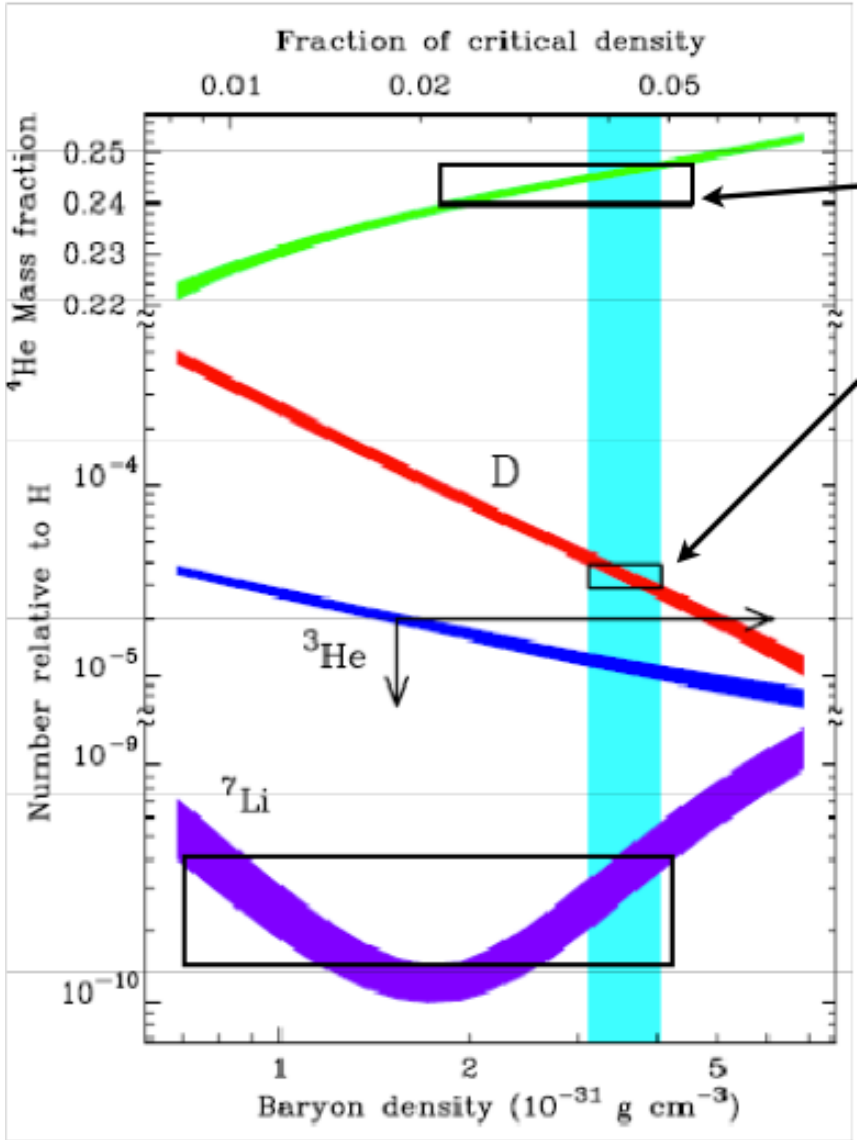
**The reaction rate for element formation depends on the total amount of baryons present in the Universe** (before nucleosynthesis they are mainly in the form of protons and neutrons)

**Examples**:

higher $\Omega_b$ → more He4 forms (the most stable species)

higher $\Omega_b$ → less D ou He3 form (because He4 is formed instead)

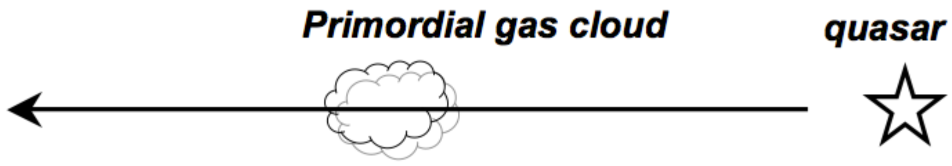
**This provides a powerful way to estimate $\Omega_b$** : we just need to be able to measure the total amount of one of these species.

