Causality, propagators and fermion quantisation

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Causality and propagators

Complex Integration and Poles

Many integrals in quantum field theory involve complex functions with singularities:

$$f(z) = \frac{1}{(z - z_1)(z - z_2)\cdots}.$$

Points where the denominator vanishes are called **poles**, $z = z_1,\ z_2,\ \ldots$

The residue theorem allows us to evaluate contour integrals by studying only the poles enclosed by the contour.

Residue Theorem

If a contour C encloses poles z_i, then

$$\oint_C dz\, f(z) = 2\pi i \sum_i \mathrm{Res}(f, z_i).$$

What is a Residue?

Simple Pole

Near a simple pole at $z = z_0$ a function behaves as

$$f(z) \sim \frac{A}{z - z_0}.$$

The coefficient A is called the **residue** $\mathrm{Res}(f, z_0) = A$.

Example

For

$$f(z) = \frac{1}{(z-a)(z-b)},$$

the residue at $z = a$ is

$$\mathrm{Res}(f, a) = \frac{1}{a - b}.$$

Contour Integration

Consider an integral of the form

$$\int_{-\infty}^{+\infty} dz\, f(z).$$

We extend the integration path into the complex plane and close it with a contour.

Upper Half-Plane

If the contour closes upward, only poles with

$$\mathrm{Im}(z) > 0$$

contribute.

Lower Half-Plane

If the contour closes downward, only poles with

$$\mathrm{Im}(z) < 0$$

contribute.

The value of the integral depends on which poles are enclosed by the contour.

Residue Theorem: Explicit Example

Example Integral

Consider

$$I = \int_{-\infty}^{+\infty} \frac{dx}{x^2 + 1}.$$

We rewrite the denominator as

$$x^2 + 1 = (x - i)(x + i).$$

The poles are therefore located at

$$z = i, \qquad z = -i.$$

Choose a contour closing in the upper half-plane.

Only the pole

$$z = i$$

lies inside the contour.

Residue at $z = i$

For

$$f(z) = \frac{1}{(z-i)(z+i)},$$

the residue is

$$\mathrm{Res}(f, i) = \lim_{z \to i} (z - i) f(z).$$

Therefore,

$$\mathrm{Res}(f, i) = \lim_{z \to i} \frac{1}{z+i} = \frac{1}{2i}.$$

Applying the residue theorem:

$$\oint dz\, f(z) = 2\pi i \left(\frac{1}{2i}\right) = \pi.$$

Since the arc contribution vanishes,

$$\boxed{\int_{-\infty}^{+\infty} \frac{dx}{x^2+1} = \pi}$$

Example with Two Different Contours

Consider

$$I(t) = \int_{-\infty}^{+\infty} dx \, \frac{e^{-ixt}}{x^2 + 1}.$$

The poles are located at $z = \pm i$. The exponential factor e^{-izt} determines which contour must be chosen for convergence.

$t > 0$

Write $z = a + ib$, then

$$e^{-izt} = e^{-iat} e^{bt}.$$

For convergence on the large arc we require

$$b < 0.$$

Therefore the contour must close in the **lower half-plane**.

The enclosed pole is

$$z = -i.$$

$t < 0$

Now

$$e^{bt} = e^{-|t|b}.$$

For convergence we require

$$b > 0.$$

Therefore the contour must close in the **upper half-plane**.

The enclosed pole is

$$z = +i.$$

Residues

For

$$f(z) = \frac{e^{-izt}}{(z-i)(z+i)},$$

$$\mathrm{Res}(f, i) = \frac{e^{t}}{2i}, \qquad \mathrm{Res}(f, -i) = -\frac{e^{-t}}{2i}.$$

Applying the residue theorem:

$$I(t) = \pi e^{-|t|}.$$

Different signs of t require different contour closures.
This is exactly the mechanism that produces time ordering in propagators.

Propagators and Causality (Scalar Field)

Question: Is the theory causal?

For causality, we require that measurements at space-like separation do not influence one another:

$$[\phi(x), \phi(y)] = 0 \qquad \text{for } (x-y)^2 < 0$$

This ensures that a measurement at x cannot affect a measurement at y if the two points are not causally connected.

