10.1 Kinematics

Let the Higgs boson of mass M_H and momentum q decay into particles A and B of masses m_1 and m_2 and momenta p_1 and p_2 respectively: $H(q) \to A(p_1) + B(p_2)$. The decay rate summed over final polarisations and colours is:

$$d\,\Gamma = \frac{1}{2\,M} \frac{d^3 p_1}{(2\,\pi)^3\, 2\,E_1} \frac{d^3 p_2}{(2\,\pi)^3\, 2\,E_2} (2\,\pi)^4\, \delta^4\,(q - p_1 - p_2)\, \left|\bar{T}\right|^2 \;, \tag{10.1}$$

with $\left|\bar{T}\right|^2$ the invariant matrix element squared, summed over final colours and polarisations. Momentum conservation imposes $p_1.p_2 = (M_H^2 - m_1^2 - m_2^2)/2$ with $p_1^2 = m_1^2$ et $p_2^2 = m_2^2$. Thus $\left|\bar{T}\right|^2$ depends only on the external masses $\left|\bar{T}(m_1^2, m_2^2, M_H^2)\right|^2$ and the integral in eq. (10.1) can be done independently of the decay channel. Using $d^3 p_2/2\,E_2 = d^4 p_2\, \delta^+(p_2^2 - m_2^2)$ and carrying out the $d^4 p_2$ integration it comes out

$$d\,\Gamma = \frac{1}{2\,M_H} \frac{\left|\bar{T}\right|^2}{(2\,\pi)^2} \int \frac{d^3 p_1}{2\,E_1}\, \delta^+\left((q - p_1)^2 - m_2^2\right). \tag{10.2}$$

Going to the rest frame of the Higgs boson, $q = (M, 0, 0, 0)$, one finds that the argument of the δ^+ function reduces to $(M^2 - 2ME_1 + m_1^2 - m_2^2)$ independent of the angles. Since all cases we consider have $m_1 = m_2$ the expressions will simplify. Using $p_1\, dp_1 = E_1\, dE_1$ all integrations are easily done to get:

$$\boxed{\Gamma = \frac{1}{16\,\pi\, M_H} \left|\bar{T}\right|^2 \sqrt{1 - \frac{4\,m^2}{M^2}}\;,} \tag{10.3}$$

with m the common mass of the decay products.

49.4.1 *Survival probability*

If a particle of mass M has mean proper lifetime τ $(= 1/\Gamma)$ and has momentum $(E, \boldsymbol{p})$, then the probability that it lives for a time t_0 or greater before decaying is given by

$$P(t_0) = e^{-t_0\,\Gamma/\gamma} = e^{-Mt_0\,\Gamma/E}\ , \tag{49.14}$$

and the probability that it travels a distance x_0 or greater is

$$P(x_0) = e^{-Mx_0\,\Gamma/|\boldsymbol{p}|}\ . \tag{49.15}$$

31st May, 2024

49.4.2 *Two-body decays*

In the rest frame of a particle of mass M, decaying into 2 particles labeled 1 and 2,

$$E_1 = \frac{M^2 - m_2^2 + m_1^2}{2M} \; , \tag{49.16}$$

$$|\boldsymbol{p}_1| = |\boldsymbol{p}_2| = \frac{1}{2M}\sqrt{\lambda(M^2, m_1^2, m_2^2)} \; , \tag{49.17}$$

and

$$d\Gamma = \frac{1}{32\pi^2}\, |\mathscr{M}|^2\, \frac{|\boldsymbol{p}_1|}{M^2}\, d\Omega \; , \tag{49.18}$$

where $\lambda(\alpha, \beta, \gamma) = \alpha^2 + \beta^2 + \gamma^2 - 2\alpha\beta - 2\alpha\gamma - 2\beta\gamma$ is the Källén function and $d\Omega = d\phi_1 d(\cos\theta_1)$ is the solid angle of particle 1. The invariant mass M can be determined from the energies and momenta using Eq. (49.2) with $M = E_{\rm cm}$.

Cross Sections

The formula for calculating the scattering cross-section (σ) for a two-to-two ($2 \rightarrow 2$) process in Quantum Field Theory (QFT) relates the scattering amplitude ($\mathcal{M}$) derived from Feynman diagrams to an experimental probability.

