Universo Primitivo

2018-2019 (1º Semestre)

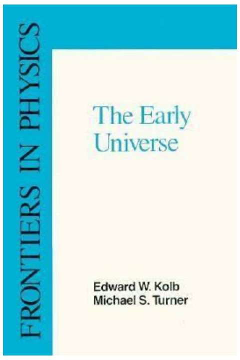
Mestrado em Física - Astronomia

Chapter 9

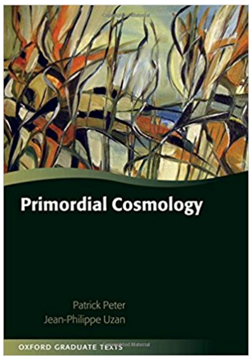
9 Inflation

- Problems of the hot Big Bang theory revisited
- Conditions for Inflation
- Distances and horizons
- Cosmological scales and horizons;
- Scalar Field Dynamics;
- Slow-roll inflation;
- Reheating

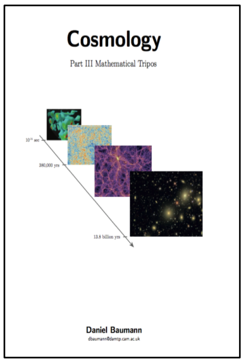
References



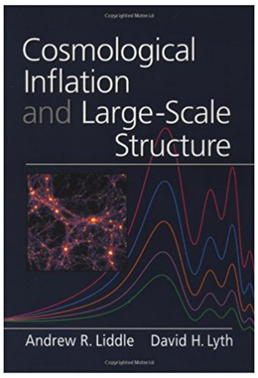
Ch. 8



Ch. 8.1; 8.2



Ch. 2



Ch. 3

Inflation

Problems of the hot Big Bang theory revisited

Friedmann-Lemaitre-Robertson-Walker **(FLRW) models** are able to describe the Universe expansion but they **imply a decelerated expansion for any fluid component with** an equation state parameter $\boldsymbol{w = p/\rho c^2 > -1/3}$.

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho + \frac{\Lambda c^2}{3} - \frac{kc^2}{a^2}$$

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}\left(\rho + 3\frac{p}{c^2}\right) \longrightarrow \boxed{p > -\frac{1}{3}\rho c^2 \;\Rightarrow\; \ddot{a} < 0}$$

Since common matter and radiation have equations of state parameters with $w > -1/3$ this leads to the fatal conclusion that the Universe's fate is to expand in a decelerated way.

This leads to a number of difficulties known as the **hot Big Bang problems** (see next slides). **A way to solve these problems is to develop a dynamical framework where the FLRW Universes may be allowed to expand in a accelerated** way, at least during some periods of the Universe's history. **These periods are called inflationary and allows one to define inflation as any phase of the universe's expansion when**:

$$\boxed{\text{Inflation} \Leftrightarrow \ddot{a} > 0}$$

Inflation

$\boxed{\text{Inflation} \Leftrightarrow \ddot{R} > 0}$

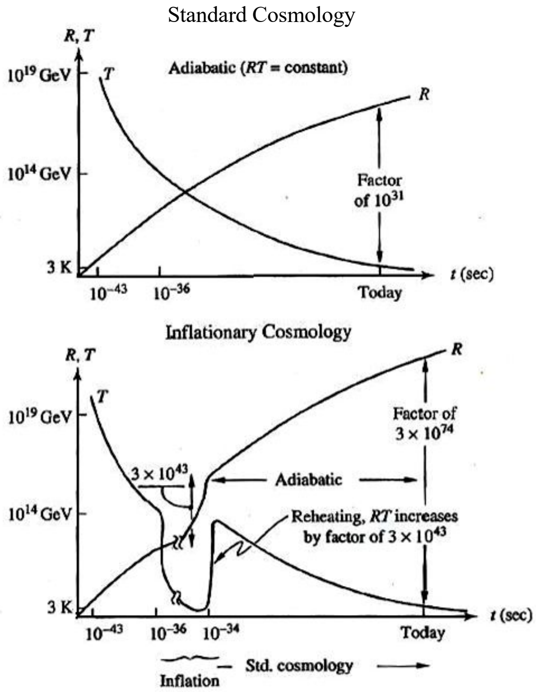


FIGURE 30.4 The evolution of the temperature of the universe and the scale factor, without and with inflation. Except for the bottom value, the temperature is given in terms of kT. (Figure adapted from Edward W. Kolb and Michael S. Turner, *The Early Universe* (page 274), ©1990 by Addison-Wesley Publishing Company, Inc., Reading, MA. Reprinted by permission of the publisher.)

Inflation

Problems of the hot Big Bang theory revisited

FLRW models with decelerated expansions are inconsistent with some important observational evidences facts and pose a number of puzzling questions:

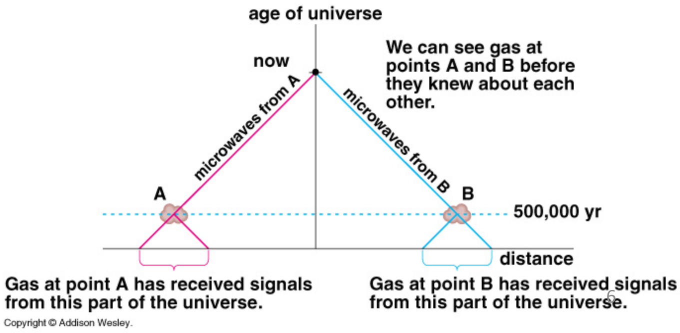
The horizon problem: The FRLW models allow one to compute the particle horizon of observer at any given time/redshift. The sky angular size of the particle horizon of an observer, θ_H, at high redshift can be approximated by:

$$\theta_H \simeq \frac{180}{\pi}\sqrt{\frac{\Omega_0}{z}}\ \mathrm{deg}$$

so an observer at $z = 1100$ (living at epoch of CMB decoupling) has a particle horizon with an angular size on our observed sky of about , $\theta_H \simeq 0.95$ deg.

This means that **there are about 54000 casual disconnect regions in the sky at CMB decoupling.**

So, why is CMB intensity spectrum so uniform temperature (2.725 °K) in all sky directions?