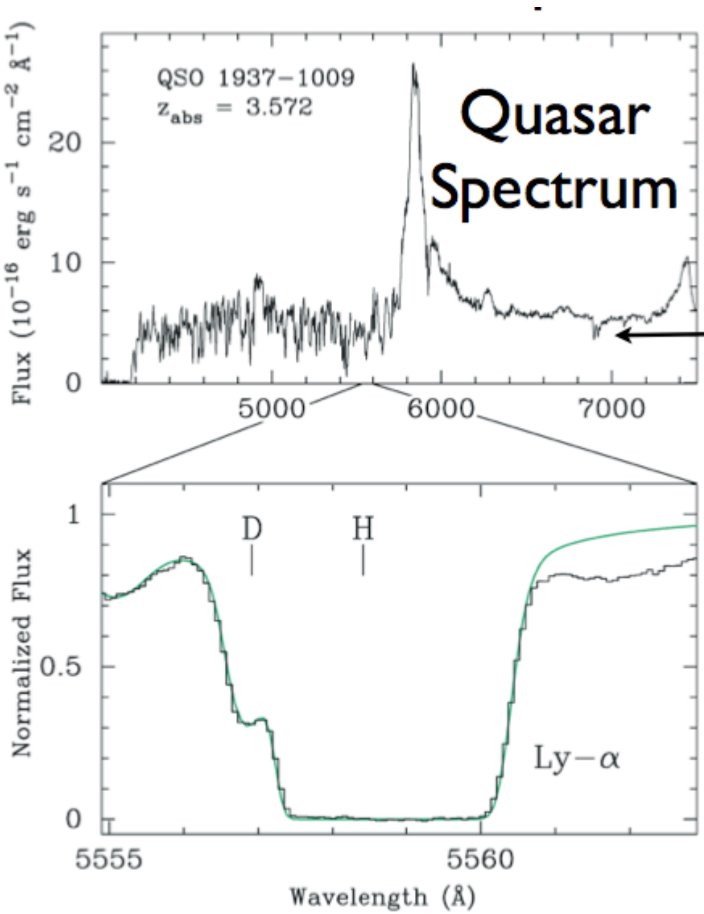



**$\Omega_b$ ~ 0.04** (with h=0.7)

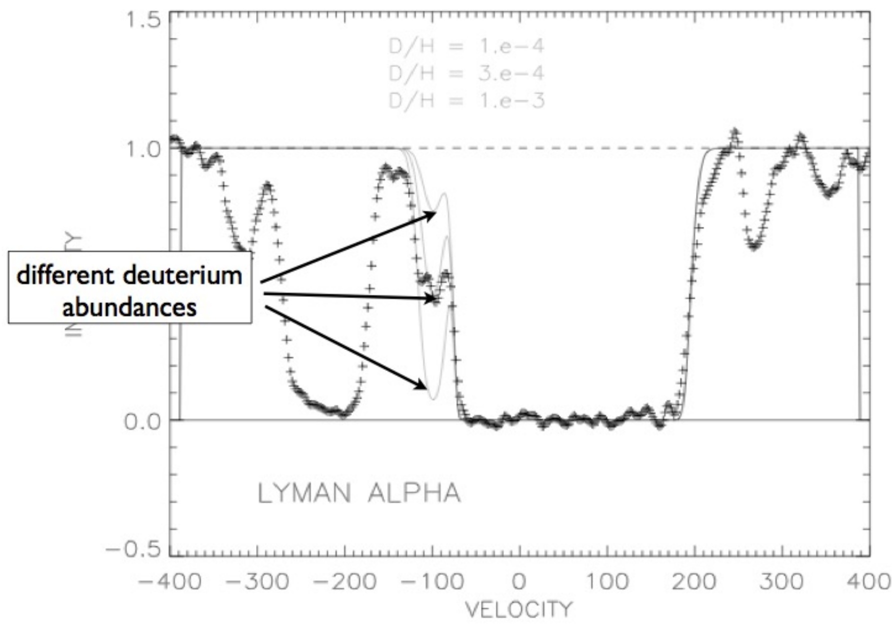
# Example: measuring the abundance of deuterium

Observe gas clouds in the early universe (where stars have not yet formed), looking for absorption features of rare elements (deuterium) on the spectrum of background bright sources (quasars)




to control, check if there are heavier elements




**Result: [D/H] ~3 x $10^{-5}$**

## Dark matter $\Omega_{dm}$

Galaxy clusters are very large quasi-linear structures. As such, they represent well the average densities of the whole Universe and the measurement of their masses is a good indicator of the total mass density $\Omega_m$ of the Universe.