The simplified, most common form used in the **Center of Mass (CM) frame** is:

$$\frac{d\sigma}{d\Omega} = \frac{1}{64\pi^2 s} \frac{|\vec{p}_f|}{|\vec{p}_i|} |\mathcal{M}|^2$$

Where the components are:

- $\frac{d\sigma}{d\Omega}$: The differential cross-section (scattering probability per unit solid angle Ω).
- $\mathcal{M}$: The Lorentz-invariant scattering amplitude (calculated via Feynman rules).
- s: Mandelstam variable, representing the square of the total center-of-mass energy ($s = E_{cm}^2$).
- $|\vec{p}_i|$: Magnitude of the momentum of the incoming particles in the CM frame.
- $|\vec{p}_f|$: Magnitude of the momentum of the outgoing particles in the CM frame. MIT OpenCourseWare | Free Online Co... +4

The two-body cross section may be written as

$$\frac{d\sigma}{dt} = \frac{1}{64\pi s} \frac{1}{|\boldsymbol{p}_{1\text{cm}}|^2} |\mathscr{M}|^2 . \tag{49.33}$$

In the center-of-mass frame

$$\begin{aligned} t &= (E_{1\text{cm}} - E_{3\text{cm}})^2 - (p_{1\text{cm}} - p_{3\text{cm}})^2 - 4p_{1\text{cm}}\, p_{3\text{cm}}\, \sin^2(\theta_{\text{cm}}/2) \\ &= t_0 - 4p_{1\text{cm}}\, p_{3\text{cm}}\, \sin^2(\theta_{\text{cm}}/2) , \end{aligned} \tag{49.34}$$

where θ_{cm} is the angle between particle 1 and 3. The limiting values t_0 ($\theta_{\text{cm}} = 0$) and t_1 ($\theta_{\text{cm}} = \pi$) for $2 \to 2$ scattering are

$$t_0(t_1) = \left[\frac{m_1^2 - m_3^2 - m_2^2 + m_4^2}{2\sqrt{s}}\right]^2 - (p_{1\,\text{cm}} \mp p_{3\,\text{cm}})^2 . \tag{49.35}$$

In the literature the notation t_{min} (t_{max}) for t_0 (t_1) is sometimes used, which should be discouraged since $t_0 > t_1$. The center-of-mass energies and momenta of the incoming particles are

$$E_{1\text{cm}} = \frac{s + m_1^2 - m_2^2}{2\sqrt{s}} , \qquad E_{2\text{cm}} = \frac{s + m_2^2 - m_1^2}{2\sqrt{s}} , \tag{49.36}$$

For $E_{3\text{cm}}$ and $E_{4\text{cm}}$, change m_1 to m_3 and m_2 to m_4. Then

$$p_{i\,\text{cm}} = \sqrt{E_{i\,\text{cm}}^2 - m_i^2} \text{ and } p_{1\text{cm}} = \frac{p_{1\,\text{lab}}\, m_2}{\sqrt{s}} . \tag{49.37}$$

Here the subscript lab refers to the frame where particle 2 is at rest. [For other relations see Eqs. (49.2)–(49.4).]

Infinities in QFT - renormalisation

Can an infinite quantity be made finite and measured?

"Excuse me, is this the Society for Asking Stupid Questions?"

Infinities occur when integrals coming from the computation of cross sections and decay widths, using Feynman diagrams, give rise to terms that lead to infinities in the high energy (short distance) or low energy (long distance) limits.

Infinities from low energy physics are called infrared divergences and occur when there are massless particles.

Infinities from high energy physics are called ultraviolet divergences and arise in the limit of high energy.

Can an infinite quantity be made finite and measured?

We start with the infinite integral

$$f(x) = \int_1^{+\infty} \frac{1}{x+y}\,dy = \left[\ln(x+y)\right]_1^{+\infty} = \infty$$

and now we regularise it in an obviously non-unique way

$$\bar{f}(x) = f(x) - f(0) = \int_1^{+\infty} \frac{-x}{y(x+y)}\,dy = -\ln x$$

and we get the finite quantity

$$f(x) = \bar{f}(x) + f(0)$$

Renormalised (measured) quantity.

