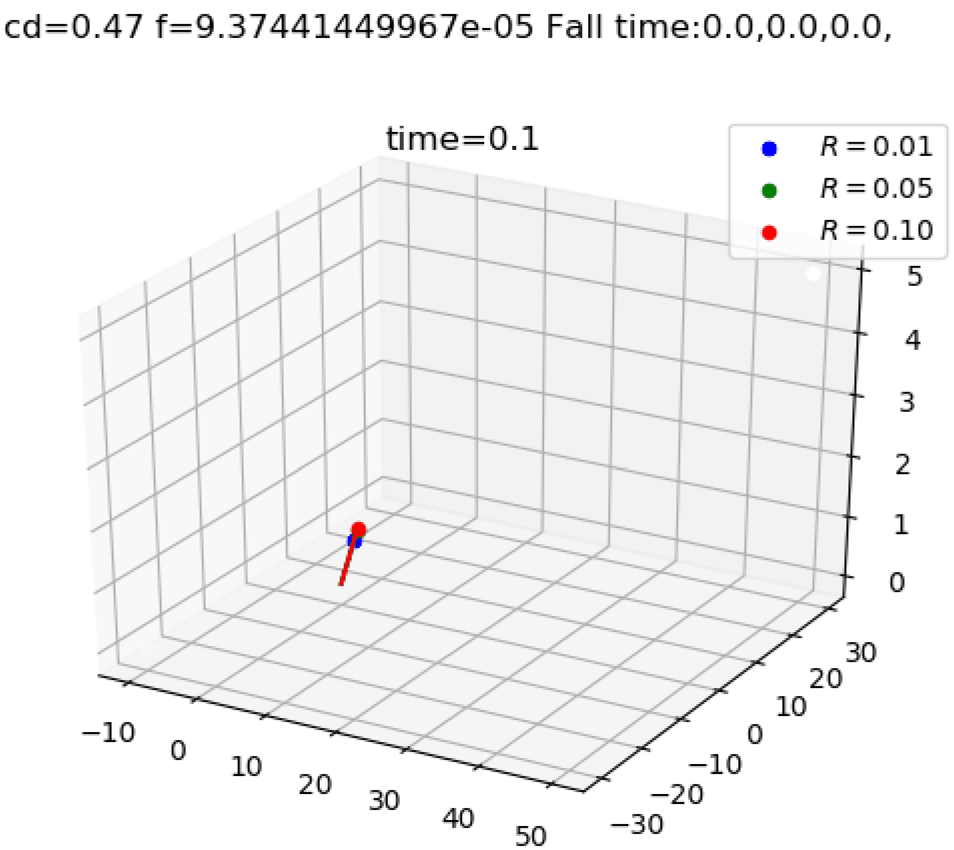
Aula 18

Equações diferenciais com condições fronteira num ponto:

viscosidade



Movimento num fluido viscoso/turbulento

sem rotação ($\Omega = 0$)

O movimento balístico (i.e. sem propulsão) num fluido (ar, água, de densidade ρ_F) é afetado pelo fluido, por dois mecanismos:

1) Impulsão, resultante do campo da pressão

Lei de Arquimedes ($\vec{k}$ aponta para cima na vertical), ρ_R densidade do projétil

$$\vec{F}_{imp} = \rho_F V g \vec{k} \Longrightarrow \vec{a}_{imp} = \frac{\vec{F}_{imp}}{m} = \frac{\rho_F}{\rho_R} g \vec{k}$$

1) Atrito, resultante da viscosidade e da turbulência

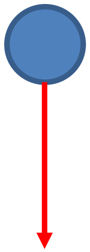
Lei empírica

$$\vec{F}_D = -c_D A \rho_F \frac{1}{2} v^2 \frac{\vec{v}}{|\vec{v}|} \Longrightarrow \vec{a}_D = -\frac{c_D A \rho_F \frac{1}{2} |\vec{v}|}{\rho_R V} \vec{v}$$

A é uma secção eficaz do projétil (πR^2 no caso de uma esfera), c_D é um coeficiente empírico (depende da forma do projétil e do regime de escoamento), V é o volume.

Experiência de Stokes

Queda livre de esferas num fluido viscoso: método de cálculo do coeficiente de viscosidade



Como o *drag* aumenta com a velocidade (para escoamentos suficientemente rápidos com v^2), a velocidade aumenta até atingir um valor constante, a velocidade terminal.

Queda livre num fluido viscoso

$$\begin{cases} \dfrac{du}{dt} = 0 \\ \dfrac{dv}{dt} = 0 \\ \dfrac{dw}{dt} = -g - Dw + I \end{cases} \wedge \begin{cases} D = \dfrac{c_D A \rho_F \frac{1}{2} |\vec{v}|}{\rho_R V} = K|w| \\ I = \dfrac{\rho_F}{\rho_R} g \end{cases}$$

Velocidade terminal:

$$\frac{dw}{dt} = 0 = -g - Dw + I \Longrightarrow K|w_{ter}|\, w_{ter} = -g + I$$