Inflation

Problems of the hot Big Bang theory revisited

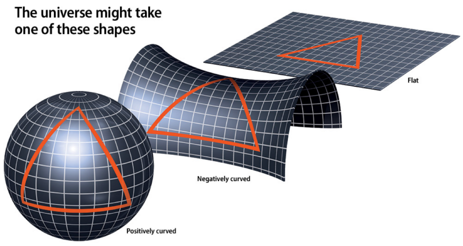
The flatness problem: At early times the Friedmann equation can be written as ($\Omega = \Omega_m + \Omega_r$):

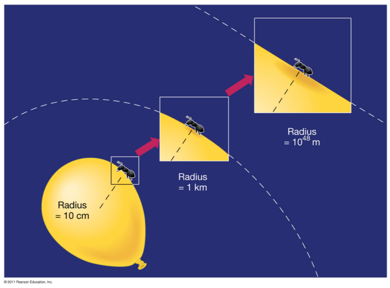
$$|\Omega(t) - 1| = \frac{|k|}{a^2(t)H^2(t)} = \frac{|k|}{\dot{a}^2(t)}$$

Since $\dot{a}(t)$ decreases with time (because $\ddot{a} < 0$) this denominator increases as $t \to 0$

So the left hand side term should approach rapidly to zero as $t \to 0$ (actually $\dot{a}(t \to 0) \to \infty$). For $t \simeq 1\times10^{-43}$ (~Planck time) Ω should deviate no more than ~ 1×10^{-60} from the unity.

So, why is the Universe "starting" with a energy density parameter so extremely close to 1?





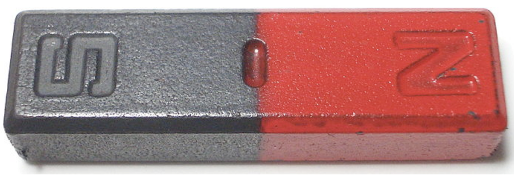
Inflation

Problems of the hot Big Bang theory revisited

The monopole and other exotic particles problem:
Quantum field theories (e.g. GUT, superstring) predict that a variety of "exotic" stable particles, such as magnetic monopoles, should be produced in the early Universe and remain in measurable amounts until the present.

No such particles have yet been observed. Why?

This either implies that the predictions from particle physics are wrong, or their densities are very small and therefore there's something missing from this evolutionary picture of the Big Bang.



Inflation

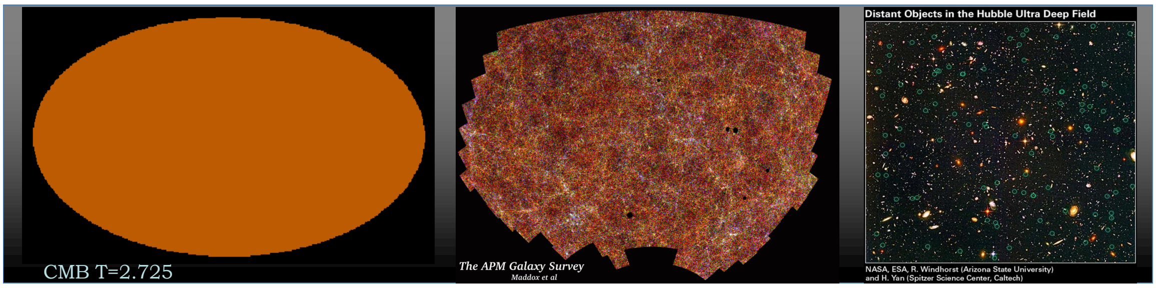
Problems of the hot Big Bang theory revisited

The origin of density fluctuations problem:
On large scales our present universe is fairly isotropic and homogeneous.

Why is that so?

At early times, that homogeneity and isotropy was even more "perfect" (due to the flattening effect effect at early times). Moreover, the FLRW universes form a very special subset of solutions of the GR equations.

So, why nature "prefers" homogeneity and isotropy from the beginning as opposed to having evolved into that stage?



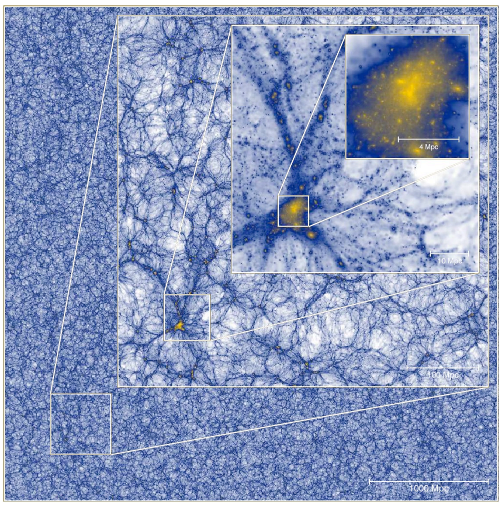
Inflation

Problems of the hot Big Bang theory revisited

The origin of density fluctuations problem:

Locally the universe is not homogeneous. It displays a complex hierarchical pattern of galaxies, clusters and super clusters.

What's the origin of cosmological structure?
Does it grew from gravitational instability?
What is the origin of the initial perturbations?



Without a mechanism to explain the existence of fluctuations one has to assume that they ``were born'' with the universe already showing the correct amplitudes on all scales, so that gravity can correctly reproduce the present-day structures?

Inflation

Conditions for Inflation

If the Universe experience periods of accelerated expansion

$$\boxed{\text{Inflation} \Leftrightarrow \ddot{a} > 0}$$

This requires that during these periods the Universe has to be dominated by a fluid component with an equation of state parameter $\boldsymbol{w < -1/3}$:

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho + \frac{\Lambda c^2}{3} - \frac{kc^2}{a^2}$$

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}\left(\rho + 3\frac{p}{c^2}\right) \longrightarrow \boxed{p < -\frac{1}{3}\rho c^2 \Rightarrow \ddot{a} > 0}$$

Let's us first look at the acceleration condition

$$\boxed{\ddot{a} > 0 \Leftrightarrow} - \frac{\ddot{a}}{\dot{a}^2} < 0 \Leftrightarrow \frac{d}{dt}\left(\dot{a}^{-1}\right) < 0 \Leftrightarrow \boxed{\frac{d}{dt}\left(aH\right)^{-1} < 0 \Leftrightarrow \frac{d}{dt}\left(cH^{-1}/a\right) < 0}$$

The quantity $\boldsymbol{R_H = cH^{-1}}$ is the **Hubble radius** ($v_H = c = HR_H$).

So **inflation** can also be defined as **any period of the universe history when the commoving Hubble radius $\boldsymbol{R_H}$ is decreasing** (shrinking).