High order contribution.

Tree-level value.

The infinity will be hidden in the experimental measured quantity, like mass.

Regularisation – IR regulator and cut-off

We now start with the infinite integral

$$f(0) = \int_0^{+\infty} \frac{1}{y^2}\, dy = \left[-\frac{1}{y} \right]_0^{+\infty} = \infty$$

When an infrared regulator x is introduced, the integral is now finite for a non-zero regulator

$$f(x) = \int_0^{+\infty} \frac{1}{y^2 + x^2}\, dy$$

When a cut-off is introduced the integral will depend on the cut-off

$$f(\Lambda) = \int_\Lambda^{+\infty} \frac{1}{y^2}\, dy$$

And the cut-off can also be introduced to handle ultraviolet divergences

Dimensional Regularisation

The simplest divergences for large momentum behave as $|p|^{-2}$ or as $|p|^{-4}$. The momentum integrals are finite only for dimensions D < 2 and D < 4 respectively.

The idea is to calculate a Feynman integral for a continuous-valued number of dimensions D for which convergence is assured. The simplest integral is

$$I(D) = \int \frac{d^D p}{(2\pi)^D} \frac{1}{p^2 - m^2} = \frac{S_D}{(2\pi)^D} \int_0^\infty dp\, p^{D-1} \frac{1}{p^2 - m^2}$$

where

$$S_D = \frac{2\pi^{D/2}}{\Gamma(D/2)}; \qquad \Gamma(z) = \int_0^\infty dt\, t^{z-1} e^{-t}$$

is the surface of a unit sphere in D dimensions.

Dimensional Regularisation

By setting D=4-ε, the integral can be written as

$$I(D)=\int\frac{d^D p}{(2\pi)^D}\frac{1}{p^2-m^2}=\frac{m^2}{(4\pi)^2}\left(\frac{4\pi}{m^2}\right)^{\varepsilon/2}\Gamma(\varepsilon/2-1)$$

and expanding in ε,

$$\left(\frac{4\pi}{m^2}\right)^{\varepsilon/2}=1+\frac{\varepsilon}{2}\ln\left(\frac{4\pi}{m^2}\right)+O(\varepsilon^2) \qquad \Gamma(\varepsilon/2-1)=\frac{2}{\varepsilon}+\psi(2)+O(\varepsilon)$$

we obtain the result,

$$I(D)=\frac{m^2}{(4\pi)^2}\left\{\frac{2}{\varepsilon}+\psi(2)+\ln\left(\frac{4\pi}{m^2}\right)\right\}$$

Note the wrong dimensions in the argument of the log. The introduction of a mass scale related to coupling constant will solve this problem.

Infrared divergences

Let us consider the integral that represents the electron self-energy (I will come back to this later). It is clear that when $k^2 \to 0$ the integral diverges

$$\int \frac{d^4k}{(2\pi)^4}\gamma^\mu \frac{i(\not{q}-\not{k}+m)}{(q-k)^2-m^2+i\varepsilon}\gamma_\mu \frac{1}{k^2}$$

Electron propagator

Photon propagator

To know where the infinities are we add a mass for the photon and when the calculation is properly done, term proportional to this mass will cancel

$$\int \frac{d^4k}{(2\pi)^4}\gamma^\mu \frac{i(\not{q}-\not{k}+m)}{(q-k)^2-m^2+i\varepsilon}\gamma_\mu \frac{1}{k^2-m_\gamma^2}$$

IR regulator

Infrared divergences

IR divergences arise from not considering all the factors in a cross-section. Although both $e^+e^- \to \mu^+\mu^-$ and $e^+e^- \to \mu^+\mu^-(\gamma)$ are IR divergent, their sum does not depend on the photon mass.

When we consider the virtual corrections

$$\sigma_V = \frac{e^2}{8\pi^2}\sigma_0 \left\{ -4\ln^2 \frac{m_\gamma^2}{Q^2} + \frac{\pi^2}{3} - \frac{7}{2} \right\}$$

together with the real emission contribution

$$\sigma_R = \frac{e^2}{8\pi^2}\sigma_0 \left\{ 4\ln^2 \frac{m_\gamma^2}{Q^2} - \frac{\pi^2}{3} + 5 \right\}$$

The measured cross section does not depend on the regulator.