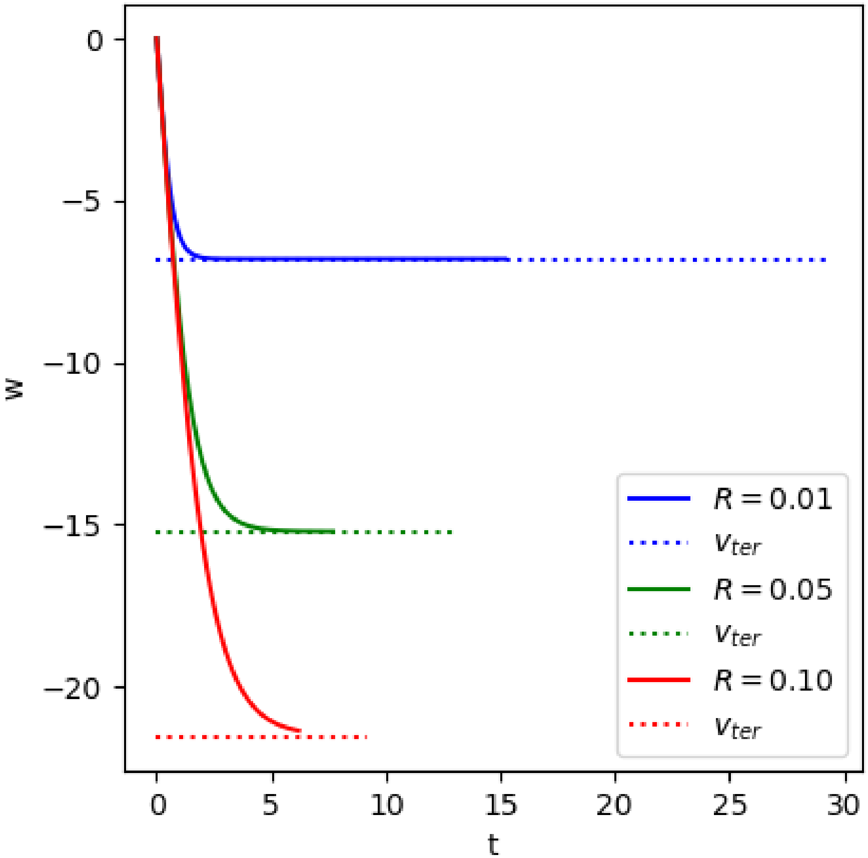
$$w_{ter} = -\sqrt{\frac{g - I}{K}}$$

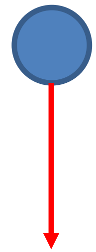
Queda livre num fluido $\Omega = 0$

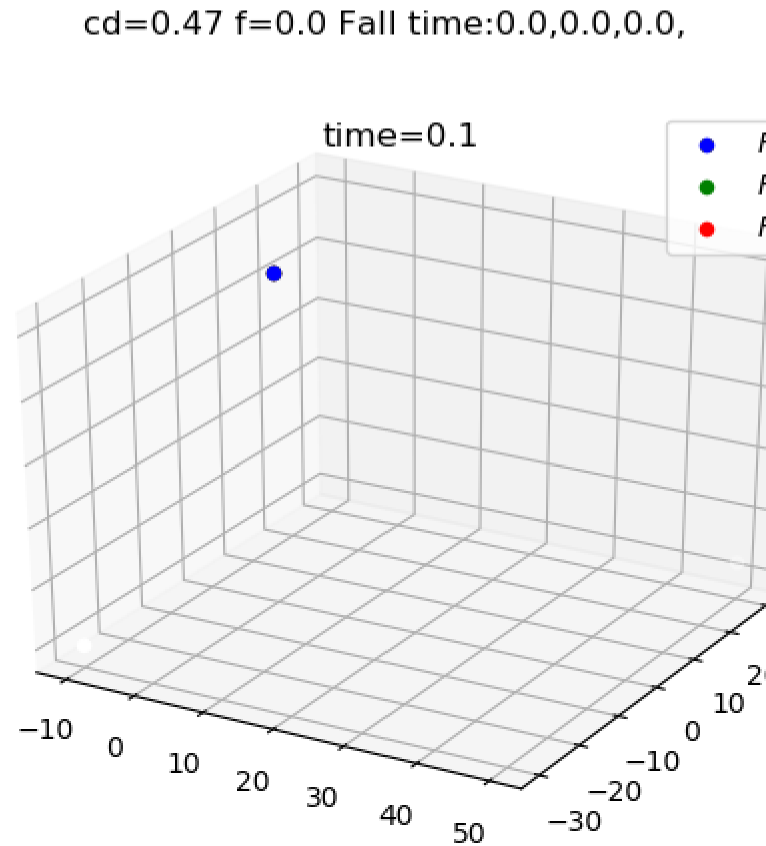
iterativo

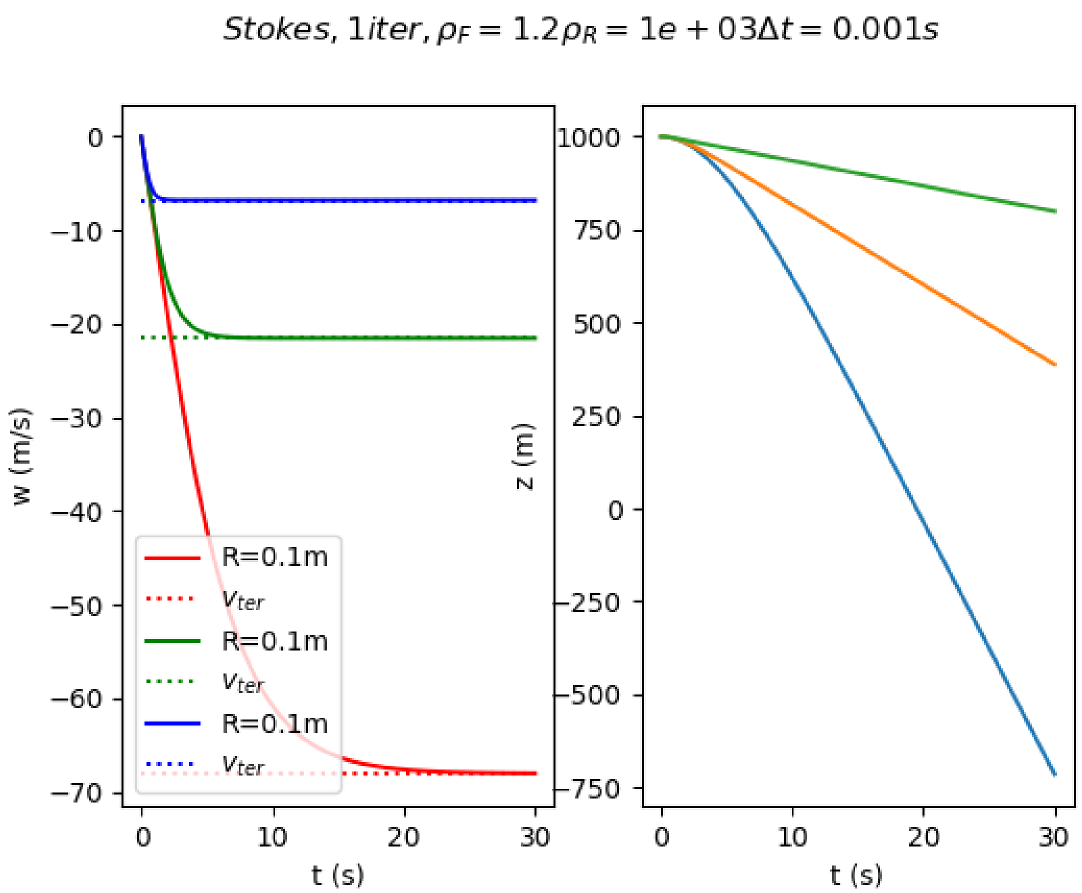
```
def stokes(z0,w0,timeInt,dt,g,R,rhoR,cd,rhoF):
    nIMP=1;massa=rhoR*4/3.*np.pi*R**3;
    Imp=rhoF*4/3.*np.pi*R**3*g/massa
    vterminal=np.sqrt(2*massa*(g-Imp)/(rhoF*np.pi*Rs[kc]**2*cd))
    tempo=np.arange(0.,timeInt,dt);n=len(tempo);
    Z=np.zeros((n),dtype='float')
    W=np.copy(Z)
    W[0]=w0;Z[0]=z0
    for kt in range(1,n):
        W[kt]=W[kt-1]-g*dt;Wh=0.5*(W[kt]+W[kt-1])
        for improve in range(nIMP):
            V2=Wh*Wh;speed=np.sqrt(V2)
            Drag=0.5*rhoF*np.pi*R**2*cd*V2/massa
            Dragz=-Drag*Wh/speed+Imp
            W[kt]=W[kt-1]+(-g+Dragz)*dt
            Wh=0.5*(W[kt]+W[kt-1])
        Z[kt]=Z[kt-1]+Wh*dt
    return Z,W,tempo,vterminal
```

