Modelação Ecológica - Ficha de trabalho 2

Tiago A. Marques

November 4, 2018

Exercício 1

Utilizando o programa R, e para o conjunto de dados dados21.txt, obtenha os seguintes elementos:

a. média, desvio-padrão, variância, mínimo, máximo e quartis das várias variáveis
b. gráficos de caixas e bigodes das variáveis originais e transformadas pela aplicação de logarítmos
c. gráficos de dispersão entre todas as variáveis
d. correlações entre todas as variáveis
e. identificação de tendências entre as várias variáveis
f. crie uma função que permita obter o número total de indivíduos e expresse-a em função da poluição para avaliar a existência de uma tendência

We begin by reading in the data

```
dados21 <- read.delim("C:/Users/tam2/Dropbox/Trabalho/DBA/ModelacaoEcologica/2018/TPs/TP3/dados21.txt")
```

We can check that the data read well

```
str(dados21)
```

```
## 'data.frame':    30 obs. of  10 variables:
##  $ station    : Factor w/ 30 levels "s1 ","s10 ","s11 ",..: 1 12 23 25 26 27 28 29 30 2 ...
##  $ a          : int  0 26 0 0 13 31 9 2 17 0 ...
##  $ b          : int  2 4 10 0 5 21 6 0 7 5 ...
##  $ c          : int  9 13 9 15 3 13 0 0 10 26 ...
##  $ d          : int  14 11 8 3 10 16 11 0 14 9 ...
##  $ e          : int  2 0 0 0 7 5 2 1 6 0 ...
##  $ Pollution  : num  4.8 2.8 5.4 8.2 3.9 2.6 4.6 5.1 3.9 10 ...
##  $ Depth      : int  72 75 59 64 61 94 53 61 68 69 ...
##  $ Temperature: num  3.5 2.5 2.7 2.9 3.1 3.5 2.9 3.3 3.4 3 ...
##  $ Sediment   : Factor w/ 3 levels "C","G","S": 3 1 1 3 1 2 3 1 1 3 ...
```

We have 30 observations for 10 variables. Of these, `station` and `sediment` are factors and hence it is not sensible to use functions that expect vectors of numbers (like `mean` or `var`) over these. Therefore, it is useful to create a second object that contains only the numeric variables, such that we can actually implement question 1a.

```
dados21NUM=dados21[,-c(1,10)]
```

1a

Then, we can calculate the functions required for each of the variables. The mean for a given variable (e.g. first variable) can be obtained as

```
mean(dados21NUM[,1])
```

```
## [1] 13.46667
```

but this would make it really cumbersome and lengthy to do it for all variables and all required summary statistics. One possible shortcut is function `summary`, which does a number of summary statistics for all variables at once

```
summary(dados21)
```

```
##     station        a               b                c        
##  s1     : 1   Min.   : 0.00   Min.   : 0.000   Min.   : 0.0  
##  s10    : 1   1st Qu.: 2.00   1st Qu.: 0.500   1st Qu.: 0.0  
##  s11    : 1   Median :12.00   Median : 6.500   Median : 8.5  
##  s12    : 1   Mean   :13.47   Mean   : 8.733   Mean   : 8.4  
##  s13    : 1   3rd Qu.:23.25   3rd Qu.:11.750   3rd Qu.:13.0  
##  s14    : 1   Max.   :42.00   Max.   :37.000   Max.   :33.0  
##  (Other):24                                                  
##        d               e            Pollution          Depth       
##  Min.   : 0.00   Min.   : 0.000   Min.   : 1.900   Min.   : 51.00  
##  1st Qu.: 7.25   1st Qu.: 0.000   1st Qu.: 2.800   1st Qu.: 61.00  
##  Median :10.00   Median : 1.500   Median : 4.300   Median : 73.50  
##  Mean   :10.90   Mean   : 2.967   Mean   : 4.517   Mean   : 74.43  
##  3rd Qu.:15.75   3rd Qu.: 6.000   3rd Qu.: 5.550   3rd Qu.: 84.75  
##  Max.   :25.00   Max.   :17.000   Max.   :10.000   Max.   :100.00  
##                                                                    
##   Temperature    Sediment
##  Min.   :2.500   C:11    
##  1st Qu.:2.900   G: 8    
##  Median :3.000   S:11    
##  Mean   :3.057           
##  3rd Qu.:3.300           
##  Max.   :3.600           
## 
```

Nonetheless, if we want to do it for each function at a time, we can actually leverage the R function `apply`. We round the numbers to 2 decimal places for easier viewing.

```
round(apply(dados21NUM,2,mean),2)
```

```
##           a           b           c           d           e   Pollution 
##       13.47        8.73        8.40       10.90        2.97        4.52 
##       Depth Temperature 
##       74.43        3.06
```

Then we do the the other functions, the variance

```
round(apply(dados21NUM,2,var),2)
```

```
##           a           b           c           d           e   Pollution 
##      157.64       83.44       73.63       44.44       15.69        4.58 
##       Depth Temperature 
##      243.84        0.08
```

the minimum

```
round(apply(dados21NUM,2,min),2)
```

```
##           a           b           c           d           e   Pollution 
##         0.0         0.0         0.0         0.0         0.0         1.9 
##       Depth Temperature 
##        51.0         2.5
```

the maximum

```
round(apply(dados21NUM,2,max),2)
```

```
##           a           b           c           d           e   Pollution 
##        42.0        37.0        33.0        25.0        17.0        10.0 
##       Depth Temperature 
##       100.0         3.6 
```

and finally the quartiles

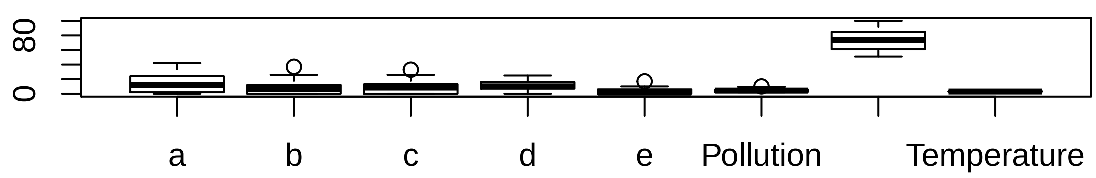
```
round(apply(dados21NUM,2,quantile,probs=c(0.25,0.50,0.75)),2)
```

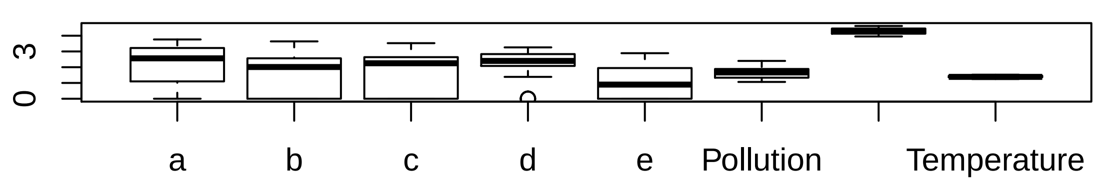
```
##         a     b    c     d   e Pollution Depth Temperature
## 25%  2.00  0.50  0.0  7.25 0.0      2.80 61.00         2.9
## 50% 12.00  6.50  8.5 10.00 1.5      4.30 73.50         3.0
## 75% 23.25 11.75 13.0 15.75 6.0      5.55 84.75         3.3
```

1b

To generate the boxplots is straightforward (note that to apply the log we used the standard trick to consider Y=log(X+1) to avoid problems if X=0)

```
par(mfrow=c(2,1))
boxplot(dados21NUM)
boxplot(log(dados21NUM+1))
```

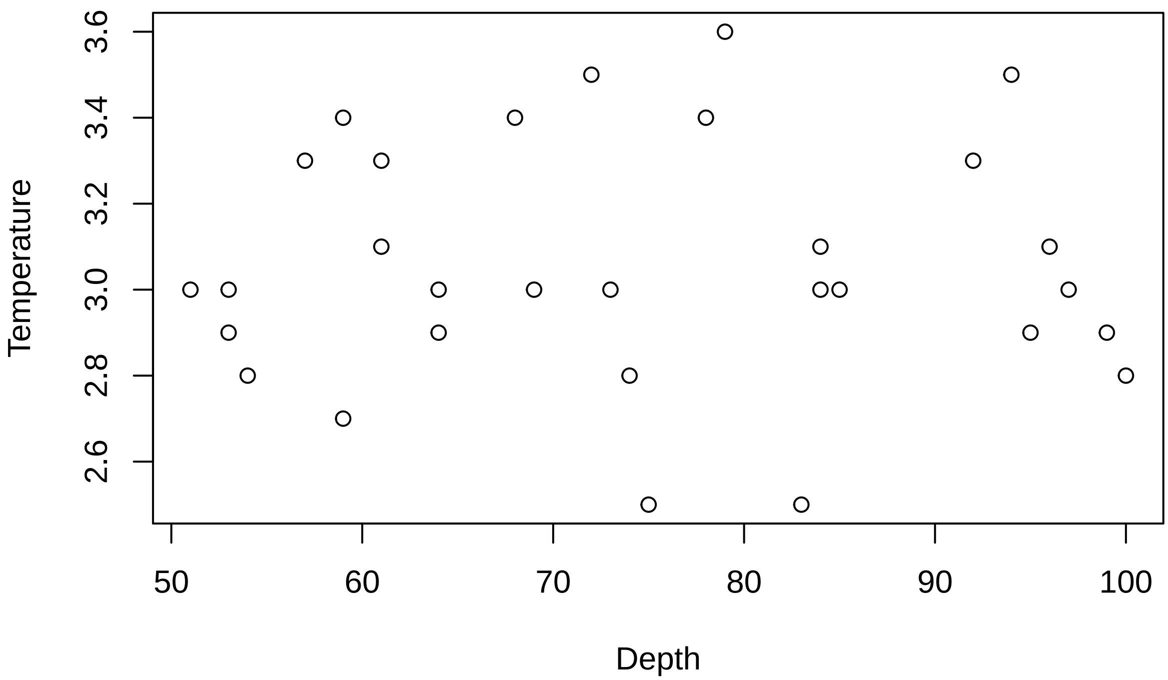




1c

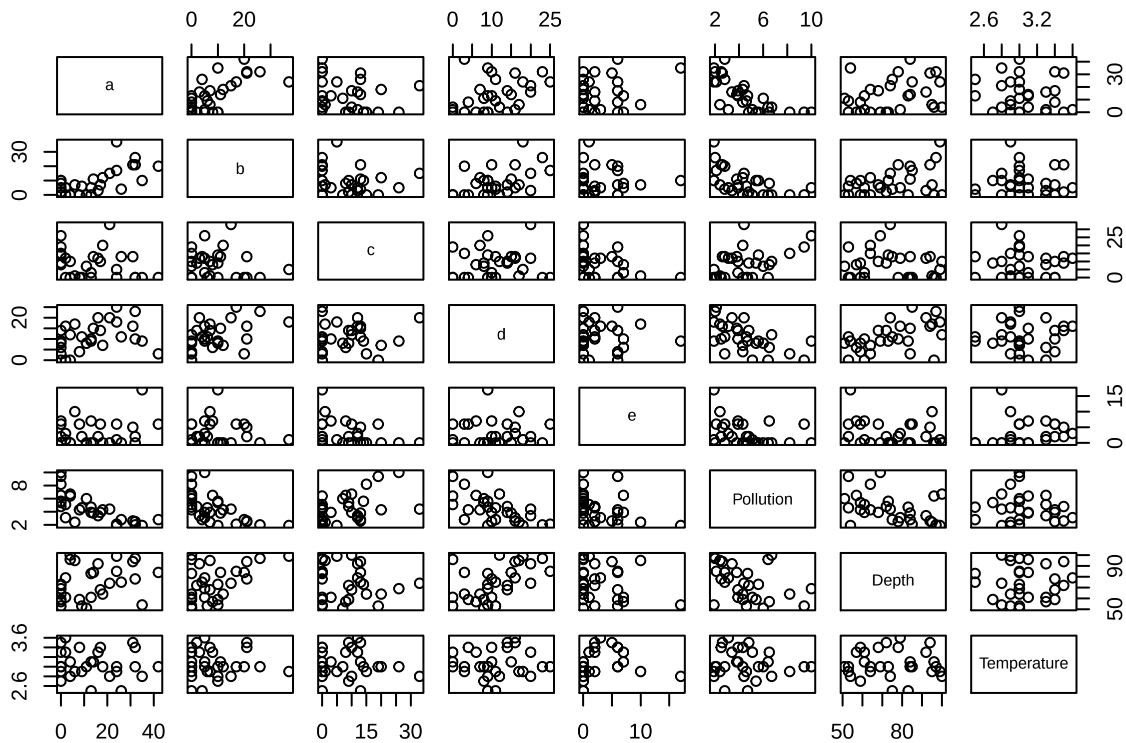
Again, while it is simple to create a dispersion plot for any two variables, e.g. for `Depth` and `Temperature`

```
with(dados21NUM,plot(Depth,Temperature))
```



this is easy to do for all combinations of pairs of variables using the function `pairs`

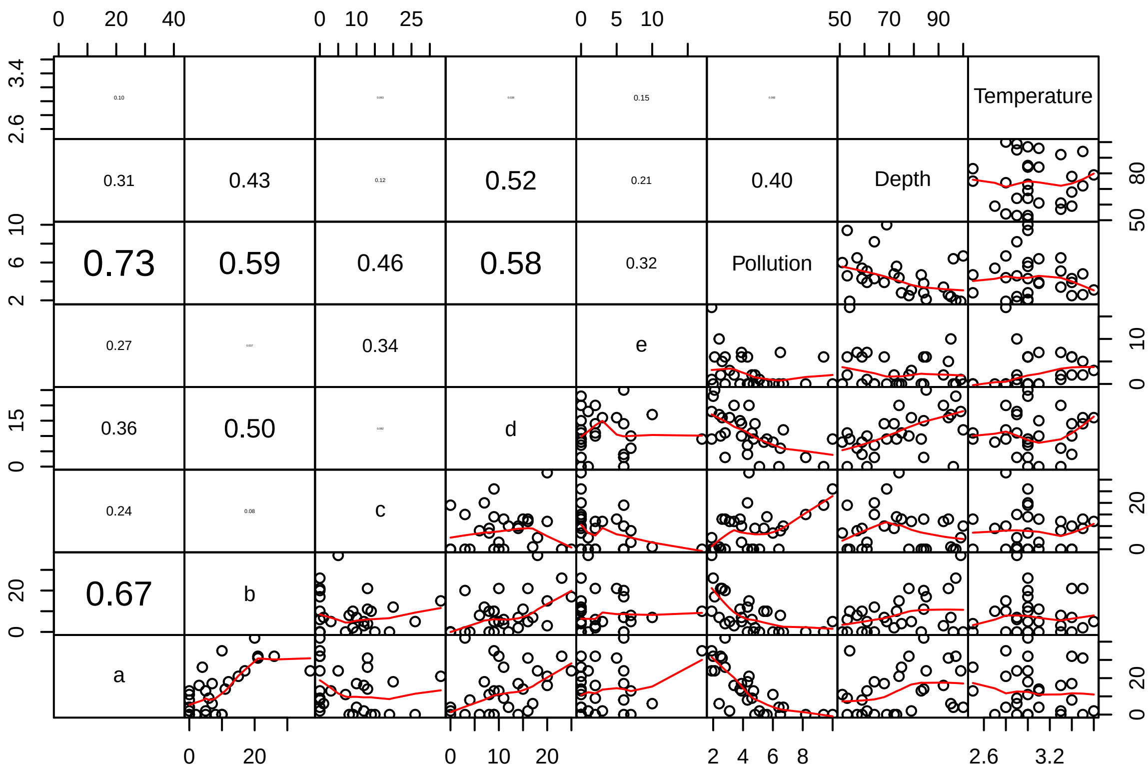
```
pairs(dados21NUM)
```



Note that the function pairs help provides additional functions to make the use of the function even more useful (we see immediately the correlation between variables and a smooth of one as a function of the other)

```
#from the ?pairs
## put histograms on the diagonal
panel.hist <- function(x, ...)
{
    usr <- par("usr"); on.exit(par(usr))
    par(usr = c(usr[1:2], 0, 1.5) )
    h <- hist(x, plot = FALSE)
    breaks <- h$breaks; nB <- length(breaks)
    y <- h$counts; y <- y/max(y)
    rect(breaks[-nB], 0, breaks[-1], y, col = "cyan", ...)
}
## put (absolute) correlations on the upper panels,
## with size proportional to the correlations.
panel.cor <- function(x, y, digits = 2, prefix = "", cex.cor, ...)
{
    usr <- par("usr"); on.exit(par(usr))
    par(usr = c(0, 1, 0, 1))
    r <- abs(cor(x, y))
    txt <- format(c(r, 0.123456789), digits = digits)[1]
    txt <- paste0(prefix, txt)
    if(missing(cex.cor)) cex.cor <- 0.8/strwidth(txt)
    text(0.5, 0.5, txt, cex = cex.cor * r)
}
pairs(dados21NUM, lower.panel = panel.smooth, upper.panel = panel.cor,
```

```
      gap=0, row1attop=FALSE)
```



As an example, we see that the largest correlation is between the abundance of species `a` and `Pollution`.