Define the commutator:

$$\Delta(x-y) \equiv [\phi(x), \phi(y)]$$

Using the mode expansion of the scalar field, one finds:

$$\Delta(x-y) = \int \frac{d^3k}{(2\pi)^3} \frac{1}{2\omega_{\mathbf{k}}} \left(e^{-ik\cdot(x-y)} - e^{ik\cdot(x-y)} \right)$$

Properties of $\Delta(x-y)$

Key properties:

1. Lorentz invariance

$$\Delta(x-y) \text{ is Lorentz invariant}$$

This follows from the invariant measure:

$$\frac{d^3k}{2\omega_{\mathbf{k}}}$$

2. Non-vanishing for time-like separation

For example, take:

$$x-y=(t,0,0,0)$$

Then:

$$[\phi(\mathbf{x},t),\phi(\mathbf{x},0)]\neq 0$$

So correlations exist inside the light cone.

3. Vanishing for space-like separation

We want to show:

$$\Delta(x-y)=0 \quad \text{if } (x-y)^2<0$$

Use Lorentz invariance: for space-like separation, go to a frame where

$$x-y=(0,\mathbf{r})$$

Then:

$$\Delta(0,\mathbf{r})=\int \frac{d^3k}{(2\pi)^3}\frac{1}{2\omega_{\mathbf{k}}}\left(e^{i\mathbf{k}\cdot\mathbf{r}}-e^{-i\mathbf{k}\cdot\mathbf{r}}\right)$$

The integrand is odd under $\mathbf{k}\to-\mathbf{k}$, hence:

$$\Delta(0,\mathbf{r})=0$$

Conclusion: Causality

Since $\Delta(x-y)$ is Lorentz invariant, it can only depend on:

$$(x-y)^2$$

We have shown:

$$\Delta(x-y) = 0 \quad \text{for one space-like configuration}$$

Therefore:

$$\boxed{\Delta(x-y) = 0 \quad \text{for all } (x-y)^2 < 0}$$

Conclusion:

- The scalar field satisfies microcausality
- Observables commute outside the light cone
- Relativistic causality is preserved

Causality and propagation outside the light cone

We can ask a different question to probe the causal structure of the theory. Suppose we start with a particle at spacetime point y. What is the probability amplitude to find it at a later point x?

$$\langle 0|\phi(x)\phi(y)|0\rangle = \int \frac{d^3\mathbf{k}\, d^3\mathbf{k}'}{(2\pi)^3} \frac{1}{\sqrt{4\omega_{\mathbf{k}}\omega_{\mathbf{k}'}}} \langle 0|a_{\mathbf{k}} a^\dagger_{\mathbf{k}'}|0\rangle e^{-ik\cdot x + ik'\cdot y}.$$

Using

$$\langle 0|a_{\mathbf{k}} a^\dagger_{\mathbf{k}'}|0\rangle = (2\pi)^3 \delta^{(3)}(\mathbf{k} - \mathbf{k}'),$$

we obtain

$$\langle 0|\phi(x)\phi(y)|0\rangle = \int \frac{d^3\mathbf{k}}{(2\pi)^3} \frac{1}{2\omega_{\mathbf{k}}} e^{-ik\cdot(x-y)} \equiv D(x-y).$$

The function $D(x-y)$ is called a propagator.

For spacelike separations $(x-y)^2 < 0$, one can show that it decays as

$$D(x-y) \sim e^{-m\sqrt{-(x-y)^2}}.$$

So it decays exponentially outside the light cone, but it is not zero.

How is this compatible with causality?

$$[\phi(x), \phi(y)] = D(x-y) - D(y-x).$$

For spacelike separations, there is no Lorentz-invariant way to order the two events. A process $x \to y$ can be viewed in another frame as $y \to x$. The two amplitudes cancel:

$$D(x-y) - D(y-x) = 0, \qquad (x-y)^2 < 0.$$

Thus,

$$\boxed{[\phi(x), \phi(y)] = 0 \qquad \text{for spacelike } x-y}$$

which is the microcausality condition.

Feynman propagator and pole prescription

Let us define the Feynman propagator as

$$\Delta_F(x-y) = \langle 0|T\phi(x)\phi(y)|0\rangle = \begin{cases} \Delta(x-y), & x^0 > y^0, \\ \Delta(y-x), & x^0 < y^0. \end{cases}$$

Here T denotes time ordering:

$$T\phi(x)\phi(y) = \begin{cases} \phi(x)\phi(y), & x^0 > y^0, \\ \phi(y)\phi(x), & y^0 > x^0. \end{cases}$$

The propagator can be written as

$$\Delta_F(x-y) = \int \frac{d^4u}{(2\pi)^4} \frac{i}{u^2 - m^2} e^{-iu\cdot(x-y)}.$$