Experiência de Stokes no ar

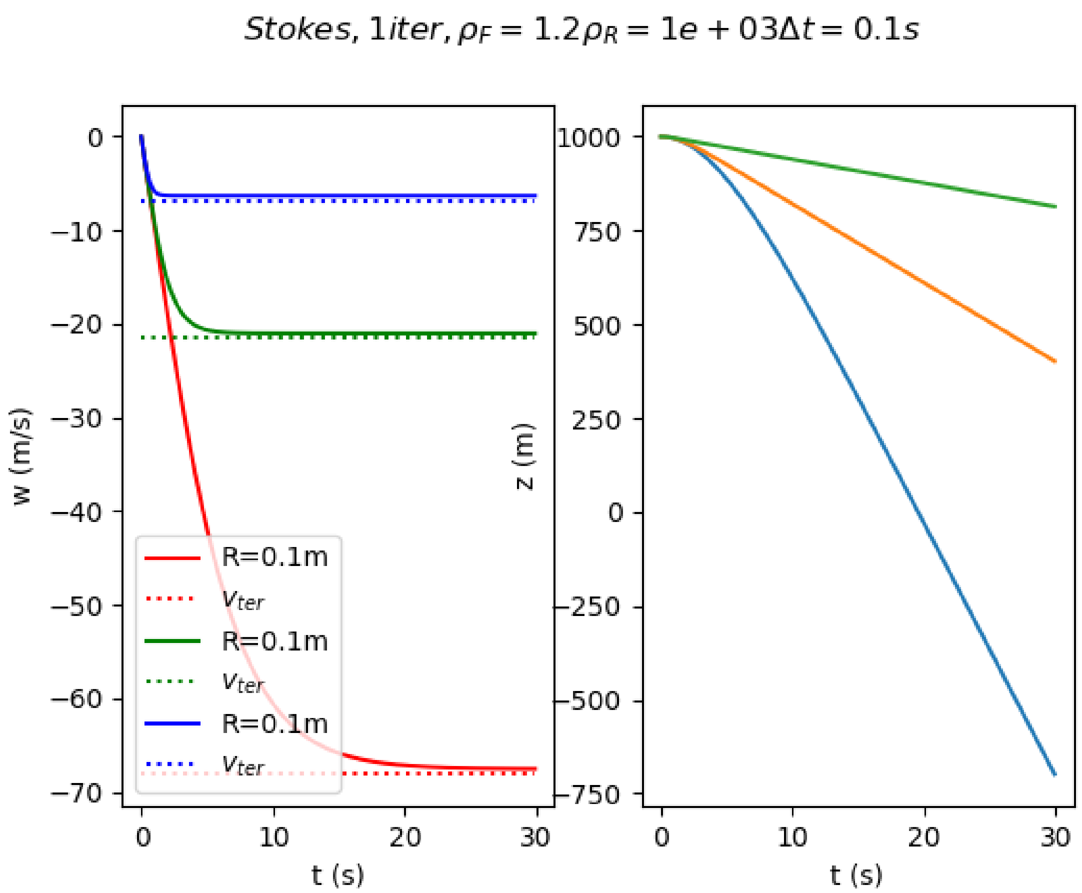




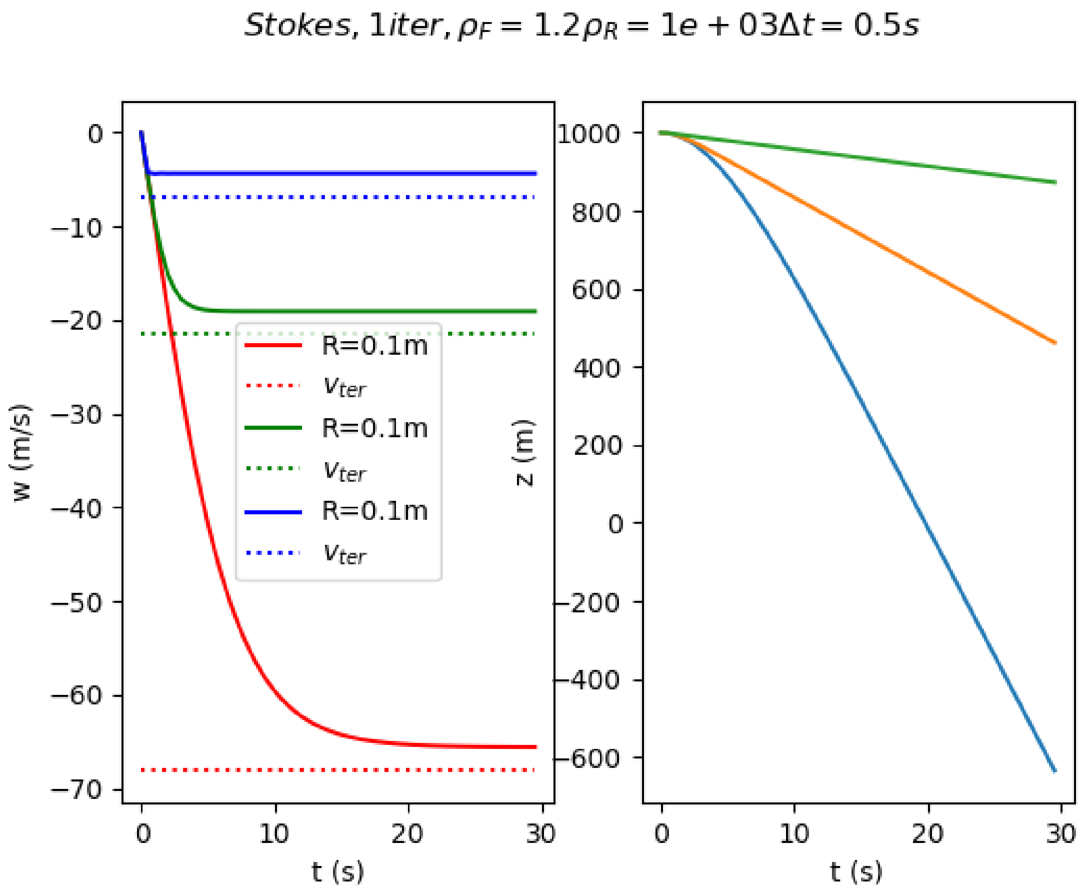




$Stokes, 1iter, \rho_F = 1.2\rho_R = 1e + 03\Delta t = 0.001s$
w (m/s)
t (s)
R=0.1m
v_{ter}
R=0.1m
v_{ter}
R=0.1m
v_{ter}
z (m)
t (s)