1d

Well, it seems like we did that using the function `pairs` above. However, since some of the correlations are so low that they do not show up in the plot above, we do it here again using function `cor` explicitly:

```
corrdados21NUM<-cor(dados21NUM)
```

We can present the output in a nice way using the function `kable` in package `knitr`

```
library(knitr)
kable(round(corrdados21NUM,2))
```

	a	b	c	d	e	Pollution	Depth	Temperature
a	1.00	0.67	-0.24	0.36	0.27	-0.73	0.31	-0.10
b	0.67	1.00	-0.08	0.50	0.04	-0.59	0.43	0.01
c	-0.24	-0.08	1.00	0.08	-0.34	0.46	-0.12	-0.06
d	0.36	0.50	0.08	1.00	0.00	-0.58	0.52	0.04
e	0.27	0.04	-0.34	0.00	1.00	-0.32	-0.21	0.15
Pollution	-0.73	-0.59	0.46	-0.58	-0.32	1.00	-0.40	-0.09
Depth	0.31	0.43	-0.12	0.52	-0.21	-0.40	1.00	0.00
Temperature	-0.10	0.01	-0.06	0.04	0.15	-0.09	0.00	1.00

1e

As before, we already looked at tendencies above!

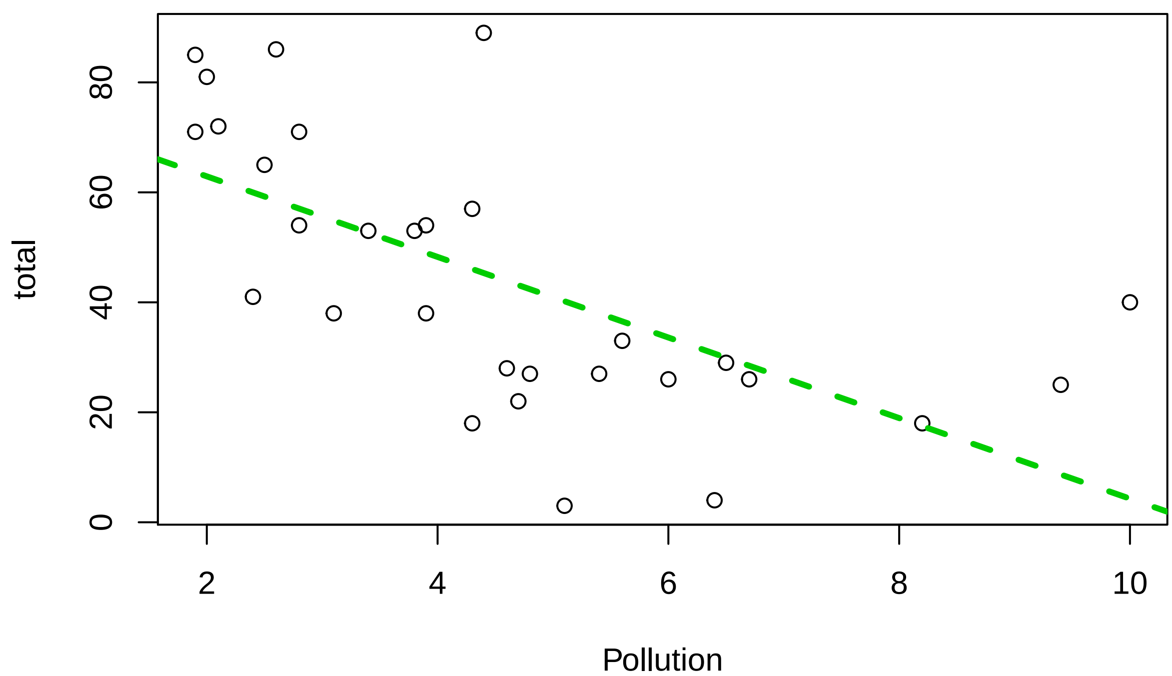
1f

We can get the total abundance (sum of species a,b,c,d,e) as

```
dados21$total=apply(dados21[,2:6],1,sum)
#equivalently
dados21$total2=rowSums(dados21[,2:6])
```

Finally, we can look at the total as a function of `Pollution`

```
with(dados21,plot(Pollution,total))
lm1<-lm(total~Pollution,data=dados21)
abline(lm1,lty=2,col=3,lwd=3)
```



We can look at the fitted model

```
summary(lm1)
```

```
##
## Call:
## lm(formula = total ~ Pollution, data = dados21)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
```

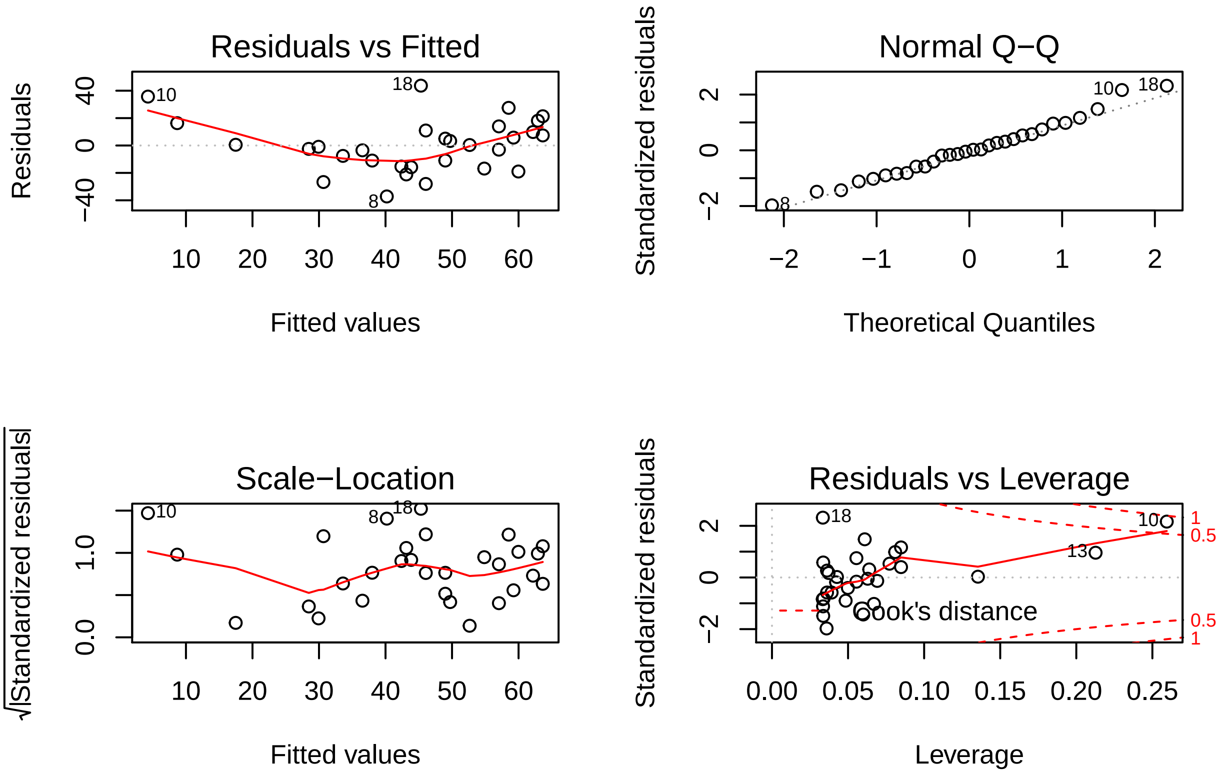
```
## -37.192 -14.292  -0.292  10.666  43.678
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept)   77.561      8.293   9.352 4.13e-10 ***
## Pollution     -7.327      1.664  -4.403 0.000142 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 19.19 on 28 degrees of freedom
## Multiple R-squared:  0.4091, Adjusted R-squared:  0.388
## F-statistic: 19.38 on 1 and 28 DF,  p-value: 0.0001416
```

The output is nonetheless not exactly printer friendly. A really nice way to report succinctly the result of a `lm` is via package `sigr`, using functions `wrapFTest` and `render`

```
#to use render
library(sigr)
lm1p <- wrapFTest(lm1)
#print(lm1p)
```

The F-Test over the regression is **F Test** summary: (R^2=0.4091, F(1,28)=19.38, p=0.0001416). Therefore we can conclude that `Pollution` is a relevant descriptor for the total abundance. As shown below by its diagnostic plots, the fitted model is not implausible, although there seems to be a tendency in the top left residual plot. With such a small sample size it is hard to say if that is relevant or not, but with more data that might be reason enough to explore additional models.

```
par(mfrow=c(2,2))
plot(lm1)
```



Exercício 2

A partir dos dados dados22.txt:

a. Construa um gráfico do peso em função do tempo para o exemplar 34.
b. Para os indivíduos sujeitos à dieta 4, construir um boxplot para cada tempo de experimentação.
c. Calcular a media do peso dos indivíduos sujeitos à dieta 4 para os vários tempos de experimentação.
d. Fazer o gráfico dos valores médios em função do tempo.
e. Faça o mesmo para a dieta 2 e junto os resultados no mesmo gráfico.
f. Insira legendas e título ao gráfico.
g. Efectue uma análise de variância aos dados e explore os resultados.
h. Efectue regressões lineares simples, para cada dieta administrada.

We begin by reading in the data

```
dados22 <- read.table("dados22.txt",header=TRUE,sep=" ")
```

We can check that the data read well

```
str(dados22)
```

```
## 'data.frame':    578 obs. of  4 variables:
##  $ weight: int  42 51 59 64 76 93 106 125 149 171 ...
##  $ Time  : int  0 2 4 6 8 10 12 14 16 18 ...
##  $ Chick : int  1 1 1 1 1 1 1 1 1 1 ...
##  $ Diet  : int  1 1 1 1 1 1 1 1 1 1 ...
```

We have 578 observations for 4 variables, all of which are numeric. We can take a look at a summary of the data

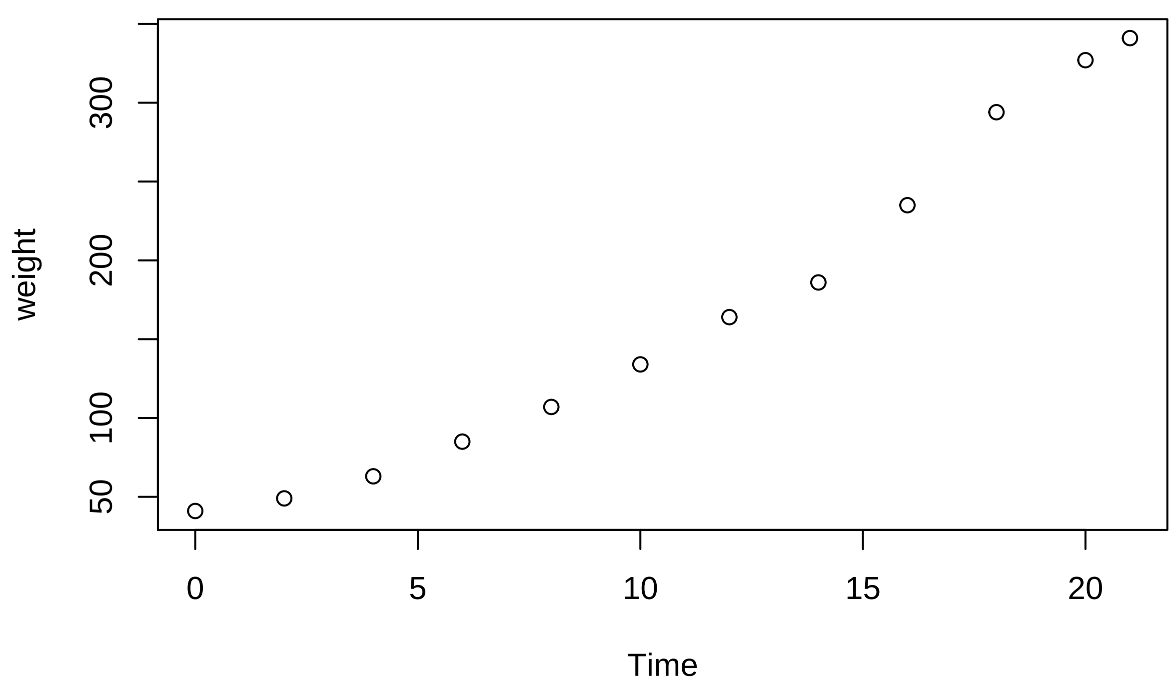
```
summary(dados22)
```

```
##      weight           Time           Chick            Diet      
##  Min.   : 35.0   Min.   : 0.00   Min.   : 1.00   Min.   :1.000  
##  1st Qu.: 63.0   1st Qu.: 4.00   1st Qu.:13.00   1st Qu.:1.000  
##  Median :103.0   Median :10.00   Median :26.00   Median :2.000  
##  Mean   :121.8   Mean   :10.72   Mean   :25.75   Mean   :2.235  
##  3rd Qu.:163.8   3rd Qu.:16.00   3rd Qu.:38.00   3rd Qu.:3.000  
##  Max.   :373.0   Max.   :21.00   Max.   :50.00   Max.   :4.000  
```

2a

We assume that the "exemplar" corresponds to `Chick`. Therefore, we can see its growth as a function of time in the following plot

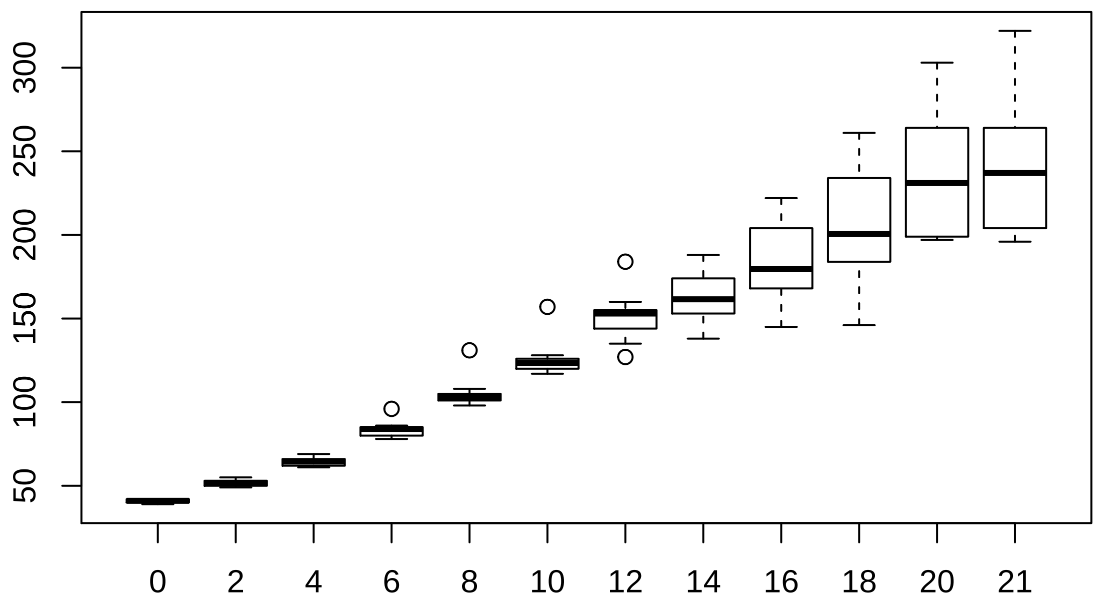
```
rowsIwant=dados22$Chick==34
with(dados22[rowsIwant,],plot(weight~Time))
```



2b

We assume that the boxplots desired are of `weight`, presumably the dependent variable

```r
with(dados22[dados22$Diet==4,],boxplot(weight~Time))
```



2c

This is simple using function `tapply`

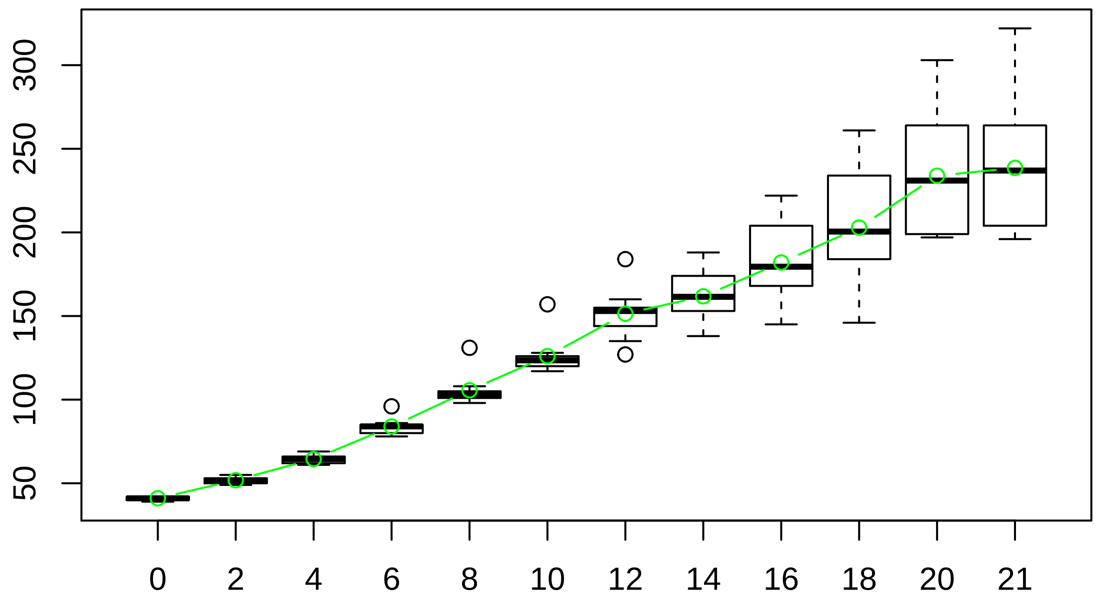
```r
means4byT=with(dados22[dados22$Diet==4,],tapply(X=weight,INDEX=Time,FUN=mean))
means4byT
```

```
##        0        2        4        6        8       10       12       14
##  41.0000  51.8000  64.5000  83.9000 105.6000 126.0000 151.4000 161.8000
##       16       18       20       21
## 182.0000 202.9000 233.8889 238.5556
```

We can overlay that over the previous plot (which answers 2d!)