However, the integral is ill-defined because there are two poles:

$$u^0 = \pm\sqrt{\mathbf{u}^2 + m^2}.$$

Writing the integral explicitly,

$$\Delta_F(x-y) = \int \frac{d^3\mathbf{u}}{(2\pi)^3} e^{i\mathbf{u}\cdot(\mathbf{x}-\mathbf{y})} \int \frac{du^0}{2\pi} \frac{i\, e^{-iu^0(x^0-y^0)}}{(u^0)^2 - \mathbf{u}^2 - m^2}.$$

To regularise the poles, we shift them away from the real axis:

$$\int_{\mathbb{R}} dx\, \frac{f(x)}{x - x_0} \quad \longrightarrow \quad \int_{\mathbb{R}} dx\, \frac{f(x)}{x - (x_0 \pm i\epsilon)}, \qquad \epsilon \to 0^+.$$

Equivalently, one can keep the poles fixed and deform the integration contour in the complex plane.

$$\boxed{\frac{i}{u^2 - m^2} \longrightarrow \frac{i}{u^2 - m^2 + i\epsilon}}$$

The Feynman prescription places the poles at

$$u^0 = +\omega_{\mathbf{u}} - i\epsilon, \qquad u^0 = -\omega_{\mathbf{u}} + i\epsilon, \qquad \omega_{\mathbf{u}} = \sqrt{\mathbf{u}^2 + m^2}.$$

Pole prescriptions and propagators

For Lorentz invariance, we should use the same regularisation for every **k**, but there are four choices $k^0 = \omega_{\mathbf{k}} \longrightarrow \omega_{\mathbf{k}} \pm i\epsilon$, $k^0 = -\omega_{\mathbf{k}} \longrightarrow -\omega_{\mathbf{k}} \pm i\epsilon$ meaning four propagators:

1. **Causal retarded propagator**

$$\Delta_R : \qquad k^0 = \pm\omega_{\mathbf{k}} - i\epsilon.$$

2. **Causal advanced propagator**

$$\Delta_A : \qquad k^0 = \pm\omega_{\mathbf{k}} + i\epsilon.$$

3. **Time-ordered propagator**

$$\Delta_F : \qquad k^0 = \pm\left(\omega_{\mathbf{k}} - i\epsilon\right).$$

4. **Anti-time-ordered propagator**

$$\Delta_{\overline{F}} : \qquad k^0 = \pm\left(\omega_{\mathbf{k}} + i\epsilon\right).$$

Example: if a particle goes from $(\mathbf{x}, t)$ to $(\mathbf{x}', t')$, then

$$\Delta_R(\mathbf{x}' - \mathbf{x}, t' - t) = 0 \qquad \text{for } t < t'.$$

Equivalently, the retarded propagator only has support when the final event lies in the future of the initial event.

$$\boxed{\Delta_R(x' - x) = 0 \quad \text{unless} \quad t' > t}$$

A causal retarded propagator is the Green's function describing the response of a field to a disturbance in such a way that influences travel only forward in time and never outside the light cone.

Poles and Causal Structure of Propagators

Consider the momentum-space denominator

$$(k^0)^2 - \omega_k^2, \qquad \omega_k = \sqrt{\mathbf{k}^2 + m^2}.$$

The poles occur at

$$k^0 = \pm\omega_k.$$

Different propagators are defined by specifying how the poles are displaced in the complex k^0 plane using the prescription

$$i\epsilon.$$

The position of the poles determines:

- which contours contribute,
- the time-ordering properties,
- and the causal structure of the propagator.

Contour Integration and Time Dependence

The propagator in position space is obtained from

$$\Delta(x) = \int \frac{dk^0}{2\pi}\, e^{-ik^0 t}\, \Delta(k).$$

The exponential factor $e^{-ik^0 t}$ determines how the contour is closed.

$t > 0$

the contour closes in the lower half-plane.
Only poles below the real axis contribute.

$t < 0$

the contour closes in the upper half-plane.
Only poles above the real axis contribute.

Consequence

The causal behavior of the propagator is entirely controlled by the location of the poles in the complex plane.

Feynman (Time-Ordered) Propagator

The Feynman propagator is

$$\Delta_F(k) = \frac{i}{(k^0)^2 - \omega_k^2 + i\epsilon}.$$

Factorizing the denominator:

$$(k^0)^2 - \omega_k^2 + i\epsilon = (k^0 - \omega_k + i\epsilon)(k^0 + \omega_k - i\epsilon).$$

Therefore the poles are located at $k^0 = \omega_k - i\epsilon, \qquad k^0 = -\omega_k + i\epsilon.$

Result

$$\Delta_F(x) \sim \theta(t)e^{-i\omega_k t} + \theta(-t)e^{+i\omega_k t}.$$

This is the time-ordered propagator.