The total mass of a cluster can be determined in 3 different ways.

Each method makes some assumptions about the state of equilibrium of the cluster

1. X-rays emission → hydrostatic equilibrium
2. Gravitational lensing → cluster symmetries
3. Dynamics of the cluster galaxies → virial theorem

## Mass from Temperature: X-ray profiles

Ionized gas in clusters - assumed to be in hydrostatic equilibrium $\frac{1}{\rho}\frac{\mathrm{d}p}{\mathrm{d}r} = -\frac{GM(r)}{r^2}$,

+ ideal gas $p = nKT$ $(n = \rho / m_p)$

$$\rightarrow \quad M(r) = -\frac{rkT}{Gm_\mathrm{p}}\frac{\mathrm{d}\ln\rho}{\mathrm{d}\ln r}$$

This is the total mass needed to keep the hot gas (with pressure p, temperature T and density ρ) in equilibrium.

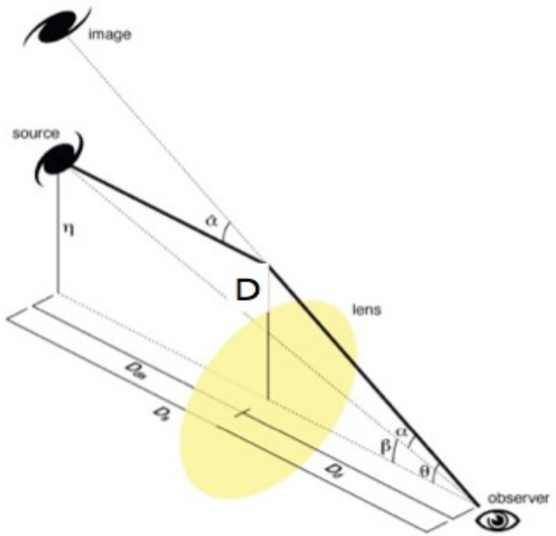
It is also needed to assume a density profile for the cluster (to be able compute dρ /dr), i.e., assume a model:

$$n(x) = \frac{n_0}{(1+x^2)^{3\beta/2}}, \quad x \equiv \frac{r}{r_\mathrm{c}}$$

$$\rightarrow \quad \frac{\mathrm{d}\ln\rho}{\mathrm{d}\ln r} = \frac{\mathrm{d}\ln n}{\mathrm{d}\ln r} = -3\beta\frac{r^2}{1+r^2} \quad \rightarrow \quad M(r) = \frac{3\beta rkT}{Gm_\mathrm{p}}\frac{r^2}{1+r^2}$$

Typical values: kT ~10 KeV; r ~1 Mpc; β = 2/3 → **M ~$10^{15}$ $M_{Sun}$**

# Mass from Gravity: Gravitational lensing




Measuring the positions of multiple images and giant arcs, we can constrain the mass distribution of the lens.

Need to model the lens. Also need to know the distance to the lens and to the source

Simple approximation: modeling the cluster as a sphere of mass M concentrated in the center, it produces a deflection of α for a light ray passing at a distance D from the center →

$$\tilde{\alpha} = \frac{4GM}{Dc^2} \qquad D = D_d D_{ds}/D_S$$

Measure the deflection α, measure the distances → get the mass **M ~$10^{15}$ $M_{Sun}$**

# Mass from kinematics: Galaxy motions

For systems that have collapsed gravitationally and are relaxed, the virial theorem is

E_kin = -1/2 E_pot

Galaxy are observed in spectroscopy → Doppler shifts are measured along the line-of-sight → the measured dispersion in the average velocity along the l-o-s is $\left\langle v_{\parallel}^2 \right\rangle$