Renormalisable theories

If a theory is renormalisable all the infinities that arise in the calculation of physical observables can be absorbed in the parameters of the theory.

All terms compatible with the symmetry of the Lagrangian must be included.

Terms with mass dimension above 4 should be discarded. Let us consider the Lagrangian for a scalar field

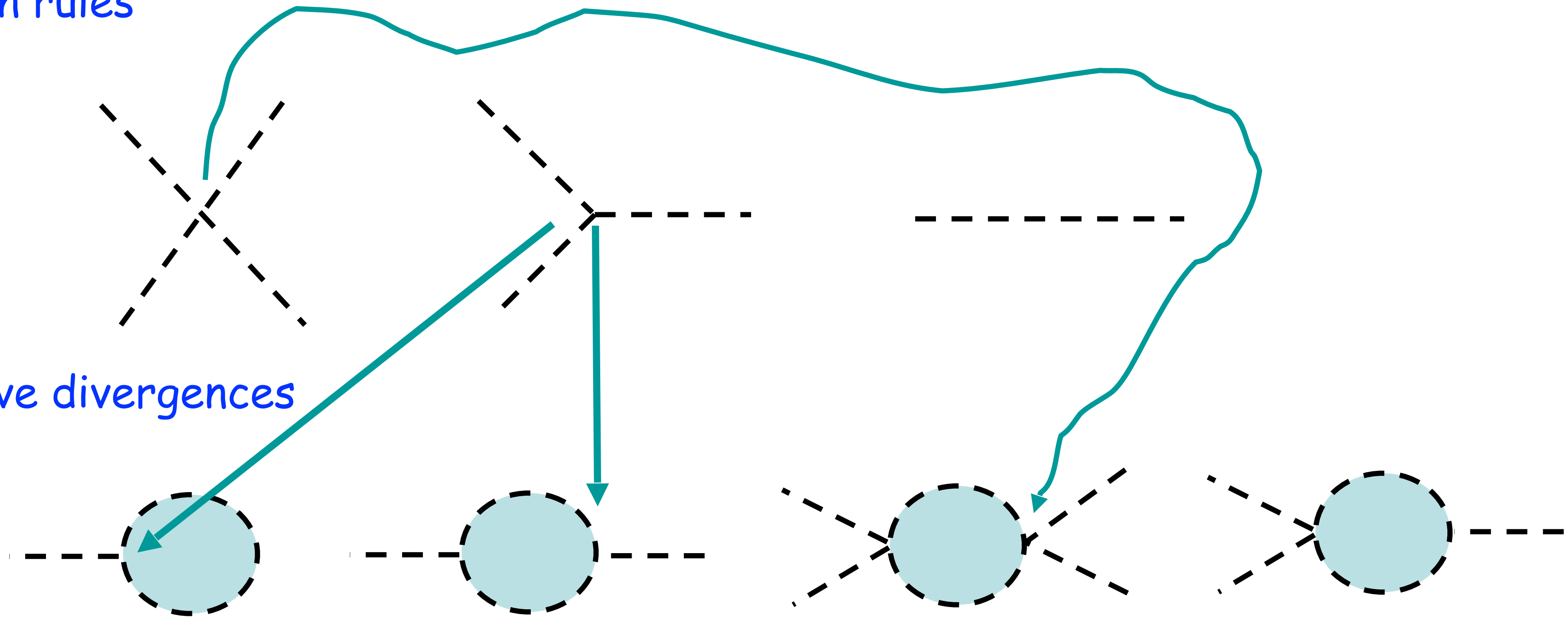
$$L = \frac{1}{2}\partial_\mu \Phi \partial^\mu \Phi - \frac{1}{2} m^2 \Phi^2 - \sum_{n \geq 3} \frac{C_n}{n!} \Phi^n \quad \Rightarrow \quad [C_n] = M^{4-n}$$

In particular the cubic coupling C_3 has dimension M, the C_4 term is dimensionless while for n above 4, C_n has a negative mass dimension.