Retarded Propagator

The retarded propagator is

$$\Delta_R(k) = \frac{i}{(k^0 + i\epsilon)^2 - \omega_k^2}.$$

$t > 0$

The contour closes below.
Poles are enclosed.

$$\Delta_R(t > 0) \neq 0$$

$t < 0$

The contour closes above.
No poles are enclosed.

$$\Delta_R(t < 0) = 0$$

Physical Meaning

The retarded propagator describes causal propagation:

effect occurs only after the source acts.

Advanced Propagator

The advanced propagator is

$$\Delta_A(k) = \frac{i}{(k^0 - i\epsilon)^2 - \omega_k^2}.$$

$t > 0$

The contour closes below.
No poles contribute.

$$\Delta_A(t > 0) = 0$$

$t < 0$

The contour closes above.
Poles contribute.

$$\Delta_A(t < 0) \neq 0$$

Physical Meaning

The advanced propagator propagates backward in time.

Anti-Time-Ordered Propagator

The anti-time-ordered propagator is

$$\Delta_D(k) = \frac{-i}{(k^0)^2 - \omega_k^2 - i\epsilon}.$$

Pole	Position
$+\omega_k$	above real axis
$-\omega_k$	below real axis

Interpretation

This propagator produces anti-time ordering:

$$\tilde{T}\{\phi(x)\phi(y)\}.$$

It reverses the Feynman pole prescription.

Summary of Pole Prescriptions

Propagator	Pole Positions	Property
Δ_F	$+\omega_k - i\epsilon$ $-\omega_k + i\epsilon$	Time ordered
Δ_D	$+\omega_k + i\epsilon$ $-\omega_k - i\epsilon$	Anti-time ordered
Δ_R	$\pm\omega_k - i\epsilon$	Retarded / causal
Δ_A	$\pm\omega_k + i\epsilon$	Advanced

The causal and time-ordering properties of propagators are completely encoded in the pole structure of the complex energy plane.

Feynman prescription from contour integration

Let us consider the Feynman pole prescription. The poles are shifted as

$$k^0 = \omega_{\mathbf{k}} - i\epsilon, \qquad k^0 = -\omega_{\mathbf{k}} + i\epsilon.$$

Equivalently,

$$(k^0 - \omega_{\mathbf{k}} + i\epsilon)(k^0 + \omega_{\mathbf{k}} - i\epsilon) \simeq (k^0)^2 - \omega_{\mathbf{k}}^2 + i\epsilon,$$

so that

$$\Delta_F(x-y) = \int \frac{d^4k}{(2\pi)^4} \frac{i}{k^2 - m^2 + i\epsilon} e^{-ik\cdot(x-y)}.$$

To evaluate the k^0-integral, consider

$$I(t,\omega) = \int_{-\infty}^{+\infty} \frac{dk^0}{2\pi} \frac{ie^{-ik^0 t}}{(k^0 - \omega + i\epsilon)(k^0 + \omega - i\epsilon)}.$$

The convergence of $e^{-ik^0 t}$ determines how we close the contour:

$$\begin{cases} t > 0: & \text{close in the lower half-plane,} \\ t < 0: & \text{close in the upper half-plane.} \end{cases}$$

The poles enclosed are therefore

$$\begin{cases} t > 0: & k^0 = \omega - i\epsilon, \\ t < 0: & k^0 = -\omega + i\epsilon. \end{cases}$$

Using the residue theorem,

$$I(t,\omega) = \begin{cases} \dfrac{e^{-i\omega t}}{2\omega}, & t > 0, \\ \dfrac{e^{+i\omega t}}{2\omega}, & t < 0. \end{cases}$$

Thus the $i\epsilon$ prescription automatically implements time ordering:

$$\boxed{\Delta_F(x-y) = \theta(x^0 - y^0) D(x-y) + \theta(y^0 - x^0) D(y-x)}$$

The Feynman propagators are defined by time ordering:

Choice of Contour

We evaluate

$$I(x-y) = \int \frac{dk^0}{2\pi} \frac{e^{-ik^0(x^0-y^0)}}{(k^0)^2 - \omega_k^2 + i\epsilon}.$$

The contour depends on the sign of

$$t \equiv x^0 - y^0.$$

- If $t > 0$, close the contour in the lower half-plane.
- If $t < 0$, close the contour in the upper half-plane.