Dispersion of the average velocity $\left\langle v^2 \right\rangle = \left\langle v_x^2 \right\rangle + \left\langle v_y^2 \right\rangle + \left\langle v_z^2 \right\rangle = 3\left\langle v_{\parallel}^2 \right\rangle$

$$E_{\rm kin} = \frac{1}{2}\sum_i m_i \mathbf{v}_i^2 = \frac{3}{2} M \left\langle v_{\parallel}^2 \right\rangle \qquad E_{\rm pot} = \frac{GM^2}{R_{\rm cl}} \rightarrow M = \frac{3}{G}\left\langle v_{\parallel}^2 \right\rangle R_{\rm cl}$$

Typical values: v ~1000 Km/s; $R_{cluster}$ ~1 Mpc → **M ~$10^{15}$ $M_{Sun}$**

# Mass from Luminosity: Luminous matter

Now, from the **mass-luminosity relation** for stars ($L \sim M^3$), and the measurement of the flux from the cluster, it is found that the total "luminous mass" in a $10^{15}\ M_{Sun}$ cluster is only $\sim 10^{13}\ M_{Sun}$ .

**This implies a mass-to-light ratio of M/L ~100 (the precise value is 160)**

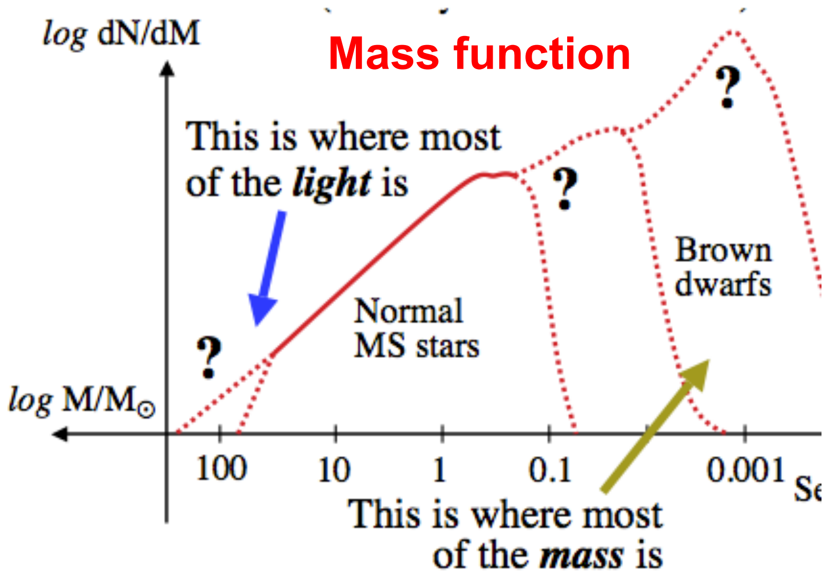
**On the other hand, the mass density in stars is known. It is also estimated from light:**