Renormalisable theories - Mass dimension in pictures

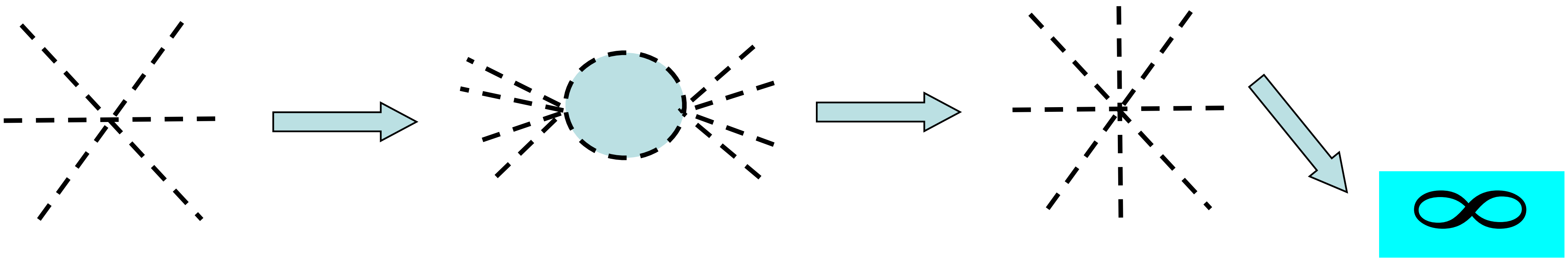
Feynman rules

Primitive divergences



Stop at ϕ^4

Higher order terms



Still…

… theories with $n > 4$ may be used as approximate effective theories (without the divergent loop graphs) for low-energy processes. The classic example is the Fermi theory of weak interactions, with Lagrangian

$$\mathscr{L}_{int} = \frac{G_F}{\sqrt{2}} \sum_{i,j} \bar{\psi}_i \gamma_\mu (1-\gamma_5)\psi_i \;\; \bar{\psi}_i \gamma_\mu (1-\gamma_5)\psi_i$$

where $[G_F] = M^{-2}$ ($G_F = 1.17 \times 10^{-5}$ GeV^{-2}). This is a good effective theory for low-energy interactions, but it cannot be used for energies above $1/(G_F)^{1/2} \approx 300$ GeV. In fact it only works for and energy well below the W boson mass. At higher energies one should use the proper electroweak theory. In QFTs which are valid for all energies, all coupling must have zero or positive energy dimensions.

Renormalisation and our favourite finite pieces

Again: renormalization is a procedure by which we make infinities of loops to disappear by absorbing them into the parameters of the Lagrangian. The subtraction scheme is a way to choose what part of the finite contribution we want to keep. Most typical are:

- Minimal Subtraction (MS): Only the divergence is removed.

- Modified MS: The divergence along with the some constant factors are removed. Of course, this is only used with dimensional regularisation.

- On-shell subtraction: Finites parts are removed such that the renormalized parameters are observables themselves.

Renormalisation of the ϕ^4 theory

We will now consider the simplest theory with all the necessary ingredients for renormalisation

$$\mathscr{L} = \frac{1}{2}[\partial_\mu \phi_0 \partial^\mu \phi_0 - m_0^2 \phi_0^2] - \frac{\lambda_0}{4!}\phi_0^4$$

The procedure is then the same for any theory: we redefine the parameters in such a way that the delta quantity will absorb the infinity from the loop

$$\rho_{i,0} = \rho_i + \delta\rho_i$$ **for the parameters.**

$$\phi_{j,0} = \sqrt{Z_{\phi_j}}\,\phi_j \approx \left(1 + \frac{\delta Z_{\phi_j}}{2}\right)\phi_j$$ **for the fields.**

Note the there is no need to renormalise the wave function. You just need to do it if you want finite Green functions.