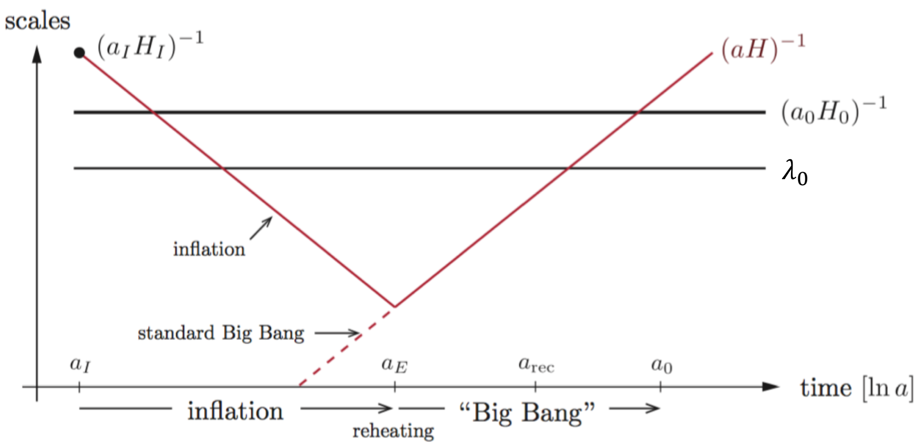
Inflation

Cosmological scales and horizons

During inflation

- any comoving cosmological scale, λ_0, is fixed in time as: $\lambda_0 = \lambda/a(t)$
- but the comoving (particle) horizon $\sim R_H = (aH)^{-1}$ decreases with time

So **during inflation scales** inside the horizon at a given time **grow faster and may become larger than (go beyond) the horizon.**



Inflation

Conditions for Inflation

The inflation conditions can be expressed in terms of other conditions. Let us first note that:

$$\frac{d}{dt}(aH)^{-1} = -\frac{\dot{a}H + a\dot{H}}{(aH)^2} = -\frac{1}{a}(1-\varepsilon)\ , \qquad \text{where} \quad \varepsilon \equiv -\frac{\dot{H}}{H^2}$$

from

$$\ddot{a} > 0 \Leftrightarrow \frac{d}{dt}(aH)^{-1} < 0 \Leftrightarrow -\frac{1}{a}(1-\epsilon) < 0$$

So we conclude that inflation happens whenever

$$\boxed{\varepsilon = -\frac{\dot{H}}{H^2} < 1}$$

$\boldsymbol{\epsilon}$ is known as the ***slowly-varying Hubble parameter***. As long as it is smaller than 1 inflation happens. The case $\boldsymbol{\epsilon = 0}$ is known as perfect inflation:

- The commoving Hubble radius is constant: $\dot{H} = 0 \Leftrightarrow H = constant$
- de Sitter Universe expansion: $\frac{\dot{a}}{a} = H \Leftrightarrow a(t) = a_i \exp(H(t - t_i))$

Inflation

Conditions for Inflation

The inflation condition can also be written as:

$$\epsilon = -\frac{\dot{H}}{H^2} = -\frac{\dot{H}/H}{\dot{a}/a} = -\frac{\mathrm{d}\ln(H)}{d\ln(a)} = -\frac{d\ln(H)}{dN} < 0$$

Where $dN = d\ln(a)$ is known as the **e-fold number**:

$$N = \int_{a_i}^{a} d\ln(a) \ = \ln\left(\frac{a}{a_i}\right)$$

The e-fold number **is used to quantify how long the inflationary period must be** in order to solve the Hot Big-Bang problems (usually $N \sim 40 - 70$).

During the inflationary period, ϵ, needs to remain small (below 1). It is then useful introduce a new parameter that measures how ϵ changes during inflation:

$$\boxed{\eta \equiv \frac{d\ln\varepsilon}{dN} = \frac{\dot{\varepsilon}}{H\varepsilon}}$$

Since ϵ needs to remain small this means that η needs to remain small.
In general one should have: $\eta < 1$ and $\epsilon < 1$

Inflation

Conditions for Inflation

The Friedmann and the continuity equations

$$H^2 = \rho/3M_{pl}^2$$
$$\dot{\rho} = -3H(\rho + p)$$

Can be combined to relate, ϵ, with the equation of state parameter.
One has:

$$\varepsilon = -\frac{\dot{H}}{H^2} = \frac{3}{2}\left(1 + \frac{P}{\rho}\right) < 1 \quad \Leftrightarrow \quad w \equiv \frac{P}{\rho} < -\frac{1}{3}$$

Combining this equation with the continuity equation It is also possible to conclude that:

$$\left|\frac{d\ln\rho}{d\ln a}\right| = 2\varepsilon < 1$$

Which shows that for small ϵ the energy density of the universe remains approximately constant. Conventional matter sources would dilute with the (exponential expansion). The energy density of whatever causes inflation needs to be an unconventional/unusual form of matter/energy.

Inflation

Basic Picture

Let us now look intuitively how the inflation condition

$$\boxed{\text{inflation} \Leftrightarrow \ddot{a} > 0 \Leftrightarrow \frac{d}{dt}(cH^{-1}/a) < 0}$$

may be used to solve the Hot Big-Bang problems

Flatness problem:
If the expansion is accelerating, $\ddot{a} > 0$, the derivative of the scale factor $\dot{a}$ is an increasing function of time. So it decreases as we go back in time

$$|\Omega(t) - 1| = \frac{|k|}{a^2(t)H^2(t)} = \frac{|k|}{\dot{a}^2(t)}$$

Is an increasing function of time, so: $\dot{a}(t \to 0) \to 0$

the flatness problem is therefore solved because…

The Universe can in principle "start" with a energy density parameter far from 1.