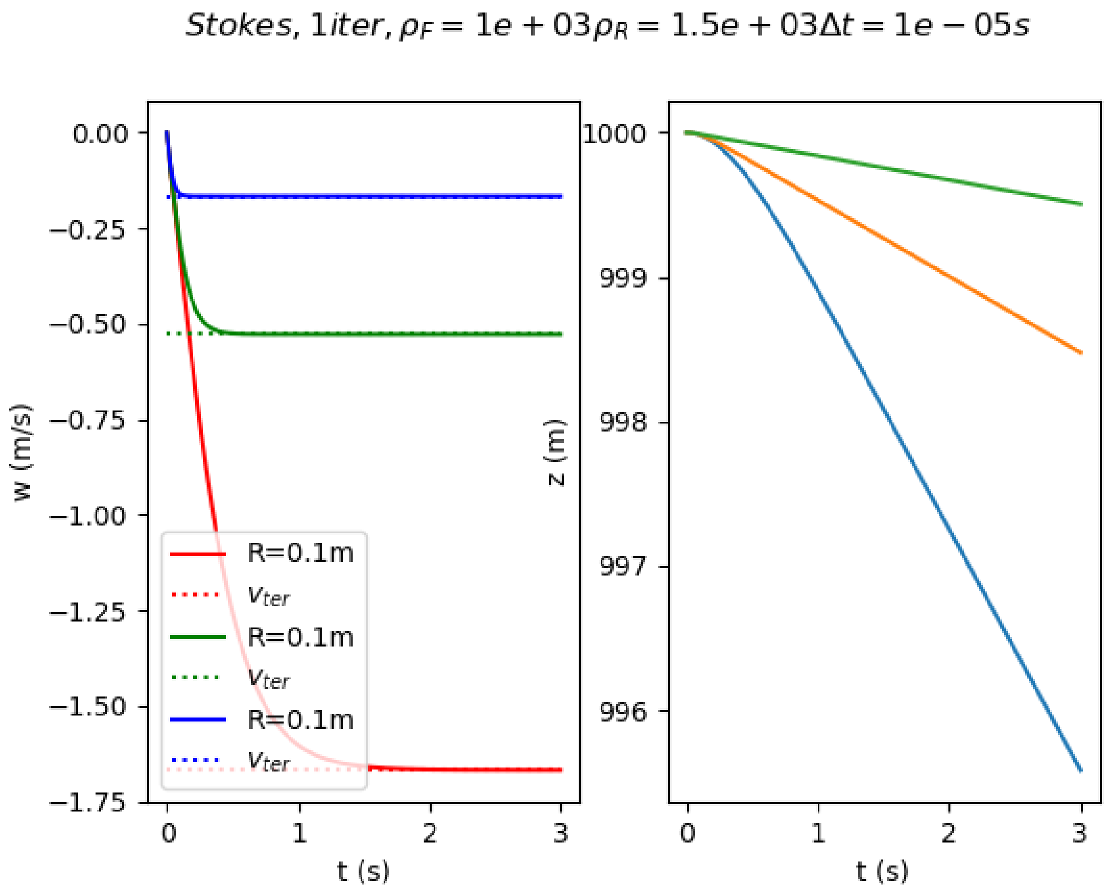


$Stokes, 1iter, \rho_F = 1.2\rho_R = 1e+03\Delta t = 0.1s$
w (m/s)
0
−10
−20
−30
−40
−50
−60
−70
0
10
20
30
t (s)
R=0.1m
v_{ter}
R=0.1m
v_{ter}
R=0.1m
v_{ter}
z (m)
1000
750
500
250
0
−250
−500
−750
0
10
20
30
t (s)