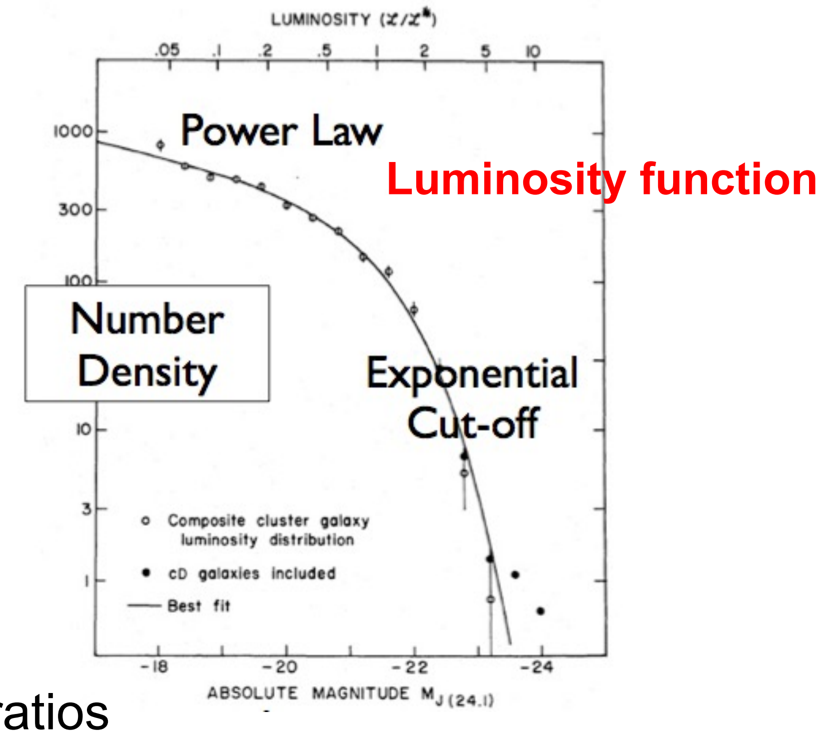
<u>Estimate mass from light:</u>

$L \sim M^3$

The mass of one star comes from its luminosity (and the luminosity is determined by flux and distance measurements)

The total mass of stars in a galaxy (in a volume) is computed taking into account the distribution of stars : **the mass (or luminosity functions).**




$M/L \sim M^{-2}$
low luminosity stars (also low mass) → high M/L ratios
high luminosity stars (also high mass) → low M/L ratios

**The result is M_stars / Volume = 3 x $10^8$ $M_{solar}$ / $Mpc^3$**

Notice that $\rho_c$ = 1.88 x $10^{-26}$ $h^2$ Kg $m^{-3}$ = 1.4 $10^{11}$ $M_{solar}$ / $Mpc^3$

**→ $\Omega_{stars}$ = 0.002**

This means that the M/L ratio of 160 found for the total mass density corresponds to

→ Ω_m_clusters = **160 x 0.002** = 0.32

**These results show that:**

**- There is a large amount of non-luminous matter** (with very high M/L ratio), much larger that the baryonic matter $\Omega_b = 0.04$ → **dark matter**, with **Ω_dm_clusters ~ 0.28**

**- The ratio between dark matter and baryonic matter densities is 7 (much lower than the ratio of 160 between dark matter and stars) →** most of baryonic matter is not in the form of stars/galaxies that contribute to the luminous matter of galaxies and clusters. (It is in the form of hot ionized gas - in clusters and in the cosmic web - ).

- Stars, which are "the light in the Universe" to our eyes, are a small contribution to the total baryonic mater in the Universe 0.002/0.04 (5%) that in turn is only 12% of the total matter in the Universe.

- **The total matter density in the Universe is less than 1 : $\Omega_{dm} + \Omega_b + \Omega_v$ ~0.3 →** **this implies there is something else missing to reach the needed total of Ω = 1 (from Friedmann eq.), and moreover that is the dominant contribution!**

**So, since the 1980-90s, much before the modern SN and CMB probes, cluster observations already gave a hint that the total matter density in the Universe was less than 1 → a missing $\Omega = 0.7$ component.**

It was first thought that this could be a hint for an **open Universe, oCDM**. Indeed, curvature can be moved to the right-side of Einstein equation and be considered as a contribution to the densities, $\Omega_K$

Is it curvature? → the Universe would need to have **negative curvature** (which would also imply it is open) in order to have $\mathbf{\Omega_K >}$ 0 (the sign of $\Omega_K$ is opposite to the sign of K).

However, later on CMB measurements found that most likely $\mathbf{\Omega_K = 0}$ → flat Universe

In the late 1990s, the SN distance measurements hinted to an accelerated expansion → the new component needs to have **w < -1/3** → **dark energy**

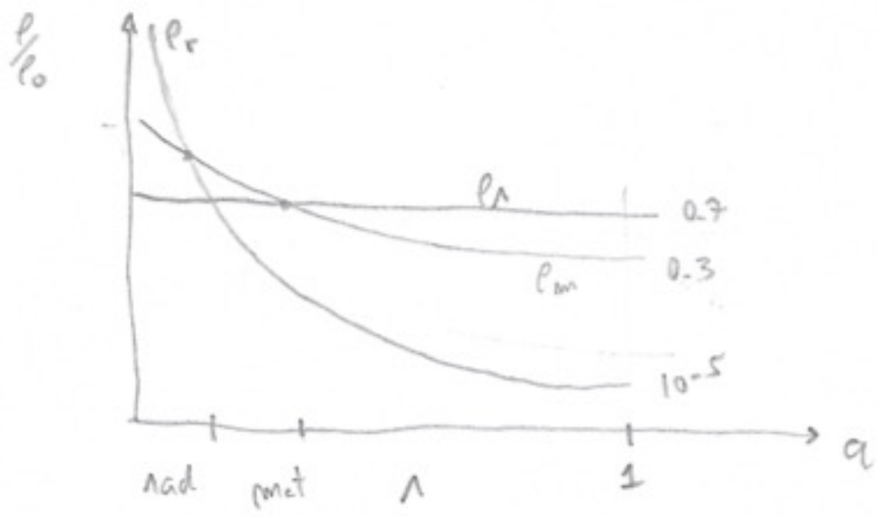
# Concordance model: characteristic times and sizes