Inflation

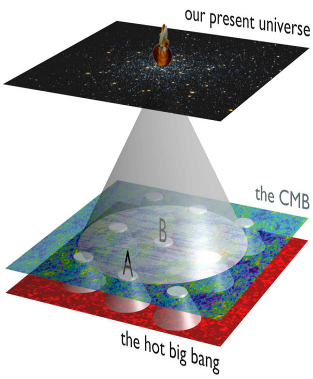
Basic Picture

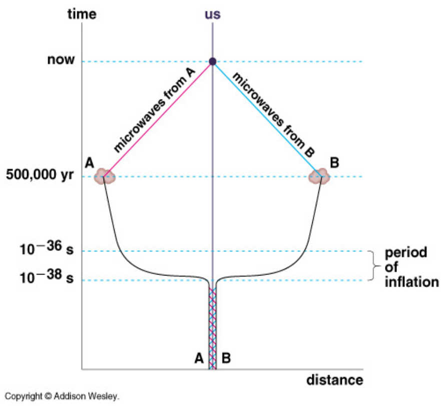
Let us now look intuitively how the inflation condition

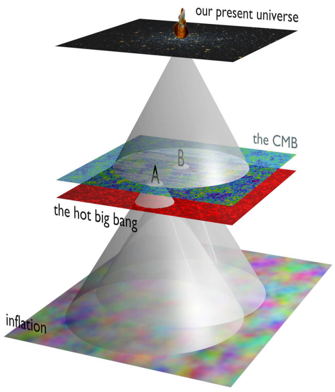
$$\boxed{\text{inflation} \Leftrightarrow \ddot{a} > 0 \Leftrightarrow \frac{d}{dt}(cH^{-1}/a) < 0}$$

may be used to solve the SMC problems

The horizon problem: If the accelerated expansion happens in a early phase of the Universe, during a long enough period, in principle, all causally disconnected sky patches of the CMB can be put in causal contact.







Inflation

Basic Picture

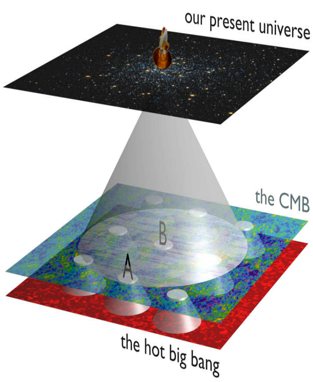
Let us now look intuitively how the inflation condition

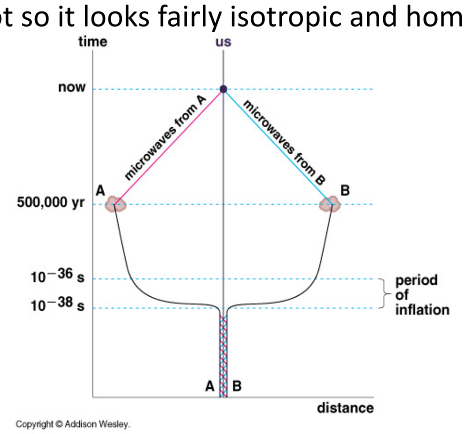
$$\boxed{\text{inflation} \Leftrightarrow \ddot{a} > 0 \Leftrightarrow \frac{d}{dt}(cH^{-1}/a) < 0}$$

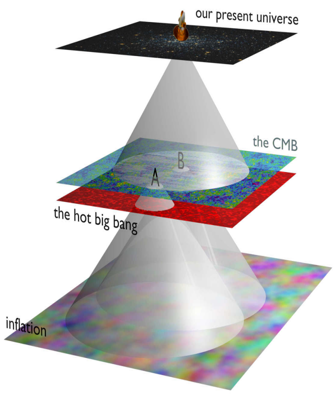
may be used to solve the SMC problems

The monopole problem: If the universe expands sufficiently after monopoles are produced their abundance can be too low to be observed.

The homogeneity problem: our visible universe comes from a causally connected region that expanded a lot so it looks fairly isotropic and homogeneous







The Theory of Inflation

Inflation also provides a mechanism for the origin of fluctuations…

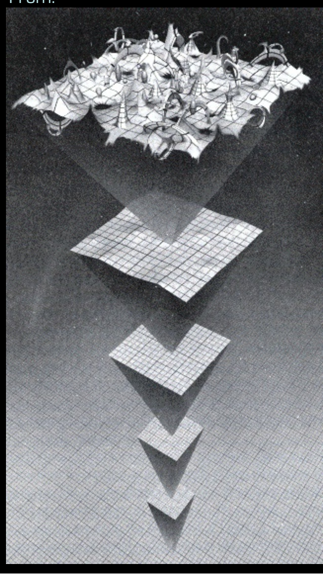
… fluctuations (density and grav. waves) are due to quantum fluctuations about the vacuum state of the inflationary potential.

The inflation (inflaton) field has energy density fluctuations allowed by the Heisenberg uncertainty principle:

$$\Delta E_{\phi} > h/(4\pi\Delta t)$$

During inflation fluctuations are "inflated" to macroscopic scales > physically connected scales become larger than the horizon scale and "freeze".

From:



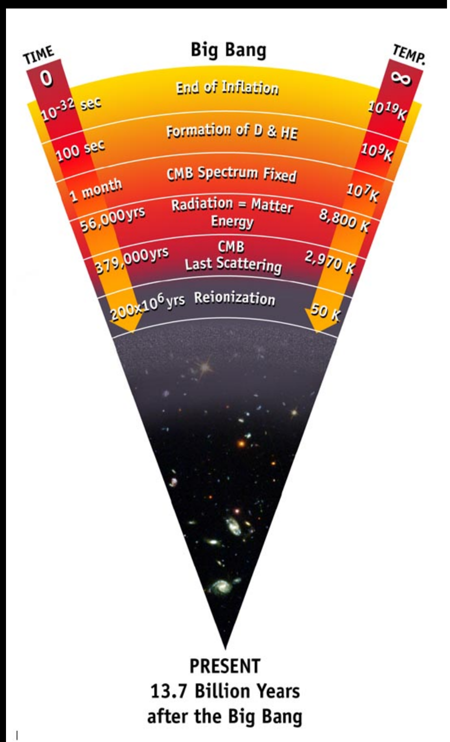
Standard Model of Cosmology (SMC)

SMC = Hot Big Bang + Inflation

FLRW models provide a description for the evolution of the "background" Universe

provides a mechanism for the origin of perturbations to the "background Universe"

fluctuations (density and grav. waves) are due to quantum fluctuations about the vacuum state of the inflationary potential.