2d

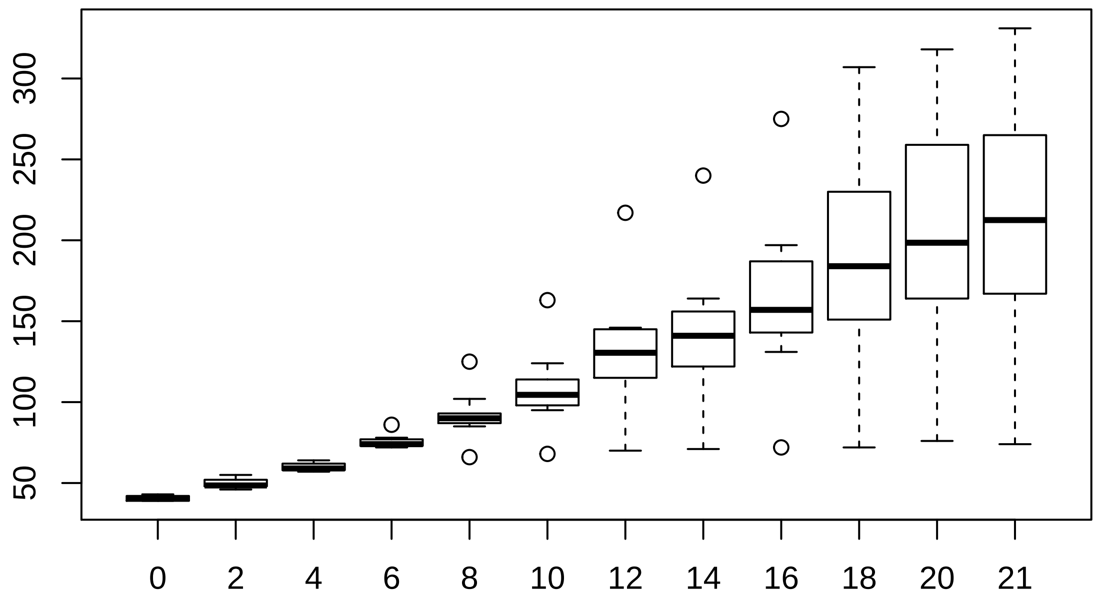
```r
with(dados22[dados22$Diet==4,],boxplot(weight~Time))
xss=as.numeric(names(means4byT))
yss=as.numeric(means4byT)
lines(1:length(xss),yss,type="b",col="green")
```



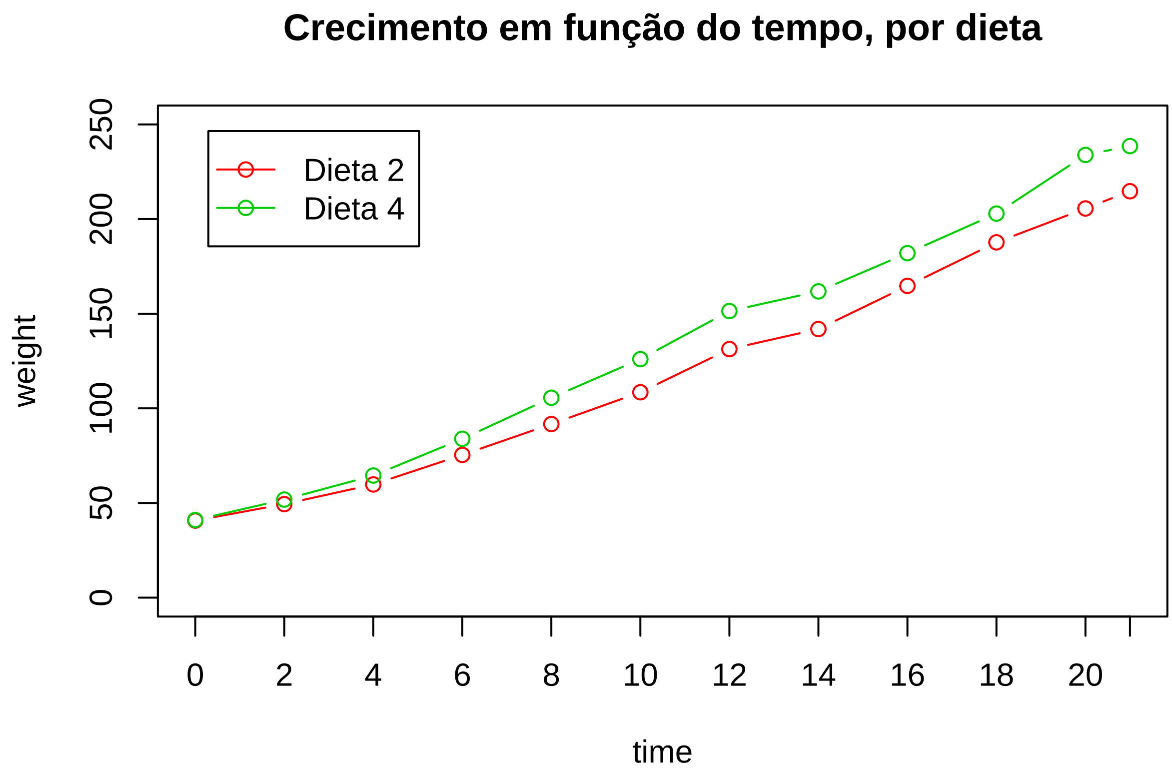
2e

Repeat for diet 2, and add details to plot (hence 2f is done here too!)

```
with(dados22[dados22$Diet==2,],boxplot(weight~Time))
```




```
means2byT=with(dados22[dados22$Diet==2,],tapply(X=weight,INDEX=Time,FUN=mean))
yss4=as.numeric(means4byT)
yss2=as.numeric(means2byT)
plot(xss,yss2,type="b",col=2,ylim=c(0,250),ylab="weight",xlab="time",
xaxt="n",main="Crecimento em função do tempo, por dieta")
axis(1,at=xss,labels = names(means4byT))
lines(xss,yss4,type="b",col=3)
legend("topleft",lty=1,pch=1,col=2:3,legend=c("Dieta 2","Dieta 4"),inset=0.05)
```



2g

While it is unclear if one should account for the repeated measurements within individual, here we ignore that component.

```
anova2Wdieta<-aov(weight~as.factor(Time)*as.factor(Diet),data=dados22)
summary(anova2Wdieta)
```

```
##                                  Df  Sum Sq Mean Sq F value   Pr(>F)
## as.factor(Time)                  11 2067050  187914 157.808  < 2e-16 ***
## as.factor(Diet)                   3  129721   43240  36.313  < 2e-16 ***
## as.factor(Time):as.factor(Diet)  33   86676    2627   2.206 0.000172 ***
## Residuals                       530  631110    1191
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

Since both factors are considered important, as is the interaction, we would now do post hoc comparisons using a Tukey test. The result, shown below, is not easy to interpret, and in fact borderline useless given the large number of comparisons. Nonetheless, for example, all diets seem to be different from each other, except diets 3 and 4 which we have no evidence to suggest are not equivalent.

Note that in a real life situation with data like this, time should be treated as a continuous covariate (leading to an ANCOVA like model), and the repeated measurements within chick would have to be accounted for. However here that is not the focus of the analysis so we keep it simple!

```
#post hoc tests
TukeyHSD(anova2Wdieta)
```

```
##   Tukey multiple comparisons of means
##     95% family-wise confidence level
##
## Fit: aov(formula = weight ~ as.factor(Time) * as.factor(Diet), data = dados22)
##
## $`as.factor(Time)`
##             diff        lwr       upr     p adj
## 2-0     8.160000 -14.496087  30.81609 0.9901305
## 4-0    18.899184  -3.872202  41.67057 0.2170973
## 6-0    33.246122  10.474736  56.01751 0.0001347
## 8-0    50.184898  27.413512  72.95628 0.0000000
## 10-0   66.776735  44.005349  89.54812 0.0000000
## 12-0   88.184898  65.413512 110.95628 0.0000000
## 14-0  102.752500  79.861628 125.64337 0.0000000
## 16-0  127.025106 104.010325 150.03989 0.0000000
## 18-0  149.131489 126.116708 172.14627 0.0000000
## 20-0  168.657391 145.514021 191.80076 0.0000000
## 21-0  177.628889 154.351972 200.90581 0.0000000
## 4-2    10.739184 -12.032202  33.51057 0.9262148
## 6-2    25.086122   2.314736  47.85751 0.0169248
## 8-2    42.024898  19.253512  64.79628 0.0000002
## 10-2   58.616735  35.845349  81.38812 0.0000000
## 12-2   80.024898  57.253512 102.79628 0.0000000
## 14-2   94.592500  71.701628 117.48337 0.0000000
## 16-2  118.865106  95.850325 141.87989 0.0000000
## 18-2  140.971489 117.956708 163.98627 0.0000000
## 20-2  160.497391 137.354021 183.64076 0.0000000
## 21-2  169.468889 146.191972 192.74581 0.0000000
## 6-4    14.346939  -8.539165  37.23304 0.6530645
## 8-4    31.285714   8.399610  54.17182 0.0005420
## 10-4   47.877551  24.991447  70.76366 0.0000000
## 12-4   69.285714  46.399610  92.17182 0.0000000
## 14-4   83.853316  60.848323 106.85831 0.0000000
## 16-4  108.125923  84.997631 131.25421 0.0000000
## 18-4  130.232306 107.104014 153.36060 0.0000000
## 20-4  149.758208 126.501954 173.01446 0.0000000
## 21-4  158.729705 135.340550 182.11886 0.0000000
## 8-6    16.938776  -5.947329  39.82488 0.3878023
## 10-6   33.530612  10.644508  56.41672 0.0001245
## 12-6   54.938776  32.052671  77.82488 0.0000000
## 14-6   69.506378  46.501384  92.51137 0.0000000
## 16-6   93.778984  70.650692 116.90728 0.0000000
## 18-6  115.885367  92.757075 139.01366 0.0000000
## 20-6  135.411269 112.155015 158.66752 0.0000000
## 21-6  144.382766 120.993611 167.77192 0.0000000
## 10-8   16.591837  -6.294267  39.47794 0.4215936
## 12-8   38.000000  15.113896  60.88610 0.0000050
## 14-8   52.567602  29.562608  75.57260 0.0000000
## 16-8   76.840208  53.711917  99.96850 0.0000000
## 18-8   98.946591  75.818300 122.07488 0.0000000
## 20-8  118.472493  95.216240 141.72875 0.0000000
## 21-8  127.443991 104.054835 150.83315 0.0000000
## 12-10  21.408163  -1.477941  44.29427 0.0921287
## 14-10  35.975765  12.970772  58.98076 0.0000257
```

```
## 16-10  60.248372  37.120080  83.37666 0.0000000
## 18-10  82.354755  59.226463 105.48305 0.0000000
## 20-10 101.880657  78.624403 125.13691 0.0000000
## 21-10 110.852154  87.462999 134.24131 0.0000000
## 14-12  14.567602  -8.437392  37.57260 0.6382474
## 16-12  38.840208  15.711917  61.96850 0.0000036
## 18-12  60.946591  37.818300  84.07488 0.0000000
## 20-12  80.472493  57.216240 103.72875 0.0000000
## 21-12  89.443991  66.054835 112.83315 0.0000000
## 16-14  24.272606   1.026663  47.51855 0.0317833
## 18-14  46.378989  23.133046  69.62493 0.0000000
## 20-14  65.904891  42.531631  89.27815 0.0000000
## 21-14  74.876389  51.370888  98.38189 0.0000000
## 18-16  22.106383  -1.261587  45.47435 0.0837035
## 20-16  41.632285  18.137659  65.12691 0.0000007
## 21-16  50.603783  26.977595  74.22997 0.0000000
## 20-18  19.525902  -3.968724  43.02053 0.2153293
## 21-18  28.497400   4.871212  52.12359 0.0048213
## 21-20   8.971498 -14.779969  32.72296 0.9854798
##
## $`as.factor(Diet)`
##          diff        lwr       upr     p adj
## 2-1 16.077232   5.984701 26.169763 0.0002723
## 3-1 36.410565  26.318034 46.503096 0.0000000
## 4-1 30.258687  20.110964 40.406411 0.0000000
## 3-2 20.333333   8.852141 31.814525 0.0000369
## 4-2 14.181455   2.651717 25.711194 0.0087378
## 4-3 -6.151878 -17.681616  5.377861 0.5155550
##
## $`as.factor(Time):as.factor(Diet)`
##                    diff          lwr          upr     p adj
## 2:1-0:1      5.85000000  -37.7785203  4.947852e+01 1.0000000
## 4:1-0:1     15.07368421  -29.1251678  5.927254e+01 1.0000000
## 6:1-0:1     25.38947368  -18.8093783  6.958833e+01 0.9807328
## 8:1-0:1     38.28421053   -5.9146415  8.248306e+01 0.2472043
## 10:1-0:1    51.65263158    7.4537796  9.585148e+01 0.0034876
## 12:1-0:1    67.12631579   22.9274638  1.113252e+02 0.0000027
## 14:1-0:1    81.98888889   37.1648453  1.268129e+02 0.0000000
## 16:1-0:1   103.24705882   57.7344335  1.487597e+02 0.0000000
## 18:1-0:1   117.54117647   72.0285512  1.630538e+02 0.0000000
## 20:1-0:1   129.01176471   83.4991394  1.745244e+02 0.0000000
## 21:1-0:1   136.35000000   90.0749661  1.826250e+02 0.0000000
## 0:2-0:1     -0.70000000  -54.1338065  5.273381e+01 1.0000000
## 2:2-0:1      8.00000000  -45.4338065  6.143381e+01 1.0000000
## 4:2-0:1     18.40000000  -35.0338065  7.183381e+01 0.9999999
## 6:2-0:1     34.00000000  -19.4338065  8.743381e+01 0.9171293
## 8:2-0:1     50.30000000   -3.1338065  1.037338e+02 0.1074719
## 10:2-0:1    67.10000000   13.6661935  1.205338e+02 0.0007038
## 12:2-0:1    89.90000000   36.4661935  1.433338e+02 0.0000001
## 14:2-0:1   100.50000000   47.0661935  1.539338e+02 0.0000000
## 16:2-0:1   123.30000000   69.8661935  1.767338e+02 0.0000000
## 18:2-0:1   146.30000000   92.8661935  1.997338e+02 0.0000000
## 20:2-0:1   164.20000000  110.7661935  2.176338e+02 0.0000000
## 21:2-0:1   173.30000000  119.8661935  2.267338e+02 0.0000000
```