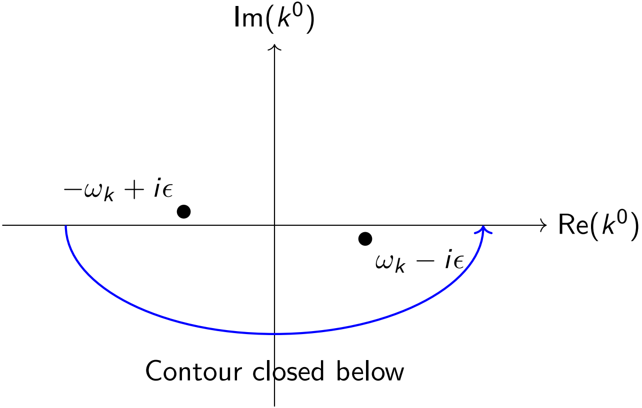
This guarantees convergence of

$$e^{-ik^0 t}.$$

Contour for $t > 0$

For $t > 0$, the enclosed pole is

$$k^0 = \omega_k - i\epsilon.$$



Residue Calculation

Using the residue theorem,

$$I(t) = -2\pi i \, \mathrm{Res}_{k^0=\omega_k - i\epsilon} \left[\frac{e^{-ik^0 t}}{(k^0 - \omega_k + i\epsilon)(k^0 + \omega_k - i\epsilon)} \right].$$

Evaluating the residue:

$$I(t) = \frac{e^{-i(\omega_k - i\epsilon)t}}{2\omega_k}.$$

Taking $\epsilon \to 0^+$,

$$I(t) = \frac{e^{-i\omega_k t}}{2\omega_k}.$$

Case $t < 0$

If $t < 0$, we close the contour in the upper half-plane.
The enclosed pole is

$$k^0 = -\omega_k + i\epsilon.$$

The result becomes

$$I(t) = \frac{e^{+i\omega_k t}}{2\omega_k}.$$

Combining both cases:

$$I(t) = \theta(t)\frac{e^{-i\omega_k t}}{2\omega_k} + \theta(-t)\frac{e^{+i\omega_k t}}{2\omega_k}.$$

Feynman Propagator

The complete propagator is

$$\Delta_F(x-y) = \int \frac{d^4k}{(2\pi)^4} \frac{i\, e^{-ik\cdot(x-y)}}{k^2 - m^2 + i\epsilon}.$$

Equivalently,

$$\Delta_F(x-y) = \theta(x^0 - y^0)\Delta^{(+)}(x-y) + \theta(y^0 - x^0)\Delta^{(-)}(x-y).$$

The $i\epsilon$ prescription determines how the contour bypasses the poles.

Retarded propagator and Green's function

For $t > t'$, the scalar propagator can be written as

$$\Delta_F(x-x') = \int \frac{d^3\mathbf{k}}{(2\pi)^3}\frac{1}{2\omega_\mathbf{k}} e^{-i\omega_\mathbf{k}(t-t')} e^{i\mathbf{k}\cdot(\mathbf{x}-\mathbf{x}')} = D(x-x').$$

The retarded propagator is

$$\Delta_R(x-x') = \begin{cases} D(x-x') - D(x'-x), & x^0 > x'^0, \\ 0, & x^0 < x'^0. \end{cases}$$

It is a Green's function for the Klein–Gordon operator:

$$\left(\partial_0^2 - \nabla^2 + m^2\right) \Delta_R(x-x') = -i\,\delta^{(4)}(x-x').$$

Green's Function Interpretation

The propagator satisfies

$$(\Box + m^2)\Delta_F(x-y) = -i\delta^{(4)}(x-y).$$

Thus Δ_F is the Green's function of the Klein–Gordon operator.