From:

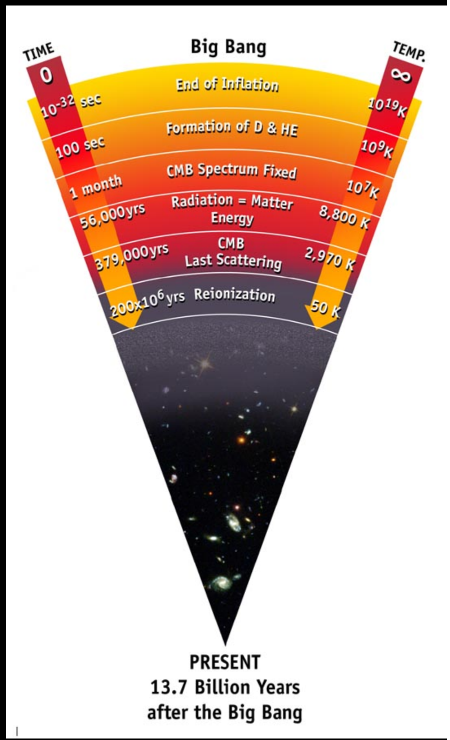
Standard Model of Cosmology (SMC)

SMC = Hot Big Bang + Inflation

After the end of inflation

$$H^2(t) = \frac{8\pi G}{3}(\rho_r + \rho_m) - \frac{kc^2}{a^2} + \frac{\Lambda c^2}{3}$$
$$= H_0^2\left[\Omega_{r0}\left(\frac{a_0}{a}\right)^4 + \Omega_{m0}\left(\frac{a_0}{a}\right)^3 + \Omega_{k0}\left(\frac{a_0}{a}\right)^2 + \Omega_{\Lambda 0}\right]$$

- Background evolution is dominated by:
  - Radiation
  - Matter
  - Dark Energy



From:

Inflation

Distances and Horizons

Let us consider the travel of light along radial ($d\theta = d\phi = 0$) geodesics in a FLRW metric

$$\begin{aligned} ds^2 &= dt^2 - a^2(t)\left[\frac{dr^2}{1-kr^2} + r^2(d\theta^2 + \sin^2\theta d\phi^2)\right], \\ &= dt^2 - a^2(t)\left[d\chi^2 + f_k(\chi)(d\theta^2 + \sin^2\theta d\phi^2)\right], \end{aligned}$$

written in a conformal way with the introduction of the conformal time $d\tau = dt/a$

$$\mathrm{d}s^2 = a^2(\tau)\left[\mathrm{d}\tau^2 - \mathrm{d}\chi^2\right]$$

(with $d\chi = dr$ for flat geometries), So light rays ($ds^2 = 0$) travel along geodesics with

$$\Delta\chi(\tau) = \pm\Delta\tau$$

From integrating this we can define the notions of:

- Particle horizon: $\chi_{\mathrm{ph}}(\tau) = \tau - \tau_i = \int_{t_i}^{t} \frac{\mathrm{d}t}{a(t)}$ with $t_i = 0$

- Event horizon: $\chi_{\mathrm{eh}}(\tau) = \tau_f - \tau = \int_{t}^{t_f} \frac{\mathrm{d}t}{a(t)}$ with $t_f = \infty$

Inflation

Distances and Horizons

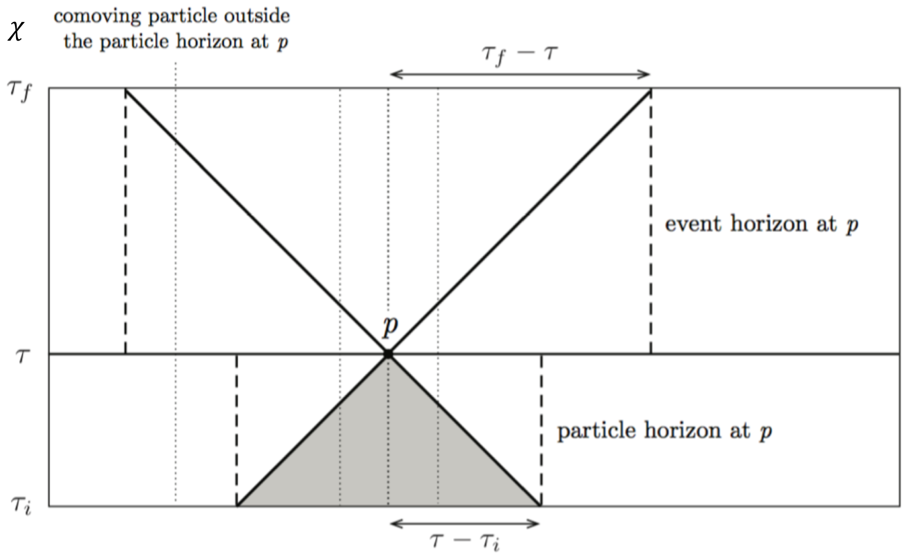


Figure 2.1: *Spacetime diagram illustrating the concept of horizons. Dotted lines show the worldlines of comoving objects. The event horizon is the maximal distance to which we can send signal. The particle horizon is the maximal distance from which we can receive signals.*

Inflation

Distances and Horizons

The particle horizon, χ_{ph}, the maximal commoving distance travelled by light until a time t, can be computed as follows:

$$\chi_{\mathrm{ph}}(\tau) = \int_{t_i}^{t} \frac{\mathrm{d}t}{a} = \int_{a_i}^{a} \frac{\mathrm{d}a}{a\dot{a}} = \int_{\ln a_i}^{\ln a} (aH)^{-1}\, \mathrm{d}\ln a$$

with $t_i = 0; a_i = 0$. The **commoving Hubble radius** inside the last integral is (see Freedman equation):

$$\boxed{(aH)^{-1} = H_0^{-1}\, a^{\frac{1}{2}(1+3w)}}$$

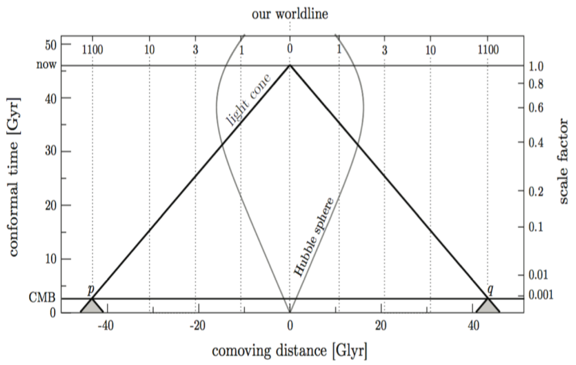
For any fluid component with an equation state parameter w. All familiar matter sources have $1 + 3w > 0$ (this is an implication of the so called strong energy condition). So in the Hot Big-Bang theory model the commoving Hubble radius is always increasing. Using the above expressions one finds (with $t_i = 0$):

$$\boxed{\chi_{\mathrm{ph}}(t) = \frac{2H_0^{-1}}{(1+3w)}\, a(t)^{\frac{1}{2}(1+3w)} = \frac{2}{(1+3w)}\, (aH)^{-1}}$$

But since **during inflation** $1 + 3w < 0$, this condition has to be violated and **the commoving Hubble radius is a decreasing function of time**.