```
## 0:3-0:1      -0.60000000  -54.0338065  5.283381e+01 1.0000000
## 2:3-0:1       9.00000000  -44.4338065  6.243381e+01 1.0000000
## 4:3-0:1      20.80000000  -32.6338065  7.423381e+01 0.9999976
## 6:3-0:1      36.50000000  -16.9338065  8.993381e+01 0.8185624
## 8:3-0:1      57.00000000    3.5661935  1.104338e+02 0.0188023
## 10:3-0:1     75.70000000   22.2661935  1.291338e+02 0.0000262
## 12:3-0:1    103.00000000   49.5661935  1.564338e+02 0.0000000
## 14:3-0:1    123.10000000   69.6661935  1.765338e+02 0.0000000
## 16:3-0:1    156.00000000  102.5661935  2.094338e+02 0.0000000
## 18:3-0:1    191.70000000  138.2661935  2.451338e+02 0.0000000
## 20:3-0:1    217.50000000  164.0661935  2.709338e+02 0.0000000
## 21:3-0:1    228.90000000  175.4661935  2.823338e+02 0.0000000
## 0:4-0:1      -0.40000000  -53.8338065  5.303381e+01 1.0000000
## 2:4-0:1      10.40000000  -43.0338065  6.383381e+01 1.0000000
## 4:4-0:1      23.10000000  -30.3338065  7.653381e+01 0.9999555
## 6:4-0:1      42.50000000  -10.9338065  9.593381e+01 0.4553911
## 8:4-0:1      64.20000000   10.7661935  1.176338e+02 0.0019387
## 10:4-0:1     84.60000000   31.1661935  1.380338e+02 0.0000006
## 12:4-0:1    110.00000000   56.5661935  1.634338e+02 0.0000000
## 14:4-0:1    120.40000000   66.9661935  1.738338e+02 0.0000000
## 16:4-0:1    140.60000000   87.1661935  1.940338e+02 0.0000000
## 18:4-0:1    161.50000000  108.0661935  2.149338e+02 0.0000000
## 20:4-0:1    192.48888889  137.1114036  2.478664e+02 0.0000000
## 21:4-0:1    197.15555556  141.7780703  2.525330e+02 0.0000000
## 4:1-2:1       9.22368421  -34.9751678  5.342254e+01 1.0000000
## 6:1-2:1      19.53947368  -24.6593783  6.373833e+01 0.9999206
## 8:1-2:1      32.43421053  -11.7646415  7.663306e+01 0.6658883
## 10:1-2:1     45.80263158    1.6037796  9.000148e+01 0.0297722
## 12:1-2:1     61.27631579   17.0774638  1.054752e+02 0.0000502
## 14:1-2:1     76.13888889   31.3148453  1.209629e+02 0.0000000
## 16:1-2:1     97.39705882   51.8844335  1.429097e+02 0.0000000
## 18:1-2:1    111.69117647   66.1785512  1.572038e+02 0.0000000
## 20:1-2:1    123.16176471   77.6491394  1.686744e+02 0.0000000
## 21:1-2:1    130.50000000   84.2249661  1.767750e+02 0.0000000
## 0:2-2:1      -6.55000000  -59.9838065  4.688381e+01 1.0000000
## 2:2-2:1       2.15000000  -51.2838065  5.558381e+01 1.0000000
## 4:2-2:1      12.55000000  -40.8838065  6.598381e+01 1.0000000
## 6:2-2:1      28.15000000  -25.2838065  8.158381e+01 0.9958736
## 8:2-2:1      44.45000000   -8.9838065  9.788381e+01 0.3399079
## 10:2-2:1     61.25000000    7.8161935  1.146838e+02 0.0051407
## 12:2-2:1     84.05000000   30.6161935  1.374838e+02 0.0000007
## 14:2-2:1     94.65000000   41.2161935  1.480838e+02 0.0000000
## 16:2-2:1    117.45000000   64.0161935  1.708838e+02 0.0000000
## 18:2-2:1    140.45000000   87.0161935  1.938838e+02 0.0000000
## 20:2-2:1    158.35000000  104.9161935  2.117838e+02 0.0000000
## 21:2-2:1    167.45000000  114.0161935  2.208838e+02 0.0000000
## 0:3-2:1      -6.45000000  -59.8838065  4.698381e+01 1.0000000
## 2:3-2:1       3.15000000  -50.2838065  5.658381e+01 1.0000000
## 4:3-2:1      14.95000000  -38.4838065  6.838381e+01 1.0000000
## 6:3-2:1      30.65000000  -22.7838065  8.408381e+01 0.9811796
## 8:3-2:1      51.15000000   -2.2838065  1.045838e+02 0.0881506
## 10:3-2:1     69.85000000   16.4161935  1.232838e+02 0.0002569
## 12:3-2:1     97.15000000   43.7161935  1.505838e+02 0.0000000
## 14:3-2:1    117.25000000   63.8161935  1.706838e+02 0.0000000
```

```
## 16:3-2:1   150.15000000   96.7161935  2.035838e+02 0.0000000
## 18:3-2:1   185.85000000  132.4161935  2.392838e+02 0.0000000
## 20:3-2:1   211.65000000  158.2161935  2.650838e+02 0.0000000
## 21:3-2:1   223.05000000  169.6161935  2.764838e+02 0.0000000
## 0:4-2:1     -6.25000000  -59.6838065  4.718381e+01 1.0000000
## 2:4-2:1      4.55000000  -48.8838065  5.798381e+01 1.0000000
## 4:4-2:1     17.25000000  -36.1838065  7.068381e+01 1.0000000
## 6:4-2:1     36.65000000  -16.7838065  9.008381e+01 0.8112071
## 8:4-2:1     58.35000000    4.9161935  1.117838e+02 0.0126378
## 10:4-2:1    78.75000000   25.3161935  1.321838e+02 0.0000074
## 12:4-2:1   104.15000000   50.7161935  1.575838e+02 0.0000000
## 14:4-2:1   114.55000000   61.1161935  1.679838e+02 0.0000000
## 16:4-2:1   134.75000000   81.3161935  1.881838e+02 0.0000000
## 18:4-2:1   155.65000000  102.2161935  2.090838e+02 0.0000000
## 20:4-2:1   186.63888889  131.2614036  2.420164e+02 0.0000000
## 21:4-2:1   191.30555556  135.9280703  2.466830e+02 0.0000000
## 6:1-4:1     10.31578947  -34.4461279  5.507771e+01 1.0000000
## 8:1-4:1     23.21052632  -21.5513911  6.797244e+01 0.9969808
## 10:1-4:1    36.57894737   -8.1829700  8.134086e+01 0.3846014
## 12:1-4:1    52.05263158    7.2907142  9.681455e+01 0.0038580
## 14:1-4:1    66.91520468   21.5358524  1.122946e+02 0.0000073
## 16:1-4:1    88.17337461   42.1137416  1.342330e+02 0.0000000
## 18:1-4:1   102.46749226   56.4078592  1.485271e+02 0.0000000
## 20:1-4:1   113.93808050   67.8784475  1.599977e+02 0.0000000
## 21:1-4:1   121.27631579   74.4631820  1.680894e+02 0.0000000
## 0:2-4:1    -15.77368421  -69.6741704  3.812680e+01 1.0000000
## 2:2-4:1     -7.07368421  -60.9741704  4.682680e+01 1.0000000
## 4:2-4:1      3.32631579  -50.5741704  5.722680e+01 1.0000000
## 6:2-4:1     18.92631579  -34.9741704  7.282680e+01 0.9999999
## 8:2-4:1     35.22631579  -18.6741704  8.912680e+01 0.8862321
## 10:2-4:1    52.02631579   -1.8741704  1.059268e+02 0.0796044
## 12:2-4:1    74.82631579   20.9258296  1.287268e+02 0.0000483
## 14:2-4:1    85.42631579   31.5258296  1.393268e+02 0.0000006
## 16:2-4:1   108.22631579   54.3258296  1.621268e+02 0.0000000
## 18:2-4:1   131.22631579   77.3258296  1.851268e+02 0.0000000
## 20:2-4:1   149.12631579   95.2258296  2.030268e+02 0.0000000
## 21:2-4:1   158.22631579  104.3258296  2.121268e+02 0.0000000
## 0:3-4:1    -15.67368421  -69.5741704  3.822680e+01 1.0000000
## 2:3-4:1     -6.07368421  -59.9741704  4.782680e+01 1.0000000
## 4:3-4:1      5.72631579  -48.1741704  5.962680e+01 1.0000000
## 6:3-4:1     21.42631579  -32.4741704  7.532680e+01 0.9999956
## 8:3-4:1     41.92631579  -11.9741704  9.582680e+01 0.5149343
## 10:3-4:1    60.62631579    6.7258296  1.145268e+02 0.0073945
## 12:3-4:1    87.92631579   34.0258296  1.418268e+02 0.0000002
## 14:3-4:1   108.02631579   54.1258296  1.619268e+02 0.0000000
## 16:3-4:1   140.92631579   87.0258296  1.948268e+02 0.0000000
## 18:3-4:1   176.62631579  122.7258296  2.305268e+02 0.0000000
## 20:3-4:1   202.42631579  148.5258296  2.563268e+02 0.0000000
## 21:3-4:1   213.82631579  159.9258296  2.677268e+02 0.0000000
## 0:4-4:1    -15.47368421  -69.3741704  3.842680e+01 1.0000000
## 2:4-4:1     -4.67368421  -58.5741704  4.922680e+01 1.0000000
## 4:4-4:1      8.02631579  -45.8741704  6.192680e+01 1.0000000
## 6:4-4:1     27.42631579  -26.4741704  8.132680e+01 0.9979437
## 8:4-4:1     49.12631579   -4.7741704  1.030268e+02 0.1529269
```

```
## 10:4-4:1    69.52631579   15.6258296  1.234268e+02 0.0003625
## 12:4-4:1    94.92631579   41.0258296  1.488268e+02 0.0000000
## 14:4-4:1   105.32631579   51.4258296  1.592268e+02 0.0000000
## 16:4-4:1   125.52631579   71.6258296  1.794268e+02 0.0000000
## 18:4-4:1   146.42631579   92.5258296  2.003268e+02 0.0000000
## 20:4-4:1   177.41520468  121.5872851  2.332431e+02 0.0000000
## 21:4-4:1   182.08187135  126.2539517  2.379098e+02 0.0000000
## 8:1-6:1     12.89473684  -31.8671806  5.765665e+01 1.0000000
## 10:1-6:1    26.26315789  -18.4987595  7.102508e+01 0.9730835
## 12:1-6:1    41.73684211   -3.0250753  8.649876e+01 0.1197394
## 14:1-6:1    56.59941520   11.2200629  1.019788e+02 0.0008278
## 16:1-6:1    77.85758514   31.7979521  1.239172e+02 0.0000000
## 18:1-6:1    92.15170279   46.0920698  1.382113e+02 0.0000000
## 20:1-6:1   103.62229102   57.5626580  1.496819e+02 0.0000000
## 21:1-6:1   110.96052632   64.1473926  1.577737e+02 0.0000000
## 0:2-6:1    -26.08947368  -79.9899599  2.781101e+01 0.9993003
## 2:2-6:1    -17.38947368  -71.2899599  3.651101e+01 1.0000000
## 4:2-6:1     -6.98947368  -60.8899599  4.691101e+01 1.0000000
## 6:2-6:1      8.61052632  -45.2899599  6.251101e+01 1.0000000
## 8:2-6:1     24.91052632  -28.9899599  7.881101e+01 0.9997613
## 10:2-6:1    41.71052632  -12.1899599  9.561101e+01 0.5287267
## 12:2-6:1    64.51052632   10.6100401  1.184110e+02 0.0021091
## 14:2-6:1    75.11052632   21.2100401  1.290110e+02 0.0000431
## 16:2-6:1    97.91052632   44.0100401  1.518110e+02 0.0000000
## 18:2-6:1   120.91052632   67.0100401  1.748110e+02 0.0000000
## 20:2-6:1   138.81052632   84.9100401  1.927110e+02 0.0000000
## 21:2-6:1   147.91052632   94.0100401  2.018110e+02 0.0000000
## 0:3-6:1    -25.98947368  -79.8899599  2.791101e+01 0.9993583
## 2:3-6:1    -16.38947368  -70.2899599  3.751101e+01 1.0000000
## 4:3-6:1     -4.58947368  -58.4899599  4.931101e+01 1.0000000
## 6:3-6:1     11.11052632  -42.7899599  6.501101e+01 1.0000000
## 8:3-6:1     31.61052632  -22.2899599  8.551101e+01 0.9732730
## 10:3-6:1    50.31052632   -3.5899599  1.042110e+02 0.1183443
## 12:3-6:1    77.61052632   23.7100401  1.315110e+02 0.0000158
## 14:3-6:1    97.71052632   43.8100401  1.516110e+02 0.0000000
## 16:3-6:1   130.61052632   76.7100401  1.845110e+02 0.0000000
## 18:3-6:1   166.31052632  112.4100401  2.202110e+02 0.0000000
## 20:3-6:1   192.11052632  138.2100401  2.460110e+02 0.0000000
## 21:3-6:1   203.51052632  149.6100401  2.574110e+02 0.0000000
## 0:4-6:1    -25.78947368  -79.6899599  2.811101e+01 0.9994616
## 2:4-6:1    -14.98947368  -68.8899599  3.891101e+01 1.0000000
## 4:4-6:1     -2.28947368  -56.1899599  5.161101e+01 1.0000000
## 6:4-6:1     17.11052632  -36.7899599  7.101101e+01 1.0000000
## 8:4-6:1     38.81052632  -15.0899599  9.271101e+01 0.7108857
## 10:4-6:1    59.21052632    5.3100401  1.131110e+02 0.0113823
## 12:4-6:1    84.61052632   30.7100401  1.385110e+02 0.0000008
## 14:4-6:1    95.01052632   41.1100401  1.489110e+02 0.0000000
## 16:4-6:1   115.21052632   61.3100401  1.691110e+02 0.0000000
## 18:4-6:1   136.11052632   82.2100401  1.900110e+02 0.0000000
## 20:4-6:1   167.09941520  111.2714956  2.229273e+02 0.0000000
## 21:4-6:1   171.76608187  115.9381622  2.275940e+02 0.0000000
## 10:1-8:1    13.36842105  -31.3934963  5.813034e+01 1.0000000
## 12:1-8:1    28.84210526  -15.9198121  7.360402e+01 0.9035093
## 14:1-8:1    43.70467836   -1.6746740  8.908403e+01 0.0818210
```