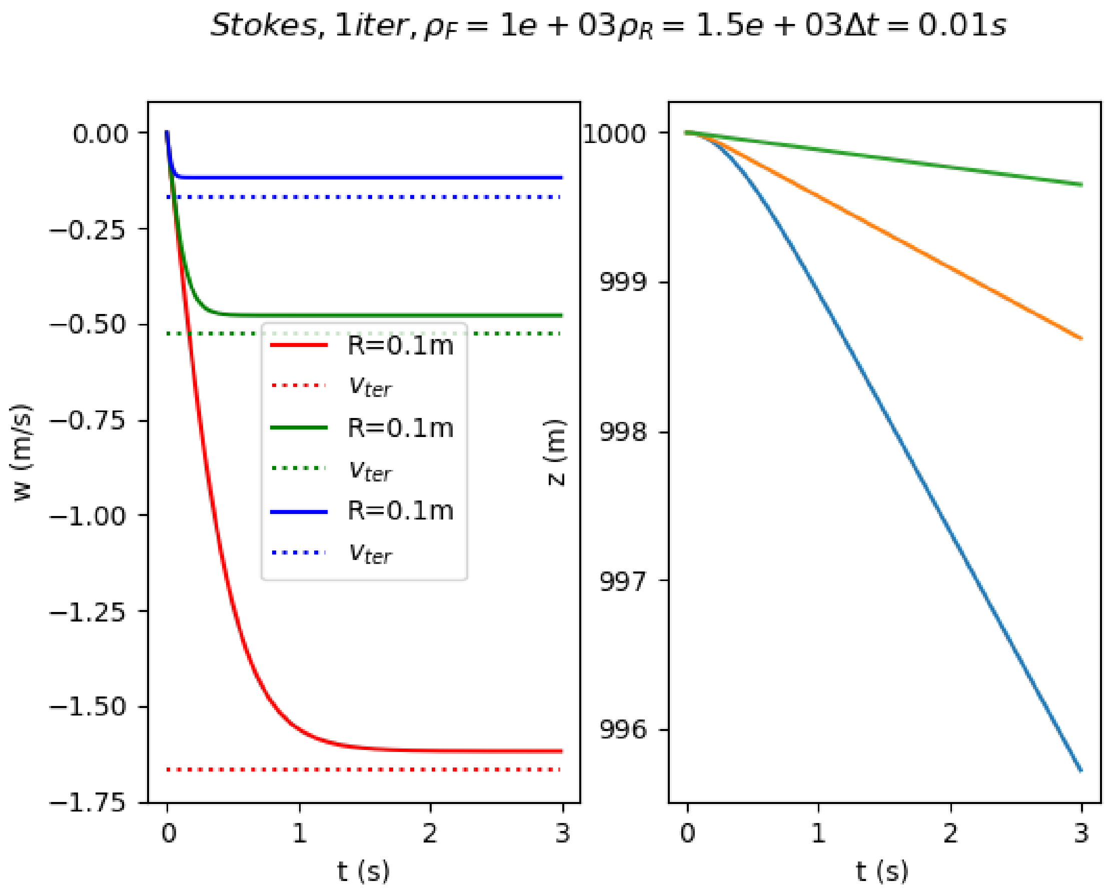


$Stokes, 1iter, \rho_F = 1.2\rho_R = 1e+03\Delta t = 0.5s$
w (m/s)
0
−10
−20
−30
−40
−50
−60
−70
R=0.1m
v_{ter}
R=0.1m
v_{ter}
R=0.1m
v_{ter}
0
10
20
30
t (s)
z (m)
1000
800
600
400
200
0
−200
−400
−600
0
10
20
30
t (s)

Queda em água (+lento, impulsão importante)



Queda em água



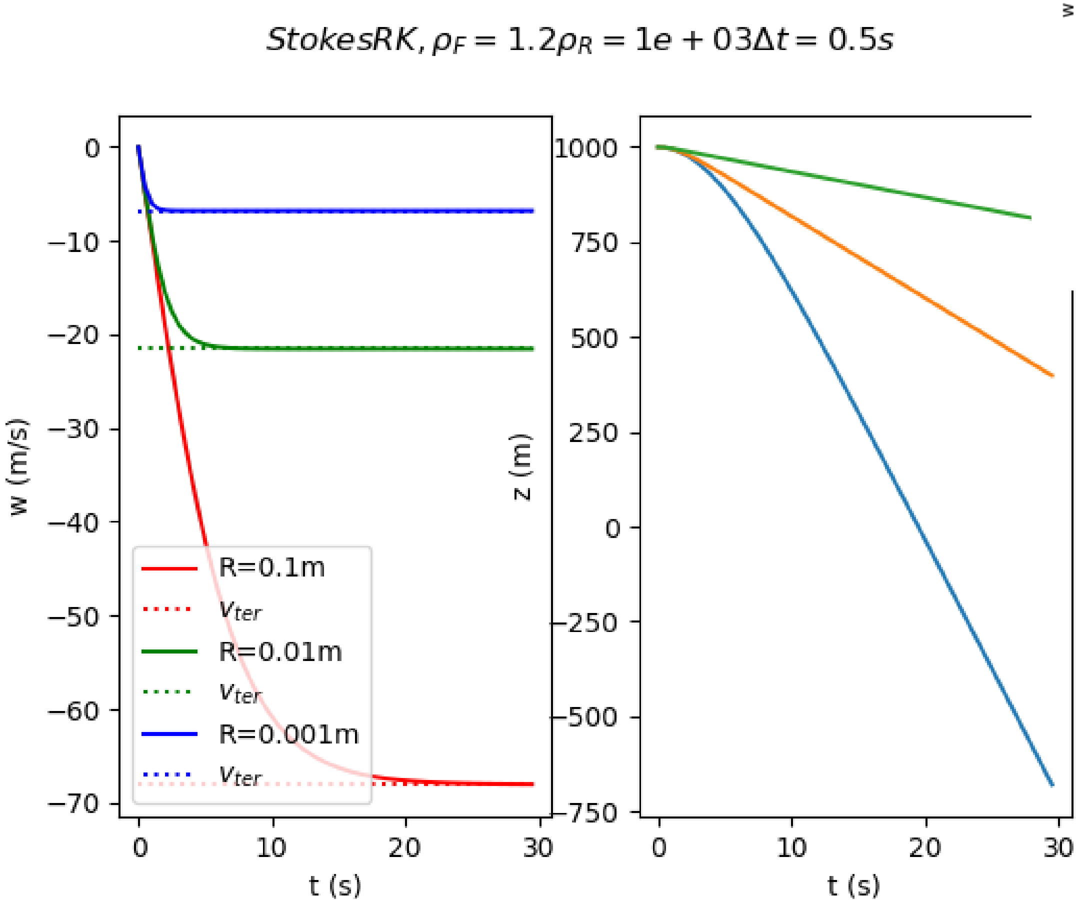
Experiência de Stokes com RungeKutta4

```
def RK4x(x,dxdt,dt): #genérico
    k1=dxdt(x)*dt
    k2=dxdt(x+k1/2)*dt
    k3=dxdt(x+k2/2.)*dt
    k4=dxdt(x+k3)*dt
    xP=x+1./6.*(k1+2*k2+2*k3+k4)
    return xP
def dwdt(w): #notar as variáveis globais
    g=9.8065
    A=np.pi*R**2
    V=4/3.*np.pi*R**3
    K=cD*A*rhoF/(2*rhoR*V)
    I=rhoF/rhoR*g
    xx=-g-K*w*abs(w)+I
    return xx
```