**The values of the density parameters determine the behavior of the homogeneous Universe (also called the zeroth-order Universe, or simply the background).**

Even though there are several open problems, the model favored by the observations is the so-called **concordance model** (given in round numbers):

$$\Lambda\text{CDM with } \mathbf{\Omega_m = 0.3,\ \Omega_\Lambda = 0.7,\ \Omega_K = 0,\ \Omega_r = 8 \times 10^{-5},\ h = 0.7}$$

Given these values and the functional forms of the densities, there is a sequence of **epochs of domination** in the evolution of the Universe: **the total density of the Universe is dominated by radiation, matter, and finally Λ.**



We can easily find the scale factor (or redshift) when the two transitions occur:

**radiation / matter $a_{eq}$**

$$\rho_m(a_{eq}) = \rho_{rad}(a_{eq})$$

$$\rho_{0m}\, a_{eq}^{-3} = \rho_{0r}\, a_{eq}^{-4}$$

$$\Omega_m\, a_{eq}^{-3} = \Omega_r\, a_{eq}^{-4}$$

$$a_{eq} = \Omega_r / \Omega_m = 2.67 \times 10^{-4}$$

$$z_{eq} = 3\,749$$

**matter / dark energy $a_\Lambda$**

$$\rho_m(a_\Lambda) = \rho_\Lambda$$

$$\Omega_m\, a_\Lambda^{-3} = \Omega_\Lambda$$

$$a_\Lambda = (\Omega_m / \Omega_\Lambda)^{1/3} = 0.75$$

$$z_\Lambda = 0.33$$

# Age of the concordance universe

Knowing the values of the cosmological parameters, we can compute **the age of the concordance universe**
(hence age, if measurable, is another quantity - like distances, curvature, transition redshifts, horizon sizes, etc - that can constrain the parameters)
For this, we just need to compute the integral found from the Friedmann eq:

$$\left(\frac{\dot{a}}{a}\right)^2 = H^2(a) \iff \frac{\dot{a}}{a} = H(a) \iff da = a\,H(a)\,dt \Rightarrow t(a) = \int_{a_i}^{a_f} \frac{da}{a\,H(a)}$$

$$\iff t(a) = \frac{1}{H_0}\int_{a_i}^{a_f} \frac{da}{a\,E(a)}$$

$\frac{1}{H_0} \downarrow t_{Hubble}$ ; $E(a) \downarrow$ função $E(a)$ $\longrightarrow$

$$E(a) = \left[\Omega_m a^{-3} + \Omega_r a^{-4} + \Omega_k a^{-2} + \Omega_\Lambda\right]^{1/2}$$

or, in terms of redshift $\quad 1+z = 1/a \rightarrow da = -\frac{dz}{(1+z)^2}$

To have a rough estimate of the age, let us compute the duration of each of the three epochs, considering the simplification that only one species is relevant during each of the epochs:

## Radiation epoch

- $0 < a < a_{eq}$

$\Omega_r$ only

$$t_{eq} = \frac{1}{H_0} \frac{1}{\sqrt{\Omega_r}} \int_{z_{eq}}^{\infty} \frac{dz}{(1+z)^3} = \frac{1}{H_0 \sqrt{\Omega_r}} \frac{1}{2} \frac{1}{(1+z)^2} \Big|_{\infty}^{z_{eq}}$$

$$= \frac{1}{2} \frac{1}{H_0 \sqrt{\Omega_r} (1+z_{eq})^2} .$$

$z_{eq} = 3749 \rightarrow t_{eq} = 4.0 \times 10^{-6}\, t_H = 55\,000$ yr

($h = 0.7 \rightarrow t_H = 13.97$ Gyr)