Renormalisation of the ϕ^4 theory

Starting with the bare Lagrangian

$$\mathscr{L} = \frac{1}{2}[\partial_\mu \phi_0 \partial^\mu \phi_0 - m_0^2 \phi_0^2] - \frac{\lambda_0}{4!}\phi_0^4$$

and redefine the parameter as

$$\phi_0 = \sqrt{Z}\phi; \qquad m_0^2 = m^2 + \delta m^2; \qquad \lambda_0 = \lambda + \delta\lambda$$

which after expanding Z leads to the renormalised Lagrangian at 1-loop

$$\mathscr{L} = \frac{1}{2}[\partial_\mu \phi \partial^\mu \phi - m^2 \phi^2] - \frac{\lambda}{4!}\phi^4 + \frac{\delta Z}{2}\partial_\mu \phi \partial^\mu \phi - \frac{\delta m^2}{2}\phi^2 - \frac{\delta\lambda}{4!}\phi^4$$

We now need to calculate the counterterms. We have only two independent infinities – primitive divergences.

Renormalisation of the ϕ^4 theory

Power counting: we have discussed the renormalisability of the theory in terms of the dimension of the coupling. The dimension of the coupling determines the dimension of the integrand. Therefore,

$$
\begin{aligned}
[\lambda] &> 0 \quad \text{super-renormalizable,} \\
[\lambda] &= 0 \quad \text{renormalizable,} \\
[\lambda] &< 0 \quad \text{non-renormalizable.}
\end{aligned}
$$

Primitive divergences in Φ⁴ theory

$$\sim \int^{\Lambda} \frac{d^4k}{k^2} \sim \Lambda^2,$$

$$\sim \int^{\Lambda} \frac{d^4k}{k^4} \sim \ln \Lambda$$

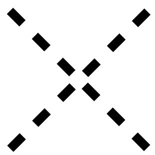
$$\sim \int^{\Lambda} \frac{d^4k}{k^6} = \text{finite}$$

In the case of a super-renormalisable theory there is some order above which all diagrams are finite.

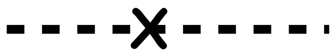
Renormalisation of the ϕ^4 theory

The Feynman rules for the renormalised Lagrangian are

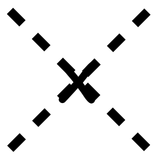
$$\frac{i}{p^2 - m^2}$$



$$-i\lambda$$



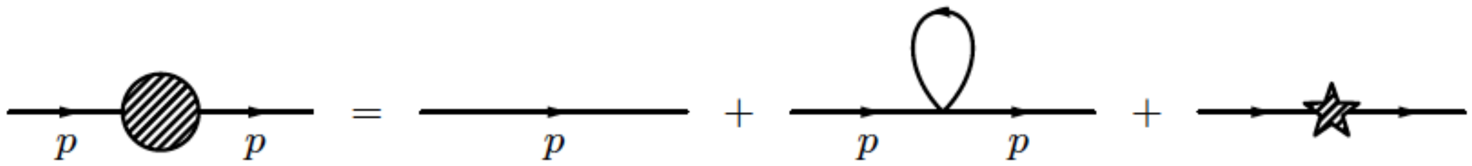
$$i(p^2\delta Z - \delta m^2)$$



$$-i\delta\lambda$$

The last two are the counterterm diagrams. Let us see now how to build the primitive divergences.

The diagrams contributing to the two point function are



and the amplitude

$$i\mathcal{M} = \frac{i}{p^2 - m_R^2 + i\epsilon} + \frac{i}{p^2 - m_R^2 + i\epsilon}\left[\frac{1}{2}(-i\lambda_R)\int \frac{d^4k}{(2\pi)^4}\frac{i}{k^2 - m_R^2 + i\epsilon}\right]\frac{i}{p^2 - m_R^2 + i\epsilon}$$
$$+ \frac{i}{p^2 - m_R^2 + i\epsilon}\left[i\left(p^2\delta_Z - \delta_m\right)\right]\frac{i}{p^2 - m_R^2 + i\epsilon}$$
$$= \frac{i}{p^2 - m_R^2 + i\epsilon} + \left(\frac{i m_R^2 \lambda_R}{32\pi^2}\left[\frac{2}{\epsilon} + 1 + \log\left(\frac{4\pi e^{-\gamma_E}\mu^2}{m_R^2}\right)\right] + i\left(p^2\delta_Z - \delta_m\right)\right)\left(\frac{i}{p^2 - m_R^2 + i\epsilon}\right)^2$$