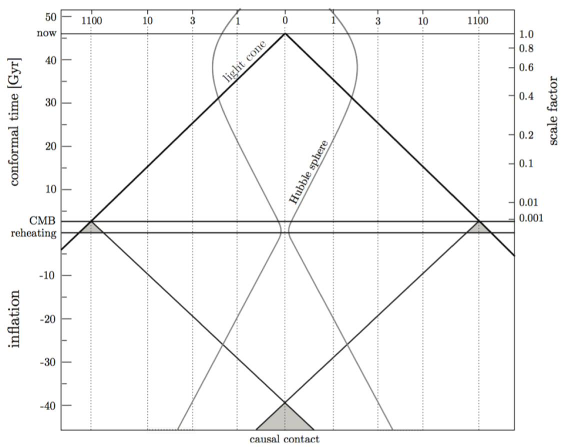
Inflation

Distances and Horizons



Hot Big-Bang evolution

Hot Big-Bang + inflation evolution



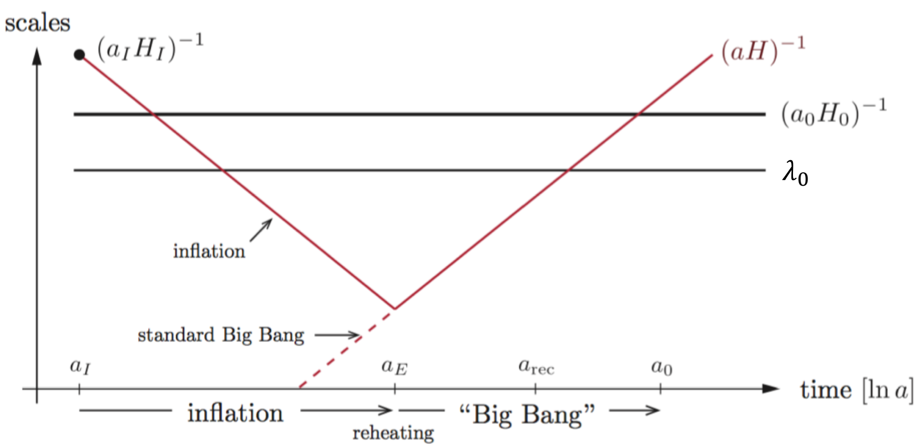
Inflation

Cosmological scales and horizons

During inflation

- any comoving cosmological scale, λ_0, is fixed in time as: $\lambda_0 = \lambda / a(t)$
- but the comoving (particle) horizon $\sim R_H = (aH)^{-1}$ decreases with time

So **during inflation scales** inside the horizon at a given time **grow faster and may become larger than (go beyond) the horizon.**

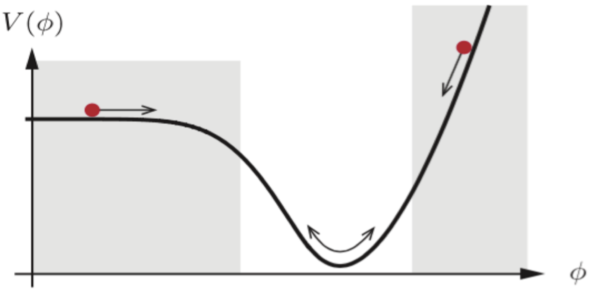


Inflation

Scalar field Dynamics

Inflation is usually modelled by a scalar field $\phi = \phi(x^i, t)$, called the **inflaton field**, that can generally be a function of position and time.

Associated with each field value there's a **potential energy**, $V(\phi)$, and If the field depends on time, the field also carries **kinetic energy**.



Using the Noether's theorem one can Prove that the energy-stress tensor of any scalar Field can be computed as:

$$T_{\mu\nu} = \partial_\mu\phi\partial_\nu\phi - g_{\mu\nu}\left(\frac{1}{2}g^{\alpha\beta}\partial_\alpha\phi\partial_\beta\phi - V(\phi)\right)$$

For a homogeneous and isotropic **FLRW** universe, without perturbations (ie inhomogeneities) **the field is only a function of time**, $\phi = \phi(t)$. Computing, $T^0_0 = \rho_\phi$, and $T^i_j = -P_\phi\, \delta^i_j$ one obtains:

$$\boxed{\rho_\phi = \frac{1}{2}\dot{\phi}^2 + V(\phi)} \qquad \boxed{P_\phi = \frac{1}{2}\dot{\phi}^2 - V(\phi)}$$

Inflation

Scalar field Dynamics: Klein-Gordan equation

Using the energy density of the inflaton filed in the **Friedmann equation** gives:

$$\boxed{H^2 = \frac{1}{3M_{\rm pl}^2}\left[\frac{1}{2}\dot{\phi}^2 + V\right]}$$

Taking the time derivative one finds:

$$2H\dot{H} = \frac{1}{3M_{\rm pl}^2}\left[\dot{\phi}\ddot{\phi} + V'\dot{\phi}\right]$$

where $V' \equiv dV/d\phi$.
Combining ρ_ϕ and p_ϕ in the **acceleration equation**, $\dot{H} = -(\rho_\phi + P_\phi)/(2M_{\rm pl}^2)$, one obtains:

$$\boxed{\dot{H} = -\frac{1}{2}\frac{\dot{\phi}^2}{M_{\rm pl}^2}}$$

This shows that the acceleration of the universe is sourced by the kinetic energy of the inflaton field. Combining these two last expressions one obtains the **Klein-Gordan** equation that describes the evolution of the inflationary field:

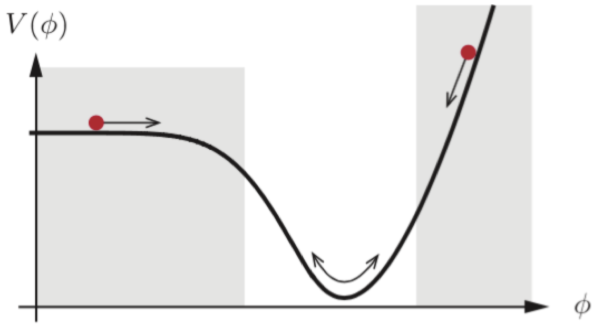
$$\boxed{\ddot{\phi} + 3H\dot{\phi} + V' = 0}$$

Inflation

Slow roll inflation

Combining the expressions: $\varepsilon \equiv -\frac{\dot{H}}{H^2}$ and $\dot{H} = -\frac{1}{2}\frac{\dot{\phi}^2}{M_{\rm pl}^2}$ gives:

$$\boxed{\varepsilon = \frac{\frac{1}{2}\dot{\phi}^2}{M_{\rm pl}^2 H^2}}$$



This means that inflation $\epsilon < 1$ only occurs if the contribution of the kinetic energy of the field to the total energy is small. When this happens the field is said to be **slow rolling**