## Matter epoch

- $a_{eq} < a < a_\Lambda$

$\Omega_m$ only

$$t_\Lambda - t_{eq} = t_H \frac{1}{\sqrt{\Omega_m}} \int_{z_\Lambda}^{z_{eq}} \frac{dz}{(1+z)^{5/2}} = \frac{t_H}{\sqrt{\Omega_m}} \frac{2}{3} \left. \frac{1}{(1+z)^{3/2}} \right|_{z_{eq}}^{z_\Lambda}$$

$z_\Lambda = 0.33 \rightarrow t_\Lambda = 0.61\ t_H = 8.52$ Gyr

## Dark energy epoch

- $a > a_\Lambda$

$$t_0 - t_\Lambda = t_H \frac{1}{\sqrt{\Omega_\Lambda}} \int_0^{z_\Lambda} \frac{dz}{1+z} = \frac{t_H}{\sqrt{\Omega_\Lambda}} \ln(1+z) \Big|_0^{z_\Lambda} = \frac{t_H}{\sqrt{\Omega_\Lambda}} \ln(1+z_\Lambda)$$

$t_\Lambda = 0.34\ t_H = 4.76$ Gyr

The radiation epoch is very short,

the matter epoch is the longest one,

the dark energy epoch did not start so recently as we might think

**age of the Universe = 0.95 $t_H$ = 13.28 Gyr**

## Characteristic sizes

**We can also compute various characteristic sizes and distances in the concordance Universe**:

(remember a comoving distance is $dx = dt/a = da/a^2H$)

- the **particle horizon $H_p$** at a given time is the distance travelled by light since the big bang up to that time.

It is thus given by (comoving):

$$H_p(a) = \int_0^a \frac{c}{a'^2\, H(a')} da'$$

- the **event horizon $H_e$** today is the maximum comoving distance that light can travel from today until the end of the Universe ($t = \infty$). This implies that light emitted today by an object farther than that distance will never reach us.

It is given by (comoving):

$$H_e(a=1) = \int_1^\infty \frac{c}{a'^2\, H(a')} da'$$

- the **size of the observable Universe** at a given time is the distance between the observer at that time and the decoupling redshift (the last scattering surface that released the CMB radiation), beyond which the Universe is opaque.

It is thus given by (comoving):

$$D_c(a) = \int_{0.00091}^{a} \frac{c}{a'^2 H(a')} da'$$

- the **Hubble radius**, given by (proper):

$$r_H(a) = \frac{c}{H(a)}$$

All these quantities are computed from the **Hubble function**, which in the concordance cosmology is given by:

$$H(a) = H_0 \left( \frac{0.3}{a^3} + \frac{8 \times 10^{-5}}{a^4} + 0.7 \right)^{1/2}$$

Using the concordance values for the density parameters, h=0.7, and

$1+z_{eq} = 3750 \rightarrow a_{eq} = 2.67 \times 10^{-4}$
$1+z_{dec} = 1101 \rightarrow a_{dec} = 9.1 \times 10^{-4}$

we can compute all these quantities.

| Feature | $a_{eq}$ | $a_{dec}$ | $a_0$ |
|---|---|---|---|
| Horizon_particle | | | |
| comoving [Mpc/h] ([Mpc]) | 73 (104) | 197 (281) | 9738 (13911) |
| proper [Mpc/h] | 0.019 | 0.18 | 9738 |
| Hubble radius | | | |
| comoving [Mpc/h] ([Mpc]) | 64 (91) | 143 (204) | 3000 (4286) |
| proper [Mpc/h] | 0.017 | 0.13 | 3000 |
| Observable Universe | | | |
| comoving = proper [Mpc/h] ([Mpc]) | - | - | 9541 (13630) |
| Horizon_event | | | |
| comoving = proper [Mpc/h] ([Mpc]) | - | - | 3422 (4889) |

Notice that an event horizon exists because the Universe is accelerating. In EdS the event horizon is infinite, all emission will eventually reach the observer.