Renormalisation of the two point function

- MS: Only cancel out the finite part. Thus

$$\delta_Z = 0, \quad \delta_m = \frac{m_R^2 \lambda_R}{16\pi^2 \epsilon}$$

In this scheme,

$$i\mathcal{M} = \frac{i}{p^2 - m_R^2 + i\epsilon} + \frac{i m_R^2 \lambda_R}{32\pi^2} \left[1 + \log\left(\frac{4\pi e^{-\gamma_E} \mu^2}{m_R^2} \right) \right] \left(\frac{i}{p^2 - m_R^2 + i\epsilon} \right)^2$$

- $\overline{\text{MS}}$: Cancel out the inifinite part and the 4π's and γ_E. Thus

$$\delta_Z = 0, \quad \delta_m = \frac{m_R^2 \lambda_R}{32\pi^2} \left[\frac{2}{\epsilon} + \log\left(4\pi e^{-\gamma_E}\right) \right]$$

In this scheme

$$i\mathcal{M} = \frac{i}{p^2 - m_R^2 + i\epsilon} + \frac{i m_R^2 \lambda_R}{32\pi^2} \left[1 + \log\left(\frac{\mu^2}{m_R^2} \right) \right] \left(\frac{i}{p^2 - m_R^2 + i\epsilon} \right)^2$$

Renormalisation of the two point function

- **On-shell subtraction:** In this scheme, we define δ_m by giving a physical interpretation to the parameter m_R. The usual thing to do would be to say that the **physical mass** of the particle is given by m_R. This is the condition that

$$= \frac{i}{p^2 - m_R^2 + i\epsilon} + \text{terms that are regular at } p^2 = m_R^2$$

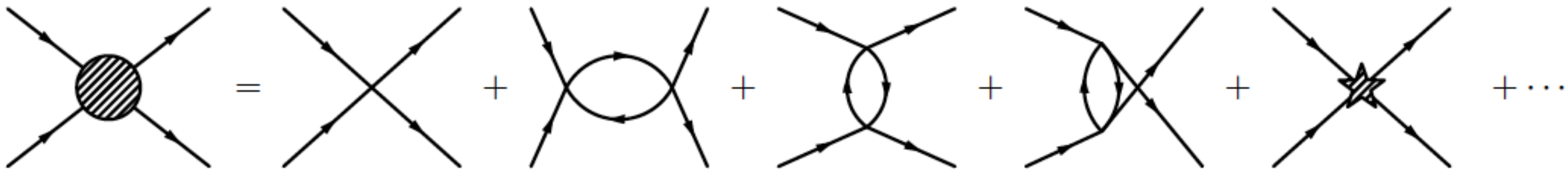
with residue i. This implies

$$\delta_Z = 0, \quad \delta_m = \frac{m_R^2 \lambda_R}{32\pi^2}\left[\frac{2}{\epsilon} + 1 + \log\left(\frac{4\pi e^{-\gamma_E}\mu^2}{m_R^2}\right)\right] \Longrightarrow i\mathcal{M} = \frac{i}{p^2 - m_R^2 + i\epsilon}$$

Notice that now that we have a physical interpretation of the parameter m_R, all dependence on the unphysical parameter has dropped out.

We have seen that in all subtraction schemes $\delta_Z = 0$ at this order. This is not true when higher order corrections are imposed.

The diagrams contributing to the four point function are



and the amplitude is

$$i\mathcal{M} = -i\lambda_R + \frac{1}{2}(-i\lambda_R)^2 \int \frac{d^4k}{(2\pi)^4} \frac{i}{k^2 - m_R^2 + i\epsilon} \frac{i}{(p_1+p_2+k)^2 - m_R^2 + i\epsilon}$$
$$+ \frac{1}{2}(-i\lambda_R)^2 \int \frac{d^4k}{(2\pi)^4} \frac{i}{k^2 - m_R^2 + i\epsilon} \frac{i}{(p_1-p_3+k)^2 - m_R^2 + i\epsilon}$$
$$+ \frac{1}{2}(-i\lambda_R)^2 \int \frac{d^4k}{(2\pi)^4} \frac{i}{k^2 - m_R^2 + i\epsilon} \frac{i}{(p_1-p_4+k)^2 - m_R^2 + i\epsilon} - i\delta_\lambda$$
$$= -i\lambda_R + (-i\lambda_R)^2 \left[iV(s) + iV(t) + iV(u)\right] - i\delta_\lambda$$