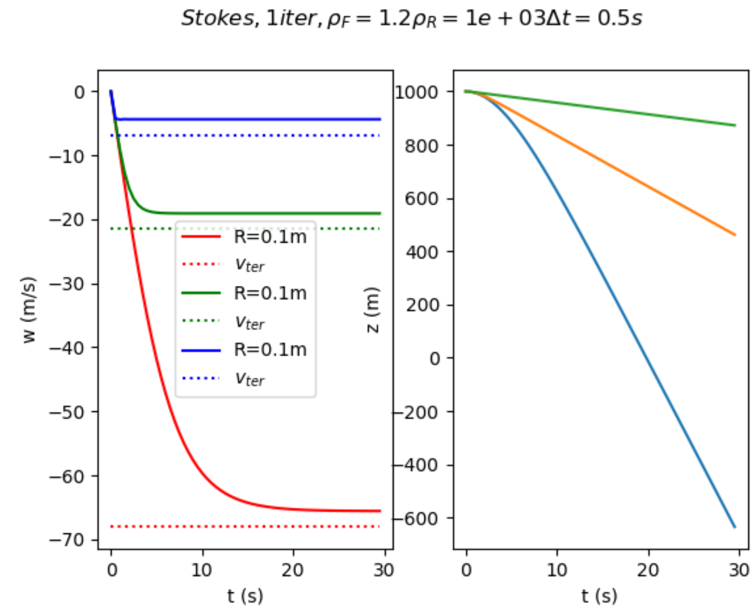
$$\frac{dw}{dt} = -g - Dw + I$$

$$D = K|w|$$

```
def stokesRK(z0,w0,timeInt,dt,g,R,rhoR,cd,rhoF):
    massa=rhoR*4/3.*np.pi*R**3
    Imp=rhoF*4/3.*np.pi*R**3*g/massa
    vterminal=np.sqrt(2*massa*(g-Imp)/\
               (rhoF*np.pi*Rs[kc]**2*cd))
    tempo=np.arange(0.,timeInt,dt);n=len(tempo);
    Z=np.zeros((n),dtype='float')
    W=np.copy(Z)
    W[0]=w0;Z[0]=z0
    for kt in range(1,n):
        W[kt]=RK4x(W[kt-1],dwdt,dt)
        Z[kt]=Z[kt-1]+0.5*(W[kt]+W[kt-1])*dt
    return Z,W,tempo,vterminal
```

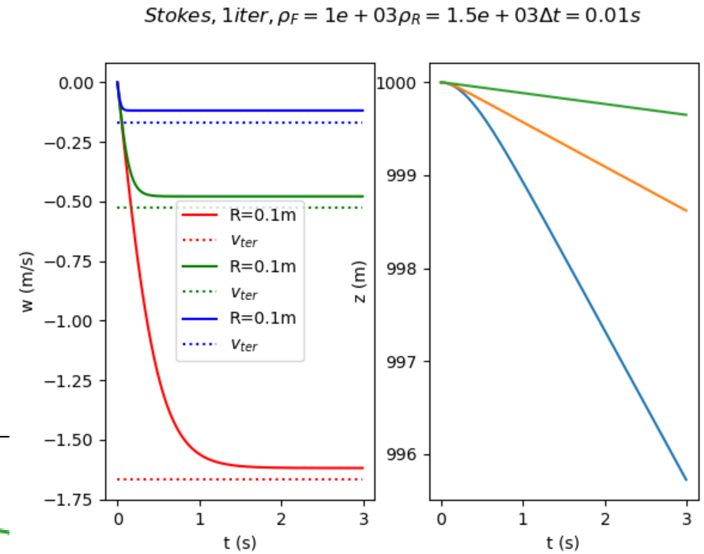


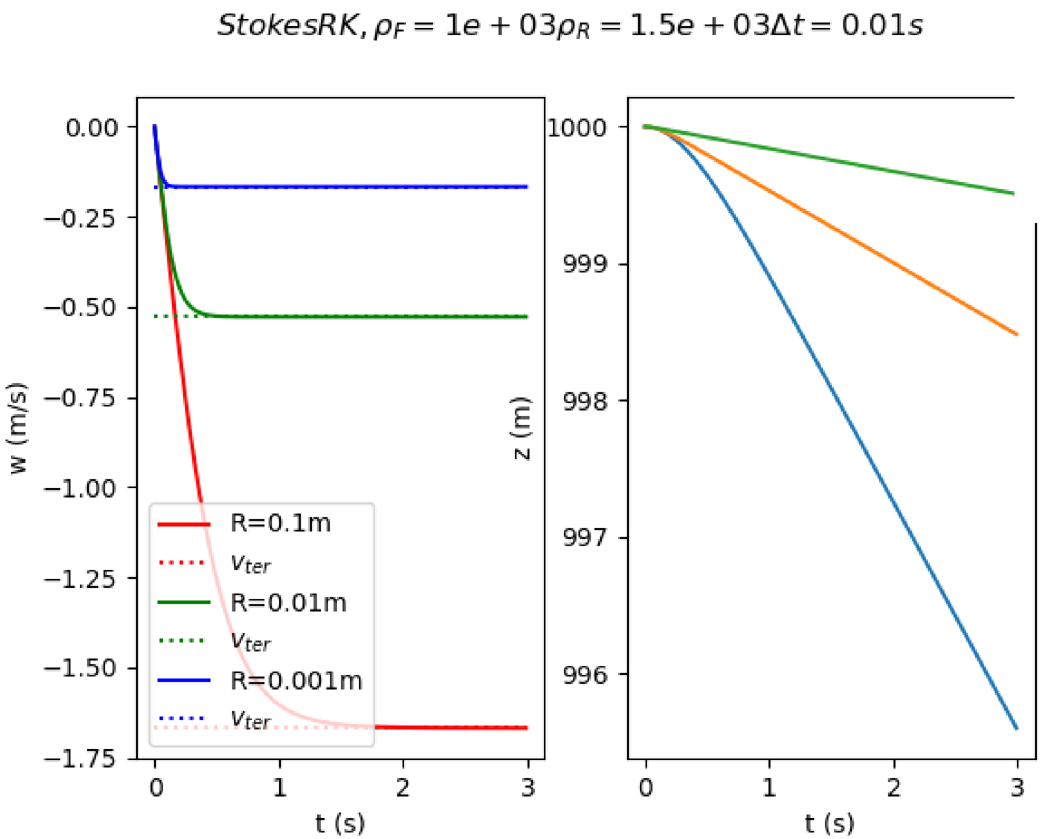
$StokesRK, \rho_F = 1.2 \rho_R = 1e+03 \Delta t = 0.5s$
w (m/s)
z (m)
t (s)
R=0.1m
v_{ter}
R=0.01m
v_{ter}
R=0.001m
v_{ter}