```
## 16:1-8:1    64.96284830   18.9032153  1.110225e+02 0.0000300
## 18:1-8:1    79.25696594   33.1973329  1.253166e+02 0.0000000
## 20:1-8:1    90.72755418   44.6679212  1.367872e+02 0.0000000
## 21:1-8:1    98.06578947   51.2526557  1.448789e+02 0.0000000
## 0:2-8:1    -38.98421053  -92.8846967  1.491628e+01 0.7005478
## 2:2-8:1    -30.28421053  -84.1846967  2.361628e+01 0.9866621
## 4:2-8:1    -19.88421053  -73.7846967  3.401628e+01 0.9999995
## 6:2-8:1     -4.28421053  -58.1846967  4.961628e+01 1.0000000
## 8:2-8:1     12.01578947  -41.8846967  6.591628e+01 1.0000000
## 10:2-8:1    28.81578947  -25.0846967  8.271628e+01 0.9945346
## 12:2-8:1    51.61578947   -2.2846967  1.055163e+02 0.0877560
## 14:2-8:1    62.21578947    8.3153033  1.161163e+02 0.0044801
## 16:2-8:1    85.01578947   31.1153033  1.389163e+02 0.0000007
## 18:2-8:1   108.01578947   54.1153033  1.619163e+02 0.0000000
## 20:2-8:1   125.91578947   72.0153033  1.798163e+02 0.0000000
## 21:2-8:1   135.01578947   81.1153033  1.889163e+02 0.0000000
## 0:3-8:1    -38.88421053  -92.7846967  1.501628e+01 0.7065148
## 2:3-8:1    -29.28421053  -83.1846967  2.461628e+01 0.9926260
## 4:3-8:1    -17.48421053  -71.3846967  3.641628e+01 1.0000000
## 6:3-8:1     -1.78421053  -55.6846967  5.211628e+01 1.0000000
## 8:3-8:1     18.71578947  -35.1846967  7.261628e+01 0.9999999
## 10:3-8:1    37.41578947  -16.4846967  9.131628e+01 0.7886815
## 12:3-8:1    64.71578947   10.8153033  1.186163e+02 0.0019683
## 14:3-8:1    84.81578947   30.9153033  1.387163e+02 0.0000007
## 16:3-8:1   117.71578947   63.8153033  1.716163e+02 0.0000000
## 18:3-8:1   153.41578947   99.5153033  2.073163e+02 0.0000000
## 20:3-8:1   179.21578947  125.3153033  2.331163e+02 0.0000000
## 21:3-8:1   190.61578947  136.7153033  2.445163e+02 0.0000000
## 0:4-8:1    -38.68421053  -92.5846967  1.521628e+01 0.7183259
## 2:4-8:1    -27.88421053  -81.7846967  2.601628e+01 0.9971184
## 4:4-8:1    -15.18421053  -69.0846967  3.871628e+01 1.0000000
## 6:4-8:1      4.21578947  -49.6846967  5.811628e+01 1.0000000
## 8:4-8:1     25.91578947  -27.9846967  7.981628e+01 0.9993982
## 10:4-8:1    46.31578947   -7.5846967  1.002163e+02 0.2644119
## 12:4-8:1    71.71578947   17.8153033  1.256163e+02 0.0001605
## 14:4-8:1    82.11578947   28.2153033  1.360163e+02 0.0000024
## 16:4-8:1   102.31578947   48.4153033  1.562163e+02 0.0000000
## 18:4-8:1   123.21578947   69.3153033  1.771163e+02 0.0000000
## 20:4-8:1   154.20467836   98.3767587  2.100326e+02 0.0000000
## 21:4-8:1   158.87134503  103.0434254  2.146993e+02 0.0000000
## 12:1-10:1   15.47368421  -29.2882332  6.023560e+01 0.9999999
## 14:1-10:1   30.33625731  -15.0430950  7.571561e+01 0.8542487
## 16:1-10:1   51.59442724    5.5347942  9.765406e+01 0.0079852
## 18:1-10:1   65.88854489   19.8289119  1.119482e+02 0.0000194
## 20:1-10:1   77.35913313   31.2995001  1.234188e+02 0.0000001
## 21:1-10:1   84.69736842   37.8842347  1.315105e+02 0.0000000
## 0:2-10:1   -52.35263158 -106.2531178  1.547855e+00 0.0735844
## 2:2-10:1   -43.65263158  -97.5531178  1.024785e+01 0.4078190
## 4:2-10:1   -33.25263158  -87.1531178  2.064785e+01 0.9444282
## 6:2-10:1   -17.65263158  -71.5531178  3.624785e+01 1.0000000
## 8:2-10:1    -1.35263158  -55.2531178  5.254785e+01 1.0000000
## 10:2-10:1   15.44736842  -38.4531178  6.934785e+01 1.0000000
## 12:2-10:1   38.24736842  -15.6531178  9.214785e+01 0.7435003
## 14:2-10:1   48.84736842   -5.0531178  1.027479e+02 0.1620986
```

```
## 16:2-10:1   71.64736842   17.7468822  1.255479e+02 0.0001647
## 18:2-10:1   94.64736842   40.7468822  1.485479e+02 0.0000000
## 20:2-10:1  112.54736842   58.6468822  1.664479e+02 0.0000000
## 21:2-10:1  121.64736842   67.7468822  1.755479e+02 0.0000000
## 0:3-10:1   -52.25263158 -106.1531178  1.647855e+00 0.0753872
## 2:3-10:1   -42.65263158  -96.5531178  1.124785e+01 0.4690034
## 4:3-10:1   -30.85263158  -84.7531178  2.304785e+01 0.9818142
## 6:3-10:1   -15.15263158  -69.0531178  3.874785e+01 1.0000000
## 8:3-10:1     5.34736842  -48.5531178  5.924785e+01 1.0000000
## 10:3-10:1   24.04736842  -29.8531178  7.794785e+01 0.9998999
## 12:3-10:1   51.34736842   -2.5531178  1.052479e+02 0.0934501
## 14:3-10:1   71.44736842   17.5468822  1.253479e+02 0.0001776
## 16:3-10:1  104.34736842   50.4468822  1.582479e+02 0.0000000
## 18:3-10:1  140.04736842   86.1468822  1.939479e+02 0.0000000
## 20:3-10:1  165.84736842  111.9468822  2.197479e+02 0.0000000
## 21:3-10:1  177.24736842  123.3468822  2.311479e+02 0.0000000
## 0:4-10:1   -52.05263158 -105.9531178  1.847855e+00 0.0791041
## 2:4-10:1   -41.25263158  -95.1531178  1.264785e+01 0.5581004
## 4:4-10:1   -28.55263158  -82.4531178  2.534785e+01 0.9954107
## 6:4-10:1    -9.15263158  -63.0531178  4.474785e+01 1.0000000
## 8:4-10:1    12.54736842  -41.3531178  6.644785e+01 1.0000000
## 10:4-10:1   32.94736842  -20.9531178  8.684785e+01 0.9510126
## 12:4-10:1   58.34736842    4.4468822  1.122479e+02 0.0146992
## 14:4-10:1   68.74736842   14.8468822  1.226479e+02 0.0004811
## 16:4-10:1   88.94736842   35.0468822  1.428479e+02 0.0000001
## 18:4-10:1  109.84736842   55.9468822  1.637479e+02 0.0000000
## 20:4-10:1  140.83625731   85.0083377  1.966642e+02 0.0000000
## 21:4-10:1  145.50292398   89.6750043  2.013308e+02 0.0000000
## 14:1-12:1   14.86257310  -30.5167792  6.024193e+01 1.0000000
## 16:1-12:1   36.12074303   -9.9388900  8.218038e+01 0.4931076
## 18:1-12:1   50.41486068    4.3552277  9.647449e+01 0.0121315
## 20:1-12:1   61.88544892   15.8258159  1.079451e+02 0.0001228
## 21:1-12:1   69.22368421   22.4105505  1.160368e+02 0.0000066
## 0:2-12:1   -67.82631579 -121.7268020 -1.392583e+01 0.0006695
## 2:2-12:1   -59.12631579 -113.0268020 -5.225830e+00 0.0116727
## 4:2-12:1   -48.72631579 -102.6268020  5.174170e+00 0.1662048
## 6:2-12:1   -33.12631579  -87.0268020  2.077417e+01 0.9472260
## 8:2-12:1   -16.82631579  -70.7268020  3.707417e+01 1.0000000
## 10:2-12:1   -0.02631579  -53.9268020  5.387417e+01 1.0000000
## 12:2-12:1   22.77368421  -31.1268020  7.667417e+01 0.9999758
## 14:2-12:1   33.37368421  -20.5268020  8.727417e+01 0.9416483
## 16:2-12:1   56.17368421    2.2731980  1.100742e+02 0.0272869
## 18:2-12:1   79.17368421   25.2731980  1.330742e+02 0.0000083
## 20:2-12:1   97.07368421   43.1731980  1.509742e+02 0.0000000
## 21:2-12:1  106.17368421   52.2731980  1.600742e+02 0.0000000
## 0:3-12:1   -67.72631579 -121.6268020 -1.382583e+01 0.0006938
## 2:3-12:1   -58.12631579 -112.0268020 -4.225830e+00 0.0156800
## 4:3-12:1   -46.32631579 -100.2268020  7.574170e+00 0.2639145
## 6:3-12:1   -30.62631579  -84.5268020  2.327417e+01 0.9838899
## 8:3-12:1   -10.12631579  -64.0268020  4.377417e+01 1.0000000
## 10:3-12:1    8.57368421  -45.3268020  6.247417e+01 1.0000000
## 12:3-12:1   35.87368421  -18.0268020  8.977417e+01 0.8609321
## 14:3-12:1   55.97368421    2.0731980  1.098742e+02 0.0288304
## 16:3-12:1   88.87368421   34.9731980  1.427742e+02 0.0000001
```

```
## 18:3-12:1  124.57368421   70.6731980  1.784742e+02 0.0000000
## 20:3-12:1  150.37368421   96.4731980  2.042742e+02 0.0000000
## 21:3-12:1  161.77368421  107.8731980  2.156742e+02 0.0000000
## 0:4-12:1   -67.52631579 -121.4268020 -1.362583e+01 0.0007448
## 2:4-12:1   -56.72631579 -110.6268020 -2.825830e+00 0.0233986
## 4:4-12:1   -44.02631579  -97.9268020  9.874170e+00 0.3857981
## 6:4-12:1   -24.62631579  -78.5268020  2.927417e+01 0.9998193
## 8:4-12:1    -2.92631579  -56.8268020  5.097417e+01 1.0000000
## 10:4-12:1   17.47368421  -36.4268020  7.137417e+01 1.0000000
## 12:4-12:1   42.87368421  -11.0268020  9.677417e+01 0.4552362
## 14:4-12:1   53.27368421   -0.6268020  1.071742e+02 0.0586229
## 16:4-12:1   73.47368421   19.5731980  1.273742e+02 0.0000819
## 18:4-12:1   94.37368421   40.4731980  1.482742e+02 0.0000000
## 20:4-12:1  125.36257310   69.5346535  1.811905e+02 0.0000000
## 21:4-12:1  130.02923977   74.2013201  1.858572e+02 0.0000000
## 16:1-14:1   21.25816993  -25.4017289  6.791807e+01 0.9998313
## 18:1-14:1   35.55228758  -11.1076113  8.221219e+01 0.5698791
## 20:1-14:1   47.02287582    0.3629770  9.368277e+01 0.0448644
## 21:1-14:1   54.36111111    6.9572520  1.017650e+02 0.0050979
## 0:2-14:1   -82.68888889 -137.1032140 -2.827456e+01 0.0000026
## 2:2-14:1   -73.98888889 -128.4032140 -1.957456e+01 0.0000879
## 4:2-14:1   -63.58888889 -118.0032140 -9.174564e+00 0.0034898
## 6:2-14:1   -47.98888889 -102.4032140  6.425436e+00 0.2107617
## 8:2-14:1   -31.68888889  -86.1032140  2.272544e+01 0.9760324
## 10:2-14:1  -14.88888889  -69.3032140  3.952544e+01 1.0000000
## 12:2-14:1    7.91111111  -46.5032140  6.232544e+01 1.0000000
## 14:2-14:1   18.51111111  -35.9032140  7.292544e+01 1.0000000
## 16:2-14:1   41.31111111  -13.1032140  9.572544e+01 0.5793914
## 18:2-14:1   64.31111111    9.8967860  1.187254e+02 0.0027594
## 20:2-14:1   82.21111111   27.7967860  1.366254e+02 0.0000032
## 21:2-14:1   91.31111111   36.8967860  1.457254e+02 0.0000001
## 0:3-14:1   -82.58888889 -137.0032140 -2.817456e+01 0.0000027
## 2:3-14:1   -72.98888889 -127.4032140 -1.857456e+01 0.0001286
## 4:3-14:1   -61.18888889 -115.6032140 -6.774564e+00 0.0074295
## 6:3-14:1   -45.48888889  -99.9032140  8.925436e+00 0.3279203
## 8:3-14:1   -24.98888889  -79.4032140  2.942544e+01 0.9997952
## 10:3-14:1   -6.28888889  -60.7032140  4.812544e+01 1.0000000
## 12:3-14:1   21.01111111  -33.4032140  7.542544e+01 0.9999981
## 14:3-14:1   41.11111111  -13.3032140  9.552544e+01 0.5920879
## 16:3-14:1   74.01111111   19.5967860  1.284254e+02 0.0000872
## 18:3-14:1  109.71111111   55.2967860  1.641254e+02 0.0000000
## 20:3-14:1  135.51111111   81.0967860  1.899254e+02 0.0000000
## 21:3-14:1  146.91111111   92.4967860  2.013254e+02 0.0000000
## 0:4-14:1   -82.38888889 -136.8032140 -2.797456e+01 0.0000030
## 2:4-14:1   -71.58888889 -126.0032140 -1.717456e+01 0.0002171
## 4:4-14:1   -58.88888889 -113.3032140 -4.474564e+00 0.0147621
## 6:4-14:1   -39.48888889  -93.9032140  1.492544e+01 0.6926254
## 8:4-14:1   -17.78888889  -72.2032140  3.662544e+01 1.0000000
## 10:4-14:1    2.61111111  -51.8032140  5.702544e+01 1.0000000
## 12:4-14:1   28.01111111  -26.4032140  8.242544e+01 0.9973920
## 14:4-14:1   38.41111111  -16.0032140  9.282544e+01 0.7546567
## 16:4-14:1   58.61111111    4.1967860  1.130254e+02 0.0159966
## 18:4-14:1   79.51111111   25.0967860  1.339254e+02 0.0000098
## 20:4-14:1  110.50000000   54.1758224  1.668242e+02 0.0000000
```

```
## 21:4-14:1  115.16666667   58.8424891  1.714908e+02 0.0000000
## 18:1-16:1   14.29411765  -33.0276568  6.161589e+01 1.0000000
## 20:1-16:1   25.76470588  -21.5570686  7.308648e+01 0.9923345
## 21:1-16:1   33.10294118  -14.9525480  8.115843e+01 0.8033080
## 0:2-16:1  -103.94705882 -158.9299921 -4.896413e+01 0.0000000
## 2:2-16:1   -95.24705882 -150.2299921 -4.026413e+01 0.0000000
## 4:2-16:1   -84.84705882 -139.8299921 -2.986413e+01 0.0000015
## 6:2-16:1   -69.24705882 -124.2299921 -1.426413e+01 0.0006560
## 8:2-16:1   -52.94705882 -107.9299921  2.035874e+00 0.0819509
## 10:2-16:1  -36.14705882  -91.1299921  1.883587e+01 0.8783981
## 12:2-16:1  -13.34705882  -68.3299921  4.163587e+01 1.0000000
## 14:2-16:1   -2.74705882  -57.7299921  5.223587e+01 1.0000000
## 16:2-16:1   20.05294118  -34.9299921  7.503587e+01 0.9999997
## 18:2-16:1   43.05294118  -11.9299921  9.803587e+01 0.4971824
## 20:2-16:1   60.95294118    5.9700079  1.159359e+02 0.0096651
## 21:2-16:1   70.05294118   15.0700079  1.250359e+02 0.0004941
## 0:3-16:1  -103.84705882 -158.8299921 -4.886413e+01 0.0000000
## 2:3-16:1   -94.24705882 -149.2299921 -3.926413e+01 0.0000000
## 4:3-16:1   -82.44705882 -137.4299921 -2.746413e+01 0.0000041
## 6:3-16:1   -66.74705882 -121.7299921 -1.176413e+01 0.0015431
## 8:3-16:1   -46.24705882 -101.2299921  8.735874e+00 0.3132197
## 10:3-16:1  -27.54705882  -82.5299921  2.743587e+01 0.9985137
## 12:3-16:1   -0.24705882  -55.2299921  5.473587e+01 1.0000000
## 14:3-16:1   19.85294118  -35.1299921  7.483587e+01 0.9999998
## 16:3-16:1   52.75294118   -2.2299921  1.077359e+02 0.0857376
## 18:3-16:1   88.45294118   33.4700079  1.434359e+02 0.0000003
## 20:3-16:1  114.25294118   59.2700079  1.692359e+02 0.0000000
## 21:3-16:1  125.65294118   70.6700079  1.806359e+02 0.0000000
## 0:4-16:1  -103.64705882 -158.6299921 -4.866413e+01 0.0000000
## 2:4-16:1   -92.84705882 -147.8299921 -3.786413e+01 0.0000000
## 4:4-16:1   -80.14705882 -135.1299921 -2.516413e+01 0.0000106
## 6:4-16:1   -60.74705882 -115.7299921 -5.764126e+00 0.0102740
## 8:4-16:1   -39.04705882  -94.0299921  1.593587e+01 0.7417455
## 10:4-16:1  -18.64705882  -73.6299921  3.633587e+01 1.0000000
## 12:4-16:1    6.75294118  -48.2299921  6.173587e+01 1.0000000
## 14:4-16:1   17.15294118  -37.8299921  7.213587e+01 1.0000000
## 16:4-16:1   37.35294118  -17.6299921  9.233587e+01 0.8281052
## 18:4-16:1   58.25294118    3.2700079  1.132359e+02 0.0210227
## 20:4-16:1   89.24183007   32.3681353  1.461155e+02 0.0000008
## 21:4-16:1   93.90849673   37.0348020  1.507822e+02 0.0000001
## 20:1-18:1   11.47058824  -35.8511862  5.879236e+01 1.0000000
## 21:1-18:1   18.80882353  -29.2466656  6.686431e+01 0.9999972
## 0:2-18:1  -118.24117647 -173.2241098 -6.325824e+01 0.0000000
## 2:2-18:1  -109.54117647 -164.5241098 -5.455824e+01 0.0000000
## 4:2-18:1   -99.14117647 -154.1241098 -4.415824e+01 0.0000000
## 6:2-18:1   -83.54117647 -138.5241098 -2.855824e+01 0.0000026
## 8:2-18:1   -67.24117647 -122.2241098 -1.225824e+01 0.0013069
## 10:2-18:1  -50.44117647 -105.4241098  4.541757e+00 0.1428416
## 12:2-18:1  -27.64117647  -82.6241098  2.734176e+01 0.9984024
## 14:2-18:1  -17.04117647  -72.0241098  3.794176e+01 1.0000000
## 16:2-18:1    5.75882353  -49.2241098  6.074176e+01 1.0000000
## 18:2-18:1   28.75882353  -26.2241098  8.374176e+01 0.9964141
## 20:2-18:1   46.65882353   -8.3241098  1.016418e+02 0.2924997
## 21:2-18:1   55.75882353    0.7758902  1.107418e+02 0.0410295
```