$$i\mathcal{M} = -i\lambda_R + \frac{i\lambda_R^2}{32\pi^2} \int_0^1 dx \left[\frac{6}{\epsilon} + \log\left(\frac{\left(4\pi e^{-\gamma_E}\mu^2\right)^3}{[m_R^2 - x(1-x)s][m_R^2 - x(1-x)t][m_R^2 - x(1-x)u]}\right)\right] - i\delta_\lambda$$

Renormalisation of the four point function

- **MS**: Only cancel out the finite part. Thus

$$\delta_\lambda = \frac{\lambda_R^2}{32\pi^2}\int_0^1 dx \frac{6}{\epsilon} = \frac{3\lambda_R^2}{16\pi^2\epsilon}$$

In this scheme

$$i\mathcal{M} = -i\lambda_R + \frac{i\lambda_R^2}{32\pi^2}\int_0^1 dx \log\left(\frac{\left(4\pi e^{-\gamma_E}\mu^2\right)^3}{[m_R^2 - x(1-x)s][m_R^2 - x(1-x)t][m_R^2 - x(1-x)u]}\right)$$

- $\overline{\textbf{MS}}$: Cancel out the inifinite part and the 4π's and γ_E. Thus

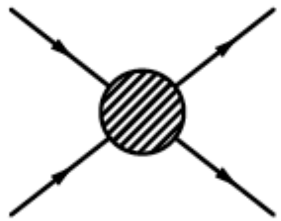
$$\delta_\lambda = \frac{\lambda_R^2}{32\pi^2}\int_0^1 dx \left[\frac{6}{\epsilon} + 3\log\left(4\pi e^{-\gamma_E}\right)\right] = \frac{\lambda_R^2}{32\pi^2}\left[\frac{6}{\epsilon} + 3\log\left(4\pi e^{-\gamma_E}\right)\right]$$

In this scheme

$$i\mathcal{M} = -i\lambda_R + \frac{i\lambda_R^2}{32\pi^2}\int_0^1 dx \log\left(\frac{\left(\mu^2\right)^3}{[m_R^2 - x(1-x)s][m_R^2 - x(1-x)t][m_R^2 - x(1-x)u]}\right)$$

Renormalisation of the four point function

- **On-shell subtraction**: In this scheme, we choose δ_λ such that the renormalized parameters are themselves the observable quantities. To define the observable quantity, we must define a **Renormalization Condition**. The usual thing to do is to perform an experiment at some scale $s = s_0,\ t = t_0,\ u = u_0$, and define the measure amplitude at that scale as the value of the coupling.



$$\xrightarrow{s=4m_P^2,\ t=u=0} -i\lambda_R$$

We then get the equation

$$-i\lambda_R = -i\lambda_R + \frac{i\lambda_R^2}{32\pi^2}\int_0^1 dx\left[\frac{6}{\epsilon} + \log\left(\frac{\left(4\pi e^{-\gamma_E}\mu^2\right)^3}{m_R^4[m_R^2 - 4x(1-x)m_P^2]}\right)\right] - i\delta_\lambda$$

$$\Longrightarrow \delta_\lambda = \frac{\lambda_R^2}{32\pi^2}\int_0^1 dx\left[\frac{6}{\epsilon} + \log\left(\frac{\left(4\pi e^{-\gamma_E}\mu^2\right)^3}{m_R^4[m_R^2 - 4x(1-x)m_P^2]}\right)\right]$$

In this scheme,

$$i\mathcal{M} = -i\lambda_R + \frac{i\lambda_R^2}{32\pi^2}\int_0^1 dx \log\left(\frac{m_R^4[m_R^2 - 4x(1-x)m_P^2]}{[m_R^2 - x(1-x)s][m_R^2 - x(1-x)t][m_R^2 - x(1-x)u]}\right)$$