$Stokes, 1iter, \rho_F = 1.2 \rho_R = 1e+03 \Delta t = 0.5s$
w (m/s)
z (m)
t (s)
R=0.1m
v_{ter}
R=0.1m
v_{ter}
R=0.1m
v_{ter}

Em água


$Stokes, 1iter, \rho_F = 1e + 03\rho_R = 1.5e + 03\Delta t = 0.01s$
w (m/s)
z (m)
t (s)
R=0.1m
v_{ter}
R=0.1m
v_{ter}
R=0.1m
v_{ter}


$StokesRK, \rho_F = 1e + 03\rho_R = 1.5e + 03\Delta t = 0.01s$
w (m/s)
z (m)
t (s)
R=0.1m
v_{ter}
R=0.01m
v_{ter}
R=0.001m
v_{ter}

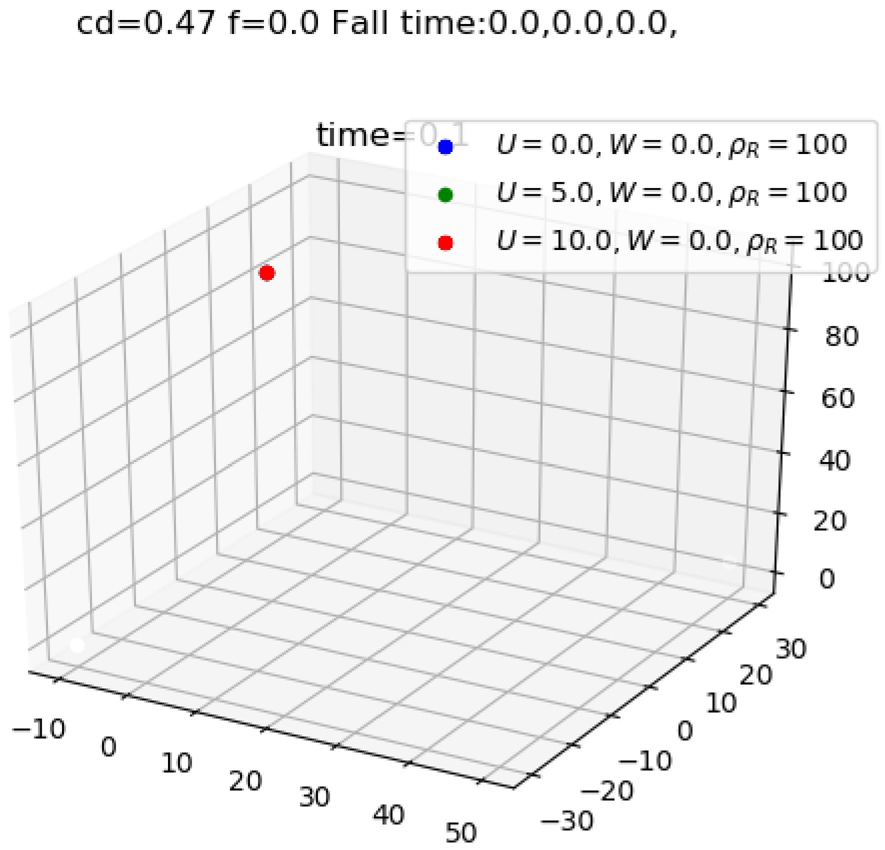
Passagem de argumentos (global)

```
z0=1000;w0=0;timeInt=3
dt=0.1
g=9.8065
cD=0.47 #global
rhoR=1500 #global
rhoF=1000 #global
cores=['red','green','blue']
for dt in [0.00001,0.001,0.1]:
    plt.figure()
    Rs=[0.1,0.01,0.001]
    for kc in range(len(cores)):
        R=Rs[kc] #global
        Z,W,tempo,vterminal=stokesRK(z0,w0,timeInt,dt,g,Rs[kc],rhoR,cd,rhoF)
        plt.subplot(1,2,1)
        plt.plot(tempo,W,color=cores[kc],label='R='+str(R)+'m')
        plt.plot([tempo[0],tempo[-1]],\
              [-vterminal,-vterminal],color=cores[kc],label=r'$v_{ter}$',linestyle=':')
        plt.xlabel('t (s)')
        plt.ylabel('w (m/s)')
        plt.subplot(1,2,2)
        plt.plot(tempo,Z)
        plt.xlabel('t (s)')
        plt.ylabel('z (m)')
    plt.subplot(1,2,1);plt.legend()
    plt.suptitle(r'$StokesRK, \rho_F=%.2g \rho_R=%.3g \Delta t=%.2g s$' % (rhoF,rhoR,dt))
```