```
## 0:3-18:1  -118.14117647 -173.1241098 -6.315824e+01 0.0000000
## 2:3-18:1  -108.54117647 -163.5241098 -5.355824e+01 0.0000000
## 4:3-18:1   -96.74117647 -151.7241098 -4.175824e+01 0.0000000
## 6:3-18:1   -81.04117647 -136.0241098 -2.605824e+01 0.0000074
## 8:3-18:1   -60.54117647 -115.5241098 -5.558243e+00 0.0109181
## 10:3-18:1  -41.84117647  -96.8241098  1.314176e+01 0.5732025
## 12:3-18:1  -14.54117647  -69.5241098  4.044176e+01 1.0000000
## 14:3-18:1    5.55882353  -49.4241098  6.054176e+01 1.0000000
## 16:3-18:1   38.45882353  -16.5241098  9.344176e+01 0.7735885
## 18:3-18:1   74.15882353   19.1758902  1.291418e+02 0.0001104
## 20:3-18:1   99.95882353   44.9758902  1.549418e+02 0.0000000
## 21:3-18:1  111.35882353   56.3758902  1.663418e+02 0.0000000
## 0:4-18:1  -117.94117647 -172.9241098 -6.295824e+01 0.0000000
## 2:4-18:1  -107.14117647 -162.1241098 -5.215824e+01 0.0000000
## 4:4-18:1   -94.44117647 -149.4241098 -3.945824e+01 0.0000000
## 6:4-18:1   -75.04117647 -130.0241098 -2.005824e+01 0.0000791
## 8:4-18:1   -53.34117647 -108.3241098  1.641757e+00 0.0746903
## 10:4-18:1  -32.94117647  -87.9241098  2.204176e+01 0.9631799
## 12:4-18:1   -7.54117647  -62.5241098  4.744176e+01 1.0000000
## 14:4-18:1    2.85882353  -52.1241098  5.784176e+01 1.0000000
## 16:4-18:1   23.05882353  -31.9241098  7.804176e+01 0.9999802
## 18:4-18:1   43.95882353  -11.0241098  9.894176e+01 0.4416763
## 20:4-18:1   74.94771242   18.0740177  1.318214e+02 0.0002078
## 21:4-18:1   79.61437908   22.7406843  1.364881e+02 0.0000377
## 21:1-20:1    7.33823529  -40.7172538  5.539372e+01 1.0000000
## 0:2-20:1  -129.71176471 -184.6946980 -7.472883e+01 0.0000000
## 2:2-20:1  -121.01176471 -175.9946980 -6.602883e+01 0.0000000
## 4:2-20:1  -110.61176471 -165.5946980 -5.562883e+01 0.0000000
## 6:2-20:1   -95.01176471 -149.9946980 -4.002883e+01 0.0000000
## 8:2-20:1   -78.71176471 -133.6946980 -2.372883e+01 0.0000189
## 10:2-20:1  -61.91176471 -116.8946980 -6.928831e+00 0.0072435
## 12:2-20:1  -39.11176471  -94.0946980  1.587117e+01 0.7381369
## 14:2-20:1  -28.51176471  -83.4946980  2.647117e+01 0.9969781
## 16:2-20:1   -5.71176471  -60.6946980  4.927117e+01 1.0000000
## 18:2-20:1   17.28823529  -37.6946980  7.227117e+01 1.0000000
## 20:2-20:1   35.18823529  -19.7946980  9.017117e+01 0.9110631
## 21:2-20:1   44.28823529  -10.6946980  9.927117e+01 0.4220021
## 0:3-20:1  -129.61176471 -184.5946980 -7.462883e+01 0.0000000
## 2:3-20:1  -120.01176471 -174.9946980 -6.502883e+01 0.0000000
## 4:3-20:1  -108.21176471 -163.1946980 -5.322883e+01 0.0000000
## 6:3-20:1   -92.51176471 -147.4946980 -3.752883e+01 0.0000001
## 8:3-20:1   -72.01176471 -126.9946980 -1.702883e+01 0.0002444
## 10:3-20:1  -53.31176471 -108.2946980  1.671169e+00 0.0752128
## 12:3-20:1  -26.01176471  -80.9946980  2.897117e+01 0.9995851
## 14:3-20:1   -5.91176471  -60.8946980  4.907117e+01 1.0000000
## 16:3-20:1   26.98823529  -27.9946980  8.197117e+01 0.9990454
## 18:3-20:1   62.68823529    7.7053020  1.176712e+02 0.0057085
## 20:3-20:1   88.48823529   33.5053020  1.434712e+02 0.0000003
## 21:3-20:1   99.88823529   44.9053020  1.548712e+02 0.0000000
## 0:4-20:1  -129.41176471 -184.3946980 -7.442883e+01 0.0000000
## 2:4-20:1  -118.61176471 -173.5946980 -6.362883e+01 0.0000000
## 4:4-20:1  -105.91176471 -160.8946980 -5.092883e+01 0.0000000
## 6:4-20:1   -86.51176471 -141.4946980 -3.152883e+01 0.0000007
## 8:4-20:1   -64.81176471 -119.7946980 -9.828831e+00 0.0029167
```

```
## 10:4-20:1  -44.41176471  -99.3946980  1.057117e+01 0.4147098
## 12:4-20:1  -19.01176471  -73.9946980  3.597117e+01 0.9999999
## 14:4-20:1   -8.61176471  -63.5946980  4.637117e+01 1.0000000
## 16:4-20:1   11.58823529  -43.3946980  6.657117e+01 1.0000000
## 18:4-20:1   32.48823529  -22.4946980  8.747117e+01 0.9700286
## 20:4-20:1   63.47712418    6.6034294  1.203508e+02 0.0085403
## 21:4-20:1   68.14379085   11.2700961  1.250175e+02 0.0020594
## 0:2-21:1  -137.05000000 -192.6656691 -8.143433e+01 0.0000000
## 2:2-21:1  -128.35000000 -183.9656691 -7.273433e+01 0.0000000
## 4:2-21:1  -117.95000000 -173.5656691 -6.233433e+01 0.0000000
## 6:2-21:1  -102.35000000 -157.9656691 -4.673433e+01 0.0000000
## 8:2-21:1   -86.05000000 -141.6656691 -3.043433e+01 0.0000014
## 10:2-21:1  -69.25000000 -124.8656691 -1.363433e+01 0.0008614
## 12:2-21:1  -46.45000000 -102.0656691  9.165669e+00 0.3301713
## 14:2-21:1  -35.85000000  -91.4656691  1.976567e+01 0.9030499
## 16:2-21:1  -13.05000000  -68.6656691  4.256567e+01 1.0000000
## 18:2-21:1    9.95000000  -45.6656691  6.556567e+01 1.0000000
## 20:2-21:1   27.85000000  -27.7656691  8.346567e+01 0.9985296
## 21:2-21:1   36.95000000  -18.6656691  9.256567e+01 0.8635387
## 0:3-21:1  -136.95000000 -192.5656691 -8.133433e+01 0.0000000
## 2:3-21:1  -127.35000000 -182.9656691 -7.173433e+01 0.0000000
## 4:3-21:1  -115.55000000 -171.1656691 -5.993433e+01 0.0000000
## 6:3-21:1   -99.85000000 -155.4656691 -4.423433e+01 0.0000000
## 8:3-21:1   -79.35000000 -134.9656691 -2.373433e+01 0.0000210
## 10:3-21:1  -60.65000000 -116.2656691 -5.034331e+00 0.0129415
## 12:3-21:1  -33.35000000  -88.9656691  2.226567e+01 0.9626976
## 14:3-21:1  -13.25000000  -68.8656691  4.236567e+01 1.0000000
## 16:3-21:1   19.65000000  -35.9656691  7.526567e+01 0.9999999
## 18:3-21:1   55.35000000   -0.2656691  1.109657e+02 0.0533996
## 20:3-21:1   81.15000000   25.5343309  1.367657e+02 0.0000103
## 21:3-21:1   92.55000000   36.9343309  1.481657e+02 0.0000001
## 0:4-21:1  -136.75000000 -192.3656691 -8.113433e+01 0.0000000
## 2:4-21:1  -125.95000000 -181.5656691 -7.033433e+01 0.0000000
## 4:4-21:1  -113.25000000 -168.8656691 -5.763433e+01 0.0000000
## 6:4-21:1   -93.85000000 -149.4656691 -3.823433e+01 0.0000000
## 8:4-21:1   -72.15000000 -127.7656691 -1.653433e+01 0.0003131
## 10:4-21:1  -51.75000000 -107.3656691  3.865669e+00 0.1225240
## 12:4-21:1  -26.35000000  -81.9656691  2.926567e+01 0.9995706
## 14:4-21:1  -15.95000000  -71.5656691  3.966567e+01 1.0000000
## 16:4-21:1    4.25000000  -51.3656691  5.986567e+01 1.0000000
## 18:4-21:1   25.15000000  -30.4656691  8.076567e+01 0.9998597
## 20:4-21:1   56.13888889   -1.3467341  1.136245e+02 0.0686422
## 21:4-21:1   60.80555556    3.3199326  1.182912e+02 0.0215819
## 2:2-0:2      8.70000000  -53.0000452  7.040005e+01 1.0000000
## 4:2-0:2     19.10000000  -42.6000452  8.080005e+01 1.0000000
## 6:2-0:2     34.70000000  -27.0000452  9.640005e+01 0.9864408
## 8:2-0:2     51.00000000  -10.7000452  1.127000e+02 0.3557009
## 10:2-0:2    67.80000000    6.0999548  1.295000e+02 0.0113183
## 12:2-0:2    90.60000000   28.8999548  1.523000e+02 0.0000084
## 14:2-0:2   101.20000000   39.4999548  1.629000e+02 0.0000001
## 16:2-0:2   124.00000000   62.2999548  1.857000e+02 0.0000000
## 18:2-0:2   147.00000000   85.2999548  2.087000e+02 0.0000000
## 20:2-0:2   164.90000000  103.1999548  2.266000e+02 0.0000000
## 21:2-0:2   174.00000000  112.2999548  2.357000e+02 0.0000000
```

```
## 0:3-0:2      0.10000000  -61.6000452  6.180005e+01 1.0000000
## 2:3-0:2      9.70000000  -52.0000452  7.140005e+01 1.0000000
## 4:3-0:2     21.50000000  -40.2000452  8.320005e+01 0.9999999
## 6:3-0:2     37.20000000  -24.5000452  9.890005e+01 0.9596483
## 8:3-0:2     57.70000000   -4.0000452  1.194000e+02 0.1158482
## 10:3-0:2    76.40000000   14.6999548  1.381000e+02 0.0009815
## 12:3-0:2   103.70000000   41.9999548  1.654000e+02 0.0000001
## 14:3-0:2   123.80000000   62.0999548  1.855000e+02 0.0000000
## 16:3-0:2   156.70000000   94.9999548  2.184000e+02 0.0000000
## 18:3-0:2   192.40000000  130.6999548  2.541000e+02 0.0000000
## 20:3-0:2   218.20000000  156.4999548  2.799000e+02 0.0000000
## 21:3-0:2   229.60000000  167.8999548  2.913000e+02 0.0000000
## 0:4-0:2      0.30000000  -61.4000452  6.200005e+01 1.0000000
## 2:4-0:2     11.10000000  -50.6000452  7.280005e+01 1.0000000
## 4:4-0:2     23.80000000  -37.9000452  8.550005e+01 0.9999982
## 6:4-0:2     43.20000000  -18.5000452  1.049000e+02 0.7715806
## 8:4-0:2     64.90000000    3.1999548  1.266000e+02 0.0235987
## 10:4-0:2    85.30000000   23.5999548  1.470000e+02 0.0000545
## 12:4-0:2   110.70000000   48.9999548  1.724000e+02 0.0000000
## 14:4-0:2   121.10000000   59.3999548  1.828000e+02 0.0000000
## 16:4-0:2   141.30000000   79.5999548  2.030000e+02 0.0000000
## 18:4-0:2   162.20000000  100.4999548  2.239000e+02 0.0000000
## 20:4-0:2   193.18888889  129.7981185  2.565797e+02 0.0000000
## 21:4-0:2   197.85555556  134.4647852  2.612463e+02 0.0000000
## 4:2-2:2     10.40000000  -51.3000452  7.210005e+01 1.0000000
## 6:2-2:2     26.00000000  -35.7000452  8.770005e+01 0.9999775
## 8:2-2:2     42.30000000  -19.4000452  1.040000e+02 0.8120545
## 10:2-2:2    59.10000000   -2.6000452  1.208000e+02 0.0874810
## 12:2-2:2    81.90000000   20.1999548  1.436000e+02 0.0001711
## 14:2-2:2    92.50000000   30.7999548  1.542000e+02 0.0000042
## 16:2-2:2   115.30000000   53.5999548  1.770000e+02 0.0000000
## 18:2-2:2   138.30000000   76.5999548  2.000000e+02 0.0000000
## 20:2-2:2   156.20000000   94.4999548  2.179000e+02 0.0000000
## 21:2-2:2   165.30000000  103.5999548  2.270000e+02 0.0000000
## 0:3-2:2     -8.60000000  -70.3000452  5.310005e+01 1.0000000
## 2:3-2:2      1.00000000  -60.7000452  6.270005e+01 1.0000000
## 4:3-2:2     12.80000000  -48.9000452  7.450005e+01 1.0000000
## 6:3-2:2     28.50000000  -33.2000452  9.020005e+01 0.9997643
## 8:3-2:2     49.00000000  -12.7000452  1.107000e+02 0.4594473
## 10:3-2:2    67.70000000    5.9999548  1.294000e+02 0.0116181
## 12:3-2:2    95.00000000   33.2999548  1.567000e+02 0.0000016
## 14:3-2:2   115.10000000   53.3999548  1.768000e+02 0.0000000
## 16:3-2:2   148.00000000   86.2999548  2.097000e+02 0.0000000
## 18:3-2:2   183.70000000  121.9999548  2.454000e+02 0.0000000
## 20:3-2:2   209.50000000  147.7999548  2.712000e+02 0.0000000
## 21:3-2:2   220.90000000  159.1999548  2.826000e+02 0.0000000
## 0:4-2:2     -8.40000000  -70.1000452  5.330005e+01 1.0000000
## 2:4-2:2      2.40000000  -59.3000452  6.410005e+01 1.0000000
## 4:4-2:2     15.10000000  -46.6000452  7.680005e+01 1.0000000
## 6:4-2:2     34.50000000  -27.2000452  9.620005e+01 0.9877165
## 8:4-2:2     56.20000000   -5.5000452  1.179000e+02 0.1539137
## 10:4-2:2    76.60000000   14.8999548  1.383000e+02 0.0009233
## 12:4-2:2   102.00000000   40.2999548  1.637000e+02 0.0000001
## 14:4-2:2   112.40000000   50.6999548  1.741000e+02 0.0000000
```