The time derivative of ϵ gives:

$$\dot{\varepsilon} = \frac{\dot{\phi}\ddot{\phi}}{M_{\rm pl}^2 H^2} - \frac{\dot{\phi}^2 \dot{H}}{M_{\rm pl}^2 H^3}$$

Which allows us to compute the η parameter as:

$$\boxed{\eta = \frac{\dot{\varepsilon}}{H\varepsilon} = 2\frac{\ddot{\phi}}{H\dot{\phi}} - 2\frac{\dot{H}}{H^2} = 2(\varepsilon - \delta)}$$

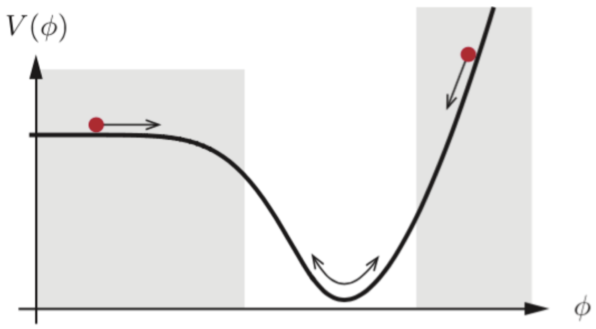
where $\delta \equiv -\ddot{\phi}/H\dot{\phi}$.

Inflation

Slow roll inflation

The conditions $\epsilon < 1$ and $|\eta| < 1$ **are a guaranty that inflation happens and persists**. Since this implies that the kinetic energy of the field is small one can assume the **slow roll inflation conditions**:

$$\{\epsilon, |\eta|\} \ll 1$$



and approximate the Friedmann and Klien Gordon equations as:

- Friedmann ($\dot{\phi}^2 \sim 0$): $H^2 \approx \frac{V}{3M_{\rm pl}^2}$
- Klein Gordon ($\ddot{\phi} \sim 0$): $3H\dot{\phi} \approx -V'$

Combining these equations (plus taking the time derivative of the Klein Gordon equation) allows one to write the $\{\epsilon, |\eta|\}$ parameters as function of the potential and its derivatives:

$$\boxed{\epsilon_{\rm v} \equiv \frac{M_{\rm pl}^2}{2}\left(\frac{V'}{V}\right)^2 \quad , \quad |\eta_{\rm v}| \equiv M_{\rm pl}^2 \frac{|V''|}{V}}$$

Inflation

Slow roll inflation

The total amount of e-folds (which gives by how much the universe expands during the inflationary period) can also be derived from our knowledge of the inflationary potential.

$$N_{\text{tot}} \equiv \int_{a_I}^{a_E} \mathrm{d}\ln a = \int_{t_I}^{t_E} H(t)\,\mathrm{d}t$$

where t_I, t_E are defined as the times when inflation begins and ends

$$\varepsilon(t_I) = \varepsilon(t_E) \equiv 1$$

The integrand function above, can be approximated by:

$$H\mathrm{d}t = \frac{H}{\dot{\phi}}\,\mathrm{d}\phi = \frac{1}{\sqrt{2\varepsilon}}\frac{|\mathrm{d}\phi|}{M_{\text{pl}}} \approx \frac{1}{\sqrt{2\epsilon_{\text{v}}}}\frac{|\mathrm{d}\phi|}{M_{\text{pl}}}$$

Which leads to:

$$\boxed{N_{\text{tot}} = \int_{\phi_I}^{\phi_E} \frac{1}{\sqrt{2\epsilon_{\text{v}}}}\frac{|\mathrm{d}\phi|}{M_{\text{pl}}}}$$

To solve the horizon problem (CMB) is possible to show that one requires at least 60 e-folds of inflationary period.

Inflation

Example: $V(\phi) = m^2\phi^2/2$

The slow roll conditions give:

$$N = 8\pi G \int_{\phi_e}^{\phi_N} \frac{V}{V'}\,d\phi = \frac{8\pi G}{2}\int_{\phi_e}^{\phi_N} \phi\,d\phi = \frac{8\pi G}{4}(\phi_N^2 - \phi_e^2)$$

So the value of the field after N e-foldings should be:

$$\phi_N^2 = \phi_e^2 + \frac{1}{2\pi G}N.$$

When inflation ends, $\epsilon = 1$, so using the definition of ϵ one has:

$$\frac{1}{16\pi G}\left(\frac{V'}{V}\right)_e^2 = 1 \qquad \Rightarrow \qquad \left(\frac{m^2\phi_e}{\frac{1}{2}m^2\phi_e^2}\right)^2 = 16\pi G$$

So one has:

$$\phi_e^2 = \frac{1}{4\pi G} \approx 0.08 m_{\text{Pl}}^2$$

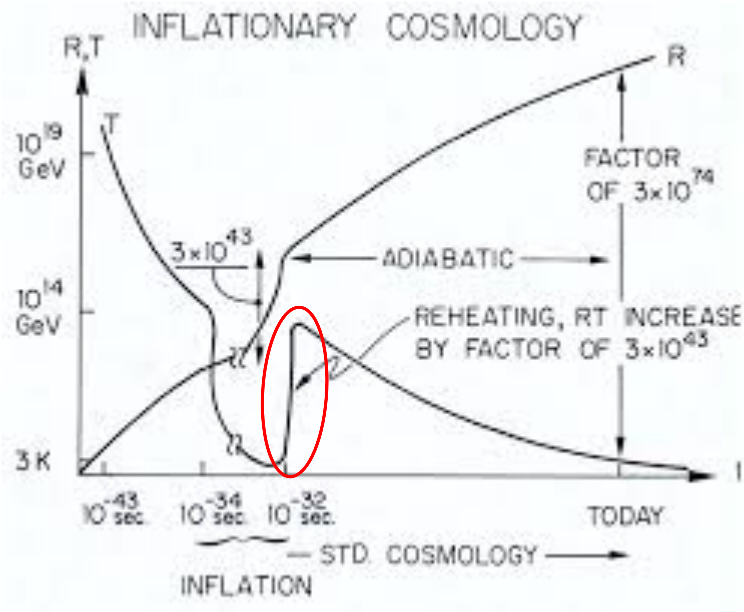
$$\phi_N^2 = \phi_e^2 + \frac{1}{2\pi G}N = \frac{m_{\text{Pl}}^2}{2\pi}\left(N + \frac{1}{2}\right) \approx \frac{m_{\text{Pl}}^2}{2\pi}N,$$