Momentum-space Green's function:

$$G(k) = \frac{i}{k^2 - m^2 + i\epsilon}.$$

Photon propagator

Electromagnetic field

The photon propagator is defined by

$$G_{\mu\nu}(x-y) = \langle 0 | T A_\mu(x) A_\nu(y) | 0 \rangle .$$

In Feynman gauge,

$$G_{\mu\nu}(x-y) = -g_{\mu\nu} \int \frac{d^4q}{(2\pi)^4} \frac{i}{q^2 + i\epsilon} e^{-iq\cdot(x-y)} .$$

In momentum space this becomes

$$G_{\mu\nu}(q) = \frac{-ig_{\mu\nu}}{q^2 + i\epsilon} .$$

More generally, the photon Green's function satisfies

$$\left[\Box g^{\mu\nu} - \left(1 - \frac{1}{\lambda}\right) \partial^\mu \partial^\nu \right] G_{\nu\rho}(x - x') = i g^\mu{}_\rho \, \delta^{(4)}(x - x') .$$

Electromagnetic Field

For the electromagnetic field,

$$G_{\mu\nu}(x-y) = \langle 0| T A_\mu(x) A_\nu(y) |0\rangle.$$

In Feynman gauge:

$$G_{\mu\nu}(k) = \frac{-i g_{\mu\nu}}{k^2 + i\epsilon}.$$

This has the same analytic structure as the scalar propagator.

Summary

- The $i\epsilon$ prescription shifts poles off the real axis.
- The contour depends on the sign of $x^0 - y^0$.
- Residue calculus gives the time-ordered propagator.
- The Feynman propagator is a Green's function.
- The same method applies to gauge fields.

Fermionic Fields: Dirac Equation

We now consider spin-$\frac{1}{2}$ fields.

Dirac equation:

$$(i\gamma^\mu \partial_\mu - m)\psi(x) = 0$$

Lagrangian:

$$\mathcal{L} = \bar{\psi}(i\gamma^\mu \partial_\mu - m)\psi$$

Key differences from scalar fields:

- Spinor structure
- Anti-commutation relations
- Two types of solutions: particles and antiparticles

Mode Expansion of Dirac Field

The Dirac field can be expanded as:

$$\psi(x) = \int \frac{d^3k}{(2\pi)^3} \frac{1}{\sqrt{2\omega_{\mathbf{k}}}} \sum_s \left(b^s_{\mathbf{k}} u^s(k) e^{-ik\cdot x} + d^{s\dagger}_{\mathbf{k}} v^s(k) e^{ik\cdot x} \right)$$

Interpretation:

- $b^\dagger$ creates particles
- $d^\dagger$ creates antiparticles
- u^s, v^s are spinors

Anti-commutation Relations

Fermionic operators satisfy:

$$\{b_{\mathbf{k}}^r, b_{\mathbf{k}'}^{s\dagger}\} = (2\pi)^3 \delta^{(3)}(\mathbf{k} - \mathbf{k}')\delta^{rs}$$

$$\{d_{\mathbf{k}}^r, d_{\mathbf{k}'}^{s\dagger}\} = (2\pi)^3 \delta^{(3)}(\mathbf{k} - \mathbf{k}')\delta^{rs}$$

Important:

- Anti-commutators $\rightarrow$ Pauli exclusion principle
- Ensures correct spin-statistics relation

Spinor Completeness Relations

The spinors satisfy:

$$\sum_s u^s(k)\bar{u}^s(k) = (\not{k} + m)$$

$$\sum_s v^s(k)\bar{v}^s(k) = (\not{k} - m)$$

where:

$$\not{k} \equiv \gamma^\mu k_\mu$$

These relations are crucial for deriving the fermion propagator.

Fermion Propagator

Define the time-ordered propagator:

$$S_F(x-y) = \langle 0 | T\{\psi(x)\bar{\psi}(y)\} | 0 \rangle$$

Using the mode expansion and spin sums, one finds:

$$S_F(x-y) = \int \frac{d^4k}{(2\pi)^4} \frac{i(\not{k}+m)}{k^2-m^2+i\epsilon} e^{-ik\cdot(x-y)}$$

Comparison: Scalar vs Fermion Propagator

Scalar:

$$D_F(k) = \frac{i}{k^2 - m^2 + i\epsilon}$$

Fermion:

$$S_F(k) = \frac{i(\not{k} + m)}{k^2 - m^2 + i\epsilon}$$

Key difference:

- Numerator encodes spin structure
- Dirac matrices appear

$$\boxed{\text{Spin} \;\Rightarrow \text{matrix-valued propagator}}$$

Fermions in Feynman Diagrams

Diagrammatic rules:

- Fermion lines have arrows (fermion number flow)
- Each internal line:

$$\frac{i(\not{k} + m)}{k^2 - m^2 + i\epsilon}$$

- External lines:

$$u^s(k), \quad \bar{u}^s(k), \quad v^s(k), \quad \bar{v}^s(k)$$

Important:

- Order of operators matters (anti-commuting fields)
- Closed fermion loops introduce a minus sign