```
## 16:4-2:2   132.60000000   70.8999548  1.943000e+02 0.0000000
## 18:4-2:2   153.50000000   91.7999548  2.152000e+02 0.0000000
## 20:4-2:2   184.48888889  121.0981185  2.478797e+02 0.0000000
## 21:4-2:2   189.15555556  125.7647852  2.525463e+02 0.0000000
## 6:2-4:2     15.60000000  -46.1000452  7.730005e+01 1.0000000
## 8:2-4:2     31.90000000  -29.8000452  9.360005e+01 0.9971529
## 10:2-4:2    48.70000000  -13.0000452  1.104000e+02 0.4758239
## 12:2-4:2    71.50000000    9.7999548  1.332000e+02 0.0041396
## 14:2-4:2    82.10000000   20.3999548  1.438000e+02 0.0001602
## 16:2-4:2   104.90000000   43.1999548  1.666000e+02 0.0000000
## 18:2-4:2   127.90000000   66.1999548  1.896000e+02 0.0000000
## 20:2-4:2   145.80000000   84.0999548  2.075000e+02 0.0000000
## 21:2-4:2   154.90000000   93.1999548  2.166000e+02 0.0000000
## 0:3-4:2    -19.00000000  -80.7000452  4.270005e+01 1.0000000
## 2:3-4:2     -9.40000000  -71.1000452  5.230005e+01 1.0000000
## 4:3-4:2      2.40000000  -59.3000452  6.410005e+01 1.0000000
## 6:3-4:2     18.10000000  -43.6000452  7.980005e+01 1.0000000
## 8:3-4:2     38.60000000  -23.1000452  1.003000e+02 0.9331352
## 10:3-4:2    57.30000000   -4.4000452  1.190000e+02 0.1251833
## 12:3-4:2    84.60000000   22.8999548  1.463000e+02 0.0000692
## 14:3-4:2   104.70000000   42.9999548  1.664000e+02 0.0000000
## 16:3-4:2   137.60000000   75.8999548  1.993000e+02 0.0000000
## 18:3-4:2   173.30000000  111.5999548  2.350000e+02 0.0000000
## 20:3-4:2   199.10000000  137.3999548  2.608000e+02 0.0000000
## 21:3-4:2   210.50000000  148.7999548  2.722000e+02 0.0000000
## 0:4-4:2    -18.80000000  -80.5000452  4.290005e+01 1.0000000
## 2:4-4:2     -8.00000000  -69.7000452  5.370005e+01 1.0000000
## 4:4-4:2      4.70000000  -57.0000452  6.640005e+01 1.0000000
## 6:4-4:2     24.10000000  -37.6000452  8.580005e+01 0.9999973
## 8:4-4:2     45.80000000  -15.9000452  1.075000e+02 0.6373750
## 10:4-4:2    66.20000000    4.4999548  1.279000e+02 0.0170802
## 12:4-4:2    91.60000000   29.8999548  1.533000e+02 0.0000058
## 14:4-4:2   102.00000000   40.2999548  1.637000e+02 0.0000001
## 16:4-4:2   122.20000000   60.4999548  1.839000e+02 0.0000000
## 18:4-4:2   143.10000000   81.3999548  2.048000e+02 0.0000000
## 20:4-4:2   174.08888889  110.6981185  2.374797e+02 0.0000000
## 21:4-4:2   178.75555556  115.3647852  2.421463e+02 0.0000000
## 8:2-6:2     16.30000000  -45.4000452  7.800005e+01 1.0000000
## 10:2-6:2    33.10000000  -28.6000452  9.480005e+01 0.9941666
## 12:2-6:2    55.90000000   -5.8000452  1.176000e+02 0.1625607
## 14:2-6:2    66.50000000    4.7999548  1.282000e+02 0.0158297
## 16:2-6:2    89.30000000   27.5999548  1.510000e+02 0.0000134
## 18:2-6:2   112.30000000   50.5999548  1.740000e+02 0.0000000
## 20:2-6:2   130.20000000   68.4999548  1.919000e+02 0.0000000
## 21:2-6:2   139.30000000   77.5999548  2.010000e+02 0.0000000
## 0:3-6:2    -34.60000000  -96.3000452  2.710005e+01 0.9870914
## 2:3-6:2    -25.00000000  -86.7000452  3.670005e+01 0.9999924
## 4:3-6:2    -13.20000000  -74.9000452  4.850005e+01 1.0000000
## 6:3-6:2      2.50000000  -59.2000452  6.420005e+01 1.0000000
## 8:3-6:2     23.00000000  -38.7000452  8.470005e+01 0.9999993
## 10:3-6:2    41.70000000  -20.0000452  1.034000e+02 0.8368121
## 12:3-6:2    69.00000000    7.2999548  1.307000e+02 0.0082342
## 14:3-6:2    89.10000000   27.3999548  1.508000e+02 0.0000144
## 16:3-6:2   122.00000000   60.2999548  1.837000e+02 0.0000000
```

```
## 18:3-6:2   157.70000000   95.9999548  2.194000e+02 0.0000000
## 20:3-6:2   183.50000000  121.7999548  2.452000e+02 0.0000000
## 21:3-6:2   194.90000000  133.1999548  2.566000e+02 0.0000000
## 0:4-6:2    -34.40000000  -96.1000452  2.730005e+01 0.9883166
## 2:4-6:2    -23.60000000  -85.3000452  3.810005e+01 0.9999986
## 4:4-6:2    -10.90000000  -72.6000452  5.080005e+01 1.0000000
## 6:4-6:2      8.50000000  -53.2000452  7.020005e+01 1.0000000
## 8:4-6:2     30.20000000  -31.5000452  9.190005e+01 0.9991029
## 10:4-6:2    50.60000000  -11.1000452  1.123000e+02 0.3755391
## 12:4-6:2    76.00000000   14.2999548  1.377000e+02 0.0011087
## 14:4-6:2    86.40000000   24.6999548  1.481000e+02 0.0000373
## 16:4-6:2   106.60000000   44.8999548  1.683000e+02 0.0000000
## 18:4-6:2   127.50000000   65.7999548  1.892000e+02 0.0000000
## 20:4-6:2   158.48888889   95.0981185  2.218797e+02 0.0000000
## 21:4-6:2   163.15555556   99.7647852  2.265463e+02 0.0000000
## 10:2-8:2    16.80000000  -44.9000452  7.850005e+01 1.0000000
## 12:2-8:2    39.60000000  -22.1000452  1.013000e+02 0.9079419
## 14:2-8:2    50.20000000  -11.5000452  1.119000e+02 0.3958775
## 16:2-8:2    73.00000000   11.2999548  1.347000e+02 0.0026979
## 18:2-8:2    96.00000000   34.2999548  1.577000e+02 0.0000011
## 20:2-8:2   113.90000000   52.1999548  1.756000e+02 0.0000000
## 21:2-8:2   123.00000000   61.2999548  1.847000e+02 0.0000000
## 0:3-8:2    -50.90000000 -112.6000452  1.080005e+01 0.3606114
## 2:3-8:2    -41.30000000 -103.0000452  2.040005e+01 0.8522565
## 4:3-8:2    -29.50000000  -91.2000452  3.220005e+01 0.9994704
## 6:3-8:2    -13.80000000  -75.5000452  4.790005e+01 1.0000000
## 8:3-8:2      6.70000000  -55.0000452  6.840005e+01 1.0000000
## 10:3-8:2    25.40000000  -36.3000452  8.710005e+01 0.9999881
## 12:3-8:2    52.70000000   -9.0000452  1.144000e+02 0.2777444
## 14:3-8:2    72.80000000   11.0999548  1.345000e+02 0.0028582
## 16:3-8:2   105.70000000   43.9999548  1.674000e+02 0.0000000
## 18:3-8:2   141.40000000   79.6999548  2.031000e+02 0.0000000
## 20:3-8:2   167.20000000  105.4999548  2.289000e+02 0.0000000
## 21:3-8:2   178.60000000  116.8999548  2.403000e+02 0.0000000
## 0:4-8:2    -50.70000000 -112.4000452  1.100005e+01 0.3705311
## 2:4-8:2    -39.90000000 -101.6000452  2.180005e+01 0.8993013
## 4:4-8:2    -27.20000000  -88.9000452  3.450005e+01 0.9999261
## 6:4-8:2     -7.80000000  -69.5000452  5.390005e+01 1.0000000
## 8:4-8:2     13.90000000  -47.8000452  7.560005e+01 1.0000000
## 10:4-8:2    34.30000000  -27.4000452  9.600005e+01 0.9888926
## 12:4-8:2    59.70000000   -2.0000452  1.214000e+02 0.0772151
## 14:4-8:2    70.10000000    8.3999548  1.318000e+02 0.0061091
## 16:4-8:2    90.30000000   28.5999548  1.520000e+02 0.0000093
## 18:4-8:2   111.20000000   49.4999548  1.729000e+02 0.0000000
## 20:4-8:2   142.18888889   78.7981185  2.055797e+02 0.0000000
## 21:4-8:2   146.85555556   83.4647852  2.102463e+02 0.0000000
## 12:2-10:2   22.80000000  -38.9000452  8.450005e+01 0.9999995
## 14:2-10:2   33.40000000  -28.3000452  9.510005e+01 0.9931020
## 16:2-10:2   56.20000000   -5.5000452  1.179000e+02 0.1539137
## 18:2-10:2   79.20000000   17.4999548  1.409000e+02 0.0004100
## 20:2-10:2   97.10000000   35.3999548  1.588000e+02 0.0000007
## 21:2-10:2  106.20000000   44.4999548  1.679000e+02 0.0000000
## 0:3-10:2   -67.70000000 -129.4000452 -5.999955e+00 0.0116181
## 2:3-10:2   -58.10000000 -119.8000452  3.600045e+00 0.1070770
```

```
## 4:3-10:2   -46.30000000 -108.0000452  1.540005e+01 0.6096992
## 6:3-10:2   -30.60000000  -92.3000452  3.110005e+01 0.9988043
## 8:3-10:2   -10.10000000  -71.8000452  5.160005e+01 1.0000000
## 10:3-10:2    8.60000000  -53.1000452  7.030005e+01 1.0000000
## 12:3-10:2   35.90000000  -25.8000452  9.760005e+01 0.9763633
## 14:3-10:2   56.00000000   -5.7000452  1.177000e+02 0.1596387
## 16:3-10:2   88.90000000   27.1999548  1.506000e+02 0.0000154
## 18:3-10:2  124.60000000   62.8999548  1.863000e+02 0.0000000
## 20:3-10:2  150.40000000   88.6999548  2.121000e+02 0.0000000
## 21:3-10:2  161.80000000  100.0999548  2.235000e+02 0.0000000
## 0:4-10:2   -67.50000000 -129.2000452 -5.799955e+00 0.0122398
## 2:4-10:2   -56.70000000 -118.4000452  5.000045e+00 0.1402846
## 4:4-10:2   -44.00000000 -105.7000452  1.770005e+01 0.7326297
## 6:4-10:2   -24.60000000  -86.3000452  3.710005e+01 0.9999952
## 8:4-10:2    -2.90000000  -64.6000452  5.880005e+01 1.0000000
## 10:4-10:2   17.50000000  -44.2000452  7.920005e+01 1.0000000
## 12:4-10:2   42.90000000  -18.8000452  1.046000e+02 0.7854926
## 14:4-10:2   53.30000000   -8.4000452  1.150000e+02 0.2529196
## 16:4-10:2   73.50000000   11.7999548  1.352000e+02 0.0023333
## 18:4-10:2   94.40000000   32.6999548  1.561000e+02 0.0000020
## 20:4-10:2  125.38888889   61.9981185  1.887797e+02 0.0000000
## 21:4-10:2  130.05555556   66.6647852  1.934463e+02 0.0000000
## 14:2-12:2   10.60000000  -51.1000452  7.230005e+01 1.0000000
## 16:2-12:2   33.40000000  -28.3000452  9.510005e+01 0.9931020
## 18:2-12:2   56.40000000   -5.3000452  1.181000e+02 0.1483458
## 20:2-12:2   74.30000000   12.5999548  1.360000e+02 0.0018448
## 21:2-12:2   83.40000000   21.6999548  1.451000e+02 0.0001039
## 0:3-12:2   -90.50000000 -152.2000452 -2.879995e+01 0.0000087
## 2:3-12:2   -80.90000000 -142.6000452 -1.919995e+01 0.0002373
## 4:3-12:2   -69.10000000 -130.8000452 -7.399955e+00 0.0080159
## 6:3-12:2   -53.40000000 -115.1000452  8.300045e+00 0.2489254
## 8:3-12:2   -32.90000000  -94.6000452  2.880005e+01 0.9947966
## 10:3-12:2  -14.20000000  -75.9000452  4.750005e+01 1.0000000
## 12:3-12:2   13.10000000  -48.6000452  7.480005e+01 1.0000000
## 14:3-12:2   33.20000000  -28.5000452  9.490005e+01 0.9938282
## 16:3-12:2   66.10000000    4.3999548  1.278000e+02 0.0175165
## 18:3-12:2  101.80000000   40.0999548  1.635000e+02 0.0000001
## 20:3-12:2  127.60000000   65.8999548  1.893000e+02 0.0000000
## 21:3-12:2  139.00000000   77.2999548  2.007000e+02 0.0000000
## 0:4-12:2   -90.30000000 -152.0000452 -2.859995e+01 0.0000093
## 2:4-12:2   -79.50000000 -141.2000452 -1.779995e+01 0.0003727
## 4:4-12:2   -66.80000000 -128.5000452 -5.099955e+00 0.0146631
## 6:4-12:2   -47.40000000 -109.1000452  1.430005e+01 0.5481161
## 8:4-12:2   -25.70000000  -87.4000452  3.600005e+01 0.9999836
## 10:4-12:2   -5.30000000  -67.0000452  5.640005e+01 1.0000000
## 12:4-12:2   20.10000000  -41.6000452  8.180005e+01 1.0000000
## 14:4-12:2   30.50000000  -31.2000452  9.220005e+01 0.9988861
## 16:4-12:2   50.70000000  -11.0000452  1.124000e+02 0.3705311
## 18:4-12:2   71.60000000    9.8999548  1.333000e+02 0.0040245
## 20:4-12:2  102.58888889   39.1981185  1.659797e+02 0.0000002
## 21:4-12:2  107.25555556   43.8647852  1.706463e+02 0.0000000
## 16:2-14:2   22.80000000  -38.9000452  8.450005e+01 0.9999995
## 18:2-14:2   45.80000000  -15.9000452  1.075000e+02 0.6373750
## 20:2-14:2   63.70000000    1.9999548  1.254000e+02 0.0315140
```

```
## 21:2-14:2   72.80000000   11.0999548  1.345000e+02 0.0028582
## 0:3-14:2  -101.10000000 -162.8000452 -3.939995e+01 0.0000002
## 2:3-14:2   -91.50000000 -153.2000452 -2.979995e+01 0.0000060
## 4:3-14:2   -79.70000000 -141.4000452 -1.799995e+01 0.0003496
## 6:3-14:2   -64.00000000 -125.7000452 -2.299955e+00 0.0293404
## 8:3-14:2   -43.50000000 -105.2000452  1.820005e+01 0.7572765
## 10:3-14:2  -24.80000000  -86.5000452  3.690005e+01 0.9999939
## 12:3-14:2    2.50000000  -59.2000452  6.420005e+01 1.0000000
## 14:3-14:2   22.60000000  -39.1000452  8.430005e+01 0.9999996
## 16:3-14:2   55.50000000   -6.2000452  1.172000e+02 0.1746496
## 18:3-14:2   91.20000000   29.4999548  1.529000e+02 0.0000067
## 20:3-14:2  117.00000000   55.2999548  1.787000e+02 0.0000000
## 21:3-14:2  128.40000000   66.6999548  1.901000e+02 0.0000000
## 0:4-14:2  -100.90000000 -162.6000452 -3.919995e+01 0.0000002
## 2:4-14:2   -90.10000000 -151.8000452 -2.839995e+01 0.0000100
## 4:4-14:2   -77.40000000 -139.1000452 -1.569995e+01 0.0007216
## 6:4-14:2   -58.00000000 -119.7000452  3.700045e+00 0.1092180
## 8:4-14:2   -36.30000000  -98.0000452  2.540005e+01 0.9719351
## 10:4-14:2  -15.90000000  -77.6000452  4.580005e+01 1.0000000
## 12:4-14:2    9.50000000  -52.2000452  7.120005e+01 1.0000000
## 14:4-14:2   19.90000000  -41.8000452  8.160005e+01 1.0000000
## 16:4-14:2   40.10000000  -21.6000452  1.018000e+02 0.8932585
## 18:4-14:2   61.00000000   -0.7000452  1.227000e+02 0.0583988
## 20:4-14:2   91.98888889   28.5981185  1.553797e+02 0.0000123
## 21:4-14:2   96.65555556   33.2647852  1.600463e+02 0.0000023
## 18:2-16:2   23.00000000  -38.7000452  8.470005e+01 0.9999993
## 20:2-16:2   40.90000000  -20.8000452  1.026000e+02 0.8668224
## 21:2-16:2   50.00000000  -11.7000452  1.117000e+02 0.4062212
## 0:3-16:2  -123.90000000 -185.6000452 -6.219995e+01 0.0000000
## 2:3-16:2  -114.30000000 -176.0000452 -5.259995e+01 0.0000000
## 4:3-16:2  -102.50000000 -164.2000452 -4.079995e+01 0.0000001
## 6:3-16:2   -86.80000000 -148.5000452 -2.509995e+01 0.0000324
## 8:3-16:2   -66.30000000 -128.0000452 -4.599955e+00 0.0166537
## 10:3-16:2  -47.60000000 -109.3000452  1.410005e+01 0.5369070
## 12:3-16:2  -20.30000000  -82.0000452  4.140005e+01 1.0000000
## 14:3-16:2   -0.20000000  -61.9000452  6.150005e+01 1.0000000
## 16:3-16:2   32.70000000  -29.0000452  9.440005e+01 0.9953681
## 18:3-16:2   68.40000000    6.6999548  1.301000e+02 0.0096635
## 20:3-16:2   94.20000000   32.4999548  1.559000e+02 0.0000022
## 21:3-16:2  105.60000000   43.8999548  1.673000e+02 0.0000000
## 0:4-16:2  -123.70000000 -185.4000452 -6.199995e+01 0.0000000
## 2:4-16:2  -112.90000000 -174.6000452 -5.119995e+01 0.0000000
## 4:4-16:2  -100.20000000 -161.9000452 -3.849995e+01 0.0000002
## 6:4-16:2   -80.80000000 -142.5000452 -1.909995e+01 0.0002452
## 8:4-16:2   -59.10000000 -120.8000452  2.600045e+00 0.0874810
## 10:4-16:2  -38.70000000 -100.4000452  2.300005e+01 0.9308605
## 12:4-16:2  -13.30000000  -75.0000452  4.840005e+01 1.0000000
## 14:4-16:2   -2.90000000  -64.6000452  5.880005e+01 1.0000000
## 16:4-16:2   17.30000000  -44.4000452  7.900005e+01 1.0000000
## 18:4-16:2   38.20000000  -23.5000452  9.990005e+01 0.9417086
## 20:4-16:2   69.18888889    5.7981185  1.325797e+02 0.0127473
## 21:4-16:2   73.85555556   10.4647852  1.372463e+02 0.0037121
## 20:2-18:2   17.90000000  -43.8000452  7.960005e+01 1.0000000
## 21:2-18:2   27.00000000  -34.7000452  8.870005e+01 0.9999389
```