For $N \simeq 70$ this gives:

$$\phi_N \approx 3.3 m_{\text{Pl}}$$

Inflation

Re-heating



Inflation

Re-heating

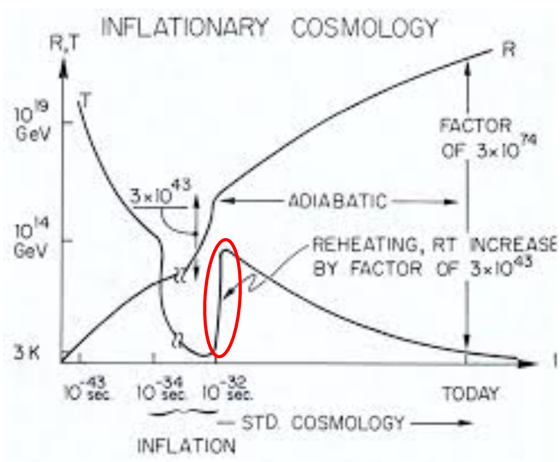
During the inflationary period most of the energy density of the Universe is given by the inflationary potential.

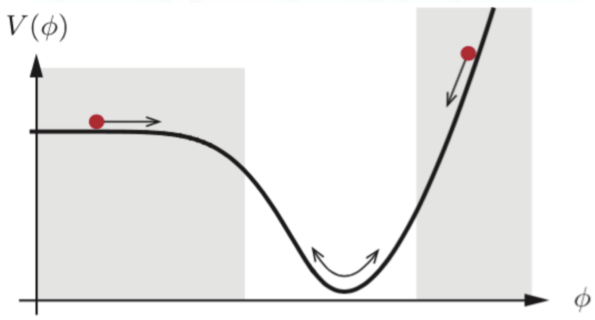
As inflation ends, the kinetic energy associated with the inflaton field is no longer negligible and the energy in the field is transferred to the matter/energy species of the fluid.

$$\dot{\rho}_\phi + 3H\rho_\phi + \Gamma\rho_\phi = 0.$$
$$\dot{\rho}_R + 3H\rho_R - \Gamma\rho_\phi = 0.$$

Where Γ is the so called **energy width** of the inflaton decay.

This process is know as **reheating** and It is followed by the hot big bang evolutionary phase of the universe.

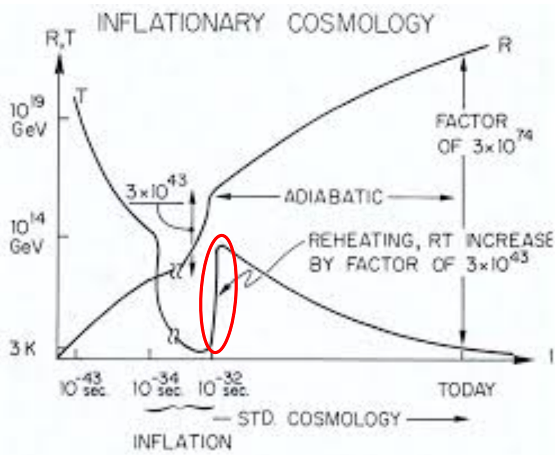




Inflation

Re-heating

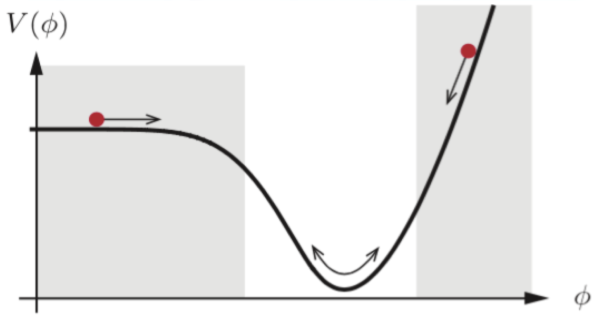
The basic idea behind reheating is that this period starts when ϕ begins to oscillate with a friction term about the minimum of the inflationary potential.



For example, taking a quadratic potential $V = m^2\phi^2/2$, the Klein-Gordon and the continuity equations give:

$$\ddot{\phi} + 3H\dot{\phi} = -m^2\phi$$

$$\dot{\rho}_\phi + 3H\rho_\phi = -3HP_\phi = -\frac{3}{2}H(m^2\phi^2 - \dot{\phi}^2)$$



Oscillations decrease in amplitude due to the friction term. By the end of the process all energy of the field is transferred, leading to the beginning of the hot Big-Bang evolution.

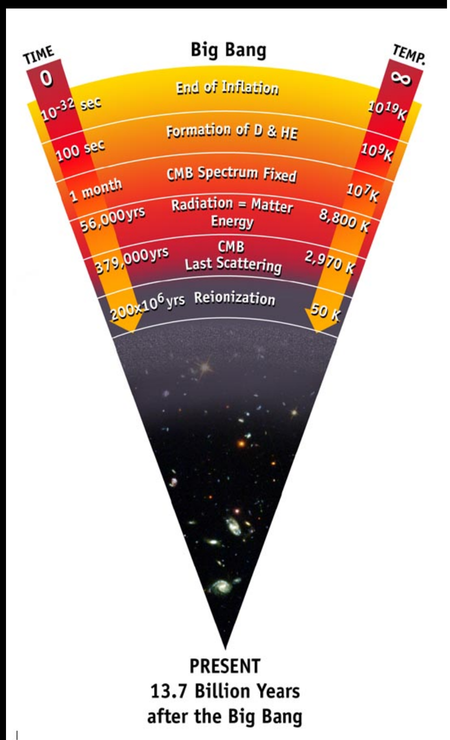
Standard Model of Cosmology (SMC)

SMC = Hot Big Bang + Inflation

After the end of inflation

$$H^2(t) = \frac{8\pi G}{3}(\rho_r + \rho_m) - \frac{kc^2}{a^2} + \frac{\Lambda c^2}{3}$$
$$= H_0^2\left[\Omega_{r0}\left(\frac{a_0}{a}\right)^4 + \Omega_{m0}\left(\frac{a_0}{a}\right)^3 + \Omega_{k0}\left(\frac{a_0}{a}\right)^2 + \Omega_{\Lambda 0}\right]$$

- Background evolution is dominated by:
  - Radiation
  - Matter
  - Dark Energy



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