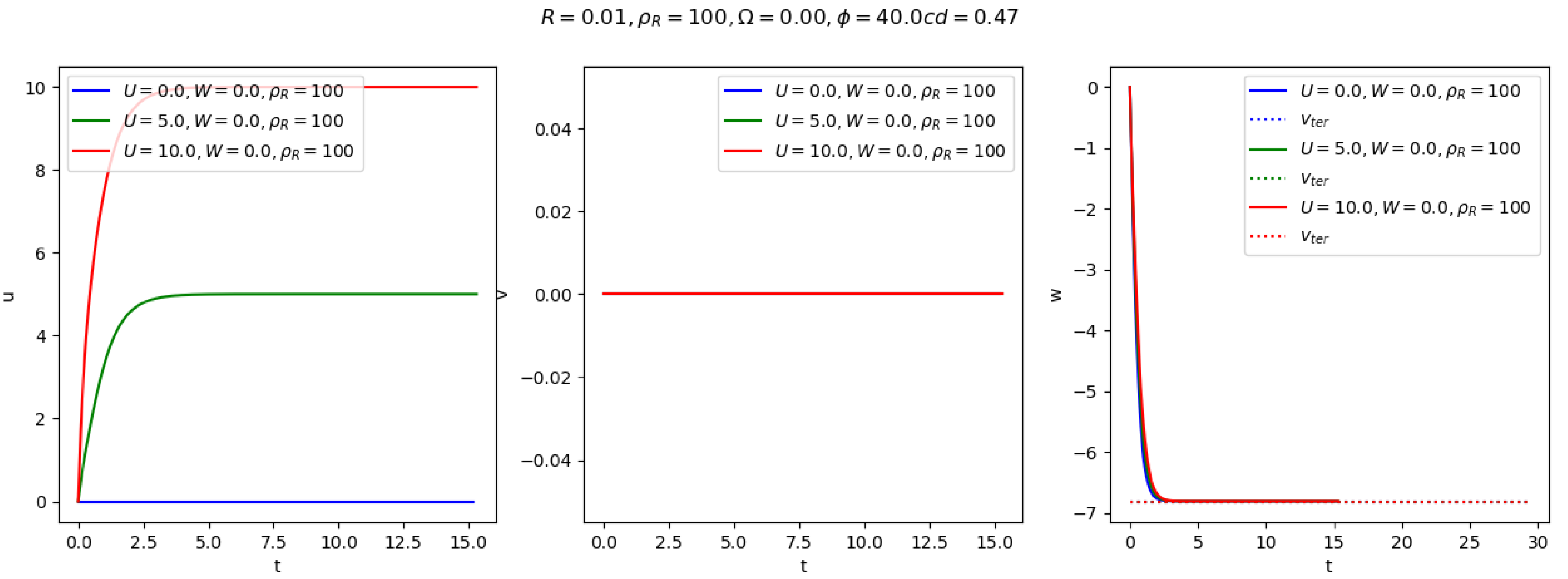
É fácil incluir o efeito do vento... (vento=$\vec{v}_0$)

$$\begin{cases} \dfrac{du}{dt} = -D(u - u_0) \\ \dfrac{dv}{dt} = -D(v - v_0) \\ \dfrac{dw}{dt} = -g - D(w - w_0) + I \end{cases} \wedge \begin{cases} D = \dfrac{c_D A \rho_F \frac{1}{2} |\vec{v} - \vec{v}_0|}{\rho_R V} \\ I = \dfrac{\rho_F}{\rho_R} g \end{cases}$$

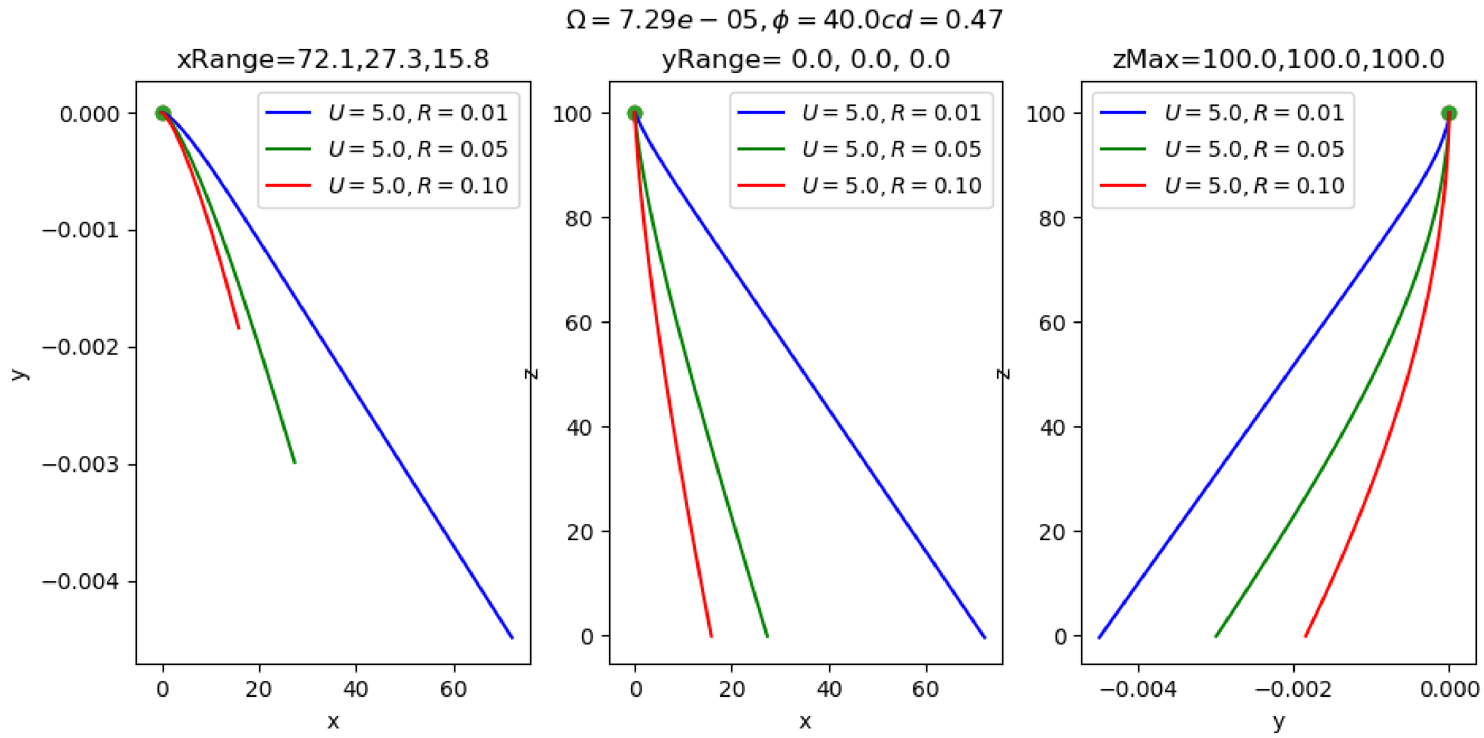
“Stokes” com vento R=1cm



“Stokes” com vento



Vento 5m/s



$\Omega = 7.29e-05, \phi = 40.0cd = 0.47$
xRange=72.1,27.3,15.8
yRange= 0.0, 0.0, 0.0
zMax=100.0,100.0,100.0
$U = 5.0, R = 0.01$
$U = 5.0, R = 0.05$
$U = 5.0, R = 0.10$
0.000
−0.001
−0.002
−0.003
−0.004
y
0
20
40
60
x
100
80
60
40
20
0
z
−0.004
−0.002
0.000