```
## 0:3-18:2  -146.90000000 -208.6000452 -8.519995e+01 0.0000000
## 2:3-18:2  -137.30000000 -199.0000452 -7.559995e+01 0.0000000
## 4:3-18:2  -125.50000000 -187.2000452 -6.379995e+01 0.0000000
## 6:3-18:2  -109.80000000 -171.5000452 -4.809995e+01 0.0000000
## 8:3-18:2   -89.30000000 -151.0000452 -2.759995e+01 0.0000134
## 10:3-18:2  -70.60000000 -132.3000452 -8.899955e+00 0.0053226
## 12:3-18:2  -43.30000000 -105.0000452  1.840005e+01 0.7668550
## 14:3-18:2  -23.20000000  -84.9000452  3.850005e+01 0.9999991
## 16:3-18:2    9.70000000  -52.0000452  7.140005e+01 1.0000000
## 18:3-18:2   45.40000000  -16.3000452  1.071000e+02 0.6592319
## 20:3-18:2   71.20000000    9.4999548  1.329000e+02 0.0045035
## 21:3-18:2   82.60000000   20.8999548  1.443000e+02 0.0001357
## 0:4-18:2  -146.70000000 -208.4000452 -8.499995e+01 0.0000000
## 2:4-18:2  -135.90000000 -197.6000452 -7.419995e+01 0.0000000
## 4:4-18:2  -123.20000000 -184.9000452 -6.149995e+01 0.0000000
## 6:4-18:2  -103.80000000 -165.5000452 -4.209995e+01 0.0000001
## 8:4-18:2   -82.10000000 -143.8000452 -2.039995e+01 0.0001602
## 10:4-18:2  -61.70000000 -123.4000452  4.516633e-05 0.0500005
## 12:4-18:2  -36.30000000  -98.0000452  2.540005e+01 0.9719351
## 14:4-18:2  -25.90000000  -87.6000452  3.580005e+01 0.9999797
## 16:4-18:2   -5.70000000  -67.4000452  5.600005e+01 1.0000000
## 18:4-18:2   15.20000000  -46.5000452  7.690005e+01 1.0000000
## 20:4-18:2   46.18888889  -17.2018815  1.095797e+02 0.6830482
## 21:4-18:2   50.85555556  -12.5352148  1.142463e+02 0.4325881
## 21:2-20:2    9.10000000  -52.6000452  7.080005e+01 1.0000000
## 0:3-20:2  -164.80000000 -226.5000452 -1.031000e+02 0.0000000
## 2:3-20:2  -155.20000000 -216.9000452 -9.349995e+01 0.0000000
## 4:3-20:2  -143.40000000 -205.1000452 -8.169995e+01 0.0000000
## 6:3-20:2  -127.70000000 -189.4000452 -6.599995e+01 0.0000000
## 8:3-20:2  -107.20000000 -168.9000452 -4.549995e+01 0.0000000
## 10:3-20:2  -88.50000000 -150.2000452 -2.679995e+01 0.0000178
## 12:3-20:2  -61.20000000 -122.9000452  5.000452e-01 0.0558839
## 14:3-20:2  -41.10000000 -102.8000452  2.060005e+01 0.8596505
## 16:3-20:2   -8.20000000  -69.9000452  5.350005e+01 1.0000000
## 18:3-20:2   27.50000000  -34.2000452  8.920005e+01 0.9999023
## 20:3-20:2   53.30000000   -8.4000452  1.150000e+02 0.2529196
## 21:3-20:2   64.70000000    2.9999548  1.264000e+02 0.0247796
## 0:4-20:2  -164.60000000 -226.3000452 -1.029000e+02 0.0000000
## 2:4-20:2  -153.80000000 -215.5000452 -9.209995e+01 0.0000000
## 4:4-20:2  -141.10000000 -202.8000452 -7.939995e+01 0.0000000
## 6:4-20:2  -121.70000000 -183.4000452 -5.999995e+01 0.0000000
## 8:4-20:2  -100.00000000 -161.7000452 -3.829995e+01 0.0000002
## 10:4-20:2  -79.60000000 -141.3000452 -1.789995e+01 0.0003609
## 12:4-20:2  -54.20000000 -115.9000452  7.500045e+00 0.2184670
## 14:4-20:2  -43.80000000 -105.5000452  1.790005e+01 0.7426038
## 16:4-20:2  -23.60000000  -85.3000452  3.810005e+01 0.9999986
## 18:4-20:2   -2.70000000  -64.4000452  5.900005e+01 1.0000000
## 20:4-20:2   28.28888889  -35.1018815  9.167966e+01 0.9998992
## 21:4-20:2   32.95555556  -30.4352148  9.634633e+01 0.9968205
## 0:3-21:2  -173.90000000 -235.6000452 -1.122000e+02 0.0000000
## 2:3-21:2  -164.30000000 -226.0000452 -1.026000e+02 0.0000000
## 4:3-21:2  -152.50000000 -214.2000452 -9.079995e+01 0.0000000
## 6:3-21:2  -136.80000000 -198.5000452 -7.509995e+01 0.0000000
## 8:3-21:2  -116.30000000 -178.0000452 -5.459995e+01 0.0000000
```

```
## 10:3-21:2  -97.60000000 -159.3000452 -3.589995e+01 0.0000006
## 12:3-21:2  -70.30000000 -132.0000452 -8.599955e+00 0.0057824
## 14:3-21:2  -50.20000000 -111.9000452  1.150005e+01 0.3958775
## 16:3-21:2  -17.30000000  -79.0000452  4.440005e+01 1.0000000
## 18:3-21:2   18.40000000  -43.3000452  8.010005e+01 1.0000000
## 20:3-21:2   44.20000000  -17.5000452  1.059000e+02 0.7225109
## 21:3-21:2   55.60000000   -6.1000452  1.173000e+02 0.1715669
## 0:4-21:2  -173.70000000 -235.4000452 -1.120000e+02 0.0000000
## 2:4-21:2  -162.90000000 -224.6000452 -1.012000e+02 0.0000000
## 4:4-21:2  -150.20000000 -211.9000452 -8.849995e+01 0.0000000
## 6:4-21:2  -130.80000000 -192.5000452 -6.909995e+01 0.0000000
## 8:4-21:2  -109.10000000 -170.8000452 -4.739995e+01 0.0000000
## 10:4-21:2  -88.70000000 -150.4000452 -2.699995e+01 0.0000166
## 12:4-21:2  -63.30000000 -125.0000452 -1.599955e+00 0.0346339
## 14:4-21:2  -52.90000000 -114.6000452  8.800045e+00 0.2693067
## 16:4-21:2  -32.70000000  -94.4000452  2.900005e+01 0.9953681
## 18:4-21:2  -11.80000000  -73.5000452  4.990005e+01 1.0000000
## 20:4-21:2   19.18888889  -44.2018815  8.257966e+01 1.0000000
## 21:4-21:2   23.85555556  -39.5352148  8.724633e+01 0.9999991
## 2:3-0:3      9.60000000  -52.1000452  7.130005e+01 1.0000000
## 4:3-0:3     21.40000000  -40.3000452  8.310005e+01 0.9999999
## 6:3-0:3     37.10000000  -24.6000452  9.880005e+01 0.9611841
## 8:3-0:3     57.60000000   -4.1000452  1.193000e+02 0.1181284
## 10:3-0:3    76.30000000   14.5999548  1.380000e+02 0.0010119
## 12:3-0:3   103.60000000   41.8999548  1.653000e+02 0.0000001
## 14:3-0:3   123.70000000   61.9999548  1.854000e+02 0.0000000
## 16:3-0:3   156.60000000   94.8999548  2.183000e+02 0.0000000
## 18:3-0:3   192.30000000  130.5999548  2.540000e+02 0.0000000
## 20:3-0:3   218.10000000  156.3999548  2.798000e+02 0.0000000
## 21:3-0:3   229.50000000  167.7999548  2.912000e+02 0.0000000
## 0:4-0:3      0.20000000  -61.5000452  6.190005e+01 1.0000000
## 2:4-0:3     11.00000000  -50.7000452  7.270005e+01 1.0000000
## 4:4-0:3     23.70000000  -38.0000452  8.540005e+01 0.9999984
## 6:4-0:3     43.10000000  -18.6000452  1.048000e+02 0.7762626
## 8:4-0:3     64.80000000    3.0999548  1.265000e+02 0.0241827
## 10:4-0:3    85.20000000   23.4999548  1.469000e+02 0.0000564
## 12:4-0:3   110.60000000   48.8999548  1.723000e+02 0.0000000
## 14:4-0:3   121.00000000   59.2999548  1.827000e+02 0.0000000
## 16:4-0:3   141.20000000   79.4999548  2.029000e+02 0.0000000
## 18:4-0:3   162.10000000  100.3999548  2.238000e+02 0.0000000
## 20:4-0:3   193.08888889  129.6981185  2.564797e+02 0.0000000
## 21:4-0:3   197.75555556  134.3647852  2.611463e+02 0.0000000
## 4:3-2:3     11.80000000  -49.9000452  7.350005e+01 1.0000000
## 6:3-2:3     27.50000000  -34.2000452  8.920005e+01 0.9999023
## 8:3-2:3     48.00000000  -13.7000452  1.097000e+02 0.5145540
## 10:3-2:3    66.70000000    4.9999548  1.284000e+02 0.0150430
## 12:3-2:3    94.00000000   32.2999548  1.557000e+02 0.0000024
## 14:3-2:3   114.10000000   52.3999548  1.758000e+02 0.0000000
## 16:3-2:3   147.00000000   85.2999548  2.087000e+02 0.0000000
## 18:3-2:3   182.70000000  120.9999548  2.444000e+02 0.0000000
## 20:3-2:3   208.50000000  146.7999548  2.702000e+02 0.0000000
## 21:3-2:3   219.90000000  158.1999548  2.816000e+02 0.0000000
## 0:4-2:3     -9.40000000  -71.1000452  5.230005e+01 1.0000000
## 2:4-2:3      1.40000000  -60.3000452  6.310005e+01 1.0000000
```

```
## 4:4-2:3      14.10000000  -47.6000452  7.580005e+01 1.0000000
## 6:4-2:3      33.50000000  -28.2000452  9.520005e+01 0.9927129
## 8:4-2:3      55.20000000   -6.5000452  1.169000e+02 0.1841422
## 10:4-2:3     75.60000000   13.8999548  1.373000e+02 0.0012513
## 12:4-2:3    101.00000000   39.2999548  1.627000e+02 0.0000002
## 14:4-2:3    111.40000000   49.6999548  1.731000e+02 0.0000000
## 16:4-2:3    131.60000000   69.8999548  1.933000e+02 0.0000000
## 18:4-2:3    152.50000000   90.7999548  2.142000e+02 0.0000000
## 20:4-2:3    183.48888889  120.0981185  2.468797e+02 0.0000000
## 21:4-2:3    188.15555556  124.7647852  2.515463e+02 0.0000000
## 6:3-4:3      15.70000000  -46.0000452  7.740005e+01 1.0000000
## 8:3-4:3      36.20000000  -25.5000452  9.790005e+01 0.9730980
## 10:3-4:3     54.90000000   -6.8000452  1.166000e+02 0.1940041
## 12:3-4:3     82.20000000   20.4999548  1.439000e+02 0.0001549
## 14:3-4:3    102.30000000   40.5999548  1.640000e+02 0.0000001
## 16:3-4:3    135.20000000   73.4999548  1.969000e+02 0.0000000
## 18:3-4:3    170.90000000  109.1999548  2.326000e+02 0.0000000
## 20:3-4:3    196.70000000  134.9999548  2.584000e+02 0.0000000
## 21:3-4:3    208.10000000  146.3999548  2.698000e+02 0.0000000
## 0:4-4:3     -21.20000000  -82.9000452  4.050005e+01 1.0000000
## 2:4-4:3     -10.40000000  -72.1000452  5.130005e+01 1.0000000
## 4:4-4:3       2.30000000  -59.4000452  6.400005e+01 1.0000000
## 6:4-4:3      21.70000000  -40.0000452  8.340005e+01 0.9999999
## 8:4-4:3      43.40000000  -18.3000452  1.051000e+02 0.7620867
## 10:4-4:3     63.80000000    2.0999548  1.255000e+02 0.0307741
## 12:4-4:3     89.20000000   27.4999548  1.509000e+02 0.0000139
## 14:4-4:3     99.60000000   37.8999548  1.613000e+02 0.0000003
## 16:4-4:3    119.80000000   58.0999548  1.815000e+02 0.0000000
## 18:4-4:3    140.70000000   78.9999548  2.024000e+02 0.0000000
## 20:4-4:3    171.68888889  108.2981185  2.350797e+02 0.0000000
## 21:4-4:3    176.35555556  112.9647852  2.397463e+02 0.0000000
## 8:3-6:3      20.50000000  -41.2000452  8.220005e+01 1.0000000
## 10:3-6:3     39.20000000  -22.5000452  1.009000e+02 0.9186783
## 12:3-6:3     66.50000000    4.7999548  1.282000e+02 0.0158297
## 14:3-6:3     86.60000000   24.8999548  1.483000e+02 0.0000348
## 16:3-6:3    119.50000000   57.7999548  1.812000e+02 0.0000000
## 18:3-6:3    155.20000000   93.4999548  2.169000e+02 0.0000000
## 20:3-6:3    181.00000000  119.2999548  2.427000e+02 0.0000000
## 21:3-6:3    192.40000000  130.6999548  2.541000e+02 0.0000000
## 0:4-6:3     -36.90000000  -98.6000452  2.480005e+01 0.9641235
## 2:4-6:3     -26.10000000  -87.8000452  3.560005e+01 0.9999750
## 4:4-6:3     -13.40000000  -75.1000452  4.830005e+01 1.0000000
## 6:4-6:3       6.00000000  -55.7000452  6.770005e+01 1.0000000
## 8:4-6:3      27.70000000  -34.0000452  8.940005e+01 0.9998828
## 10:4-6:3     48.10000000  -13.6000452  1.098000e+02 0.5089857
## 12:4-6:3     73.50000000   11.7999548  1.352000e+02 0.0023333
## 14:4-6:3     83.90000000   22.1999548  1.456000e+02 0.0000878
## 16:4-6:3    104.10000000   42.3999548  1.658000e+02 0.0000000
## 18:4-6:3    125.00000000   63.2999548  1.867000e+02 0.0000000
## 20:4-6:3    155.98888889   92.5981185  2.193797e+02 0.0000000
## 21:4-6:3    160.65555556   97.2647852  2.240463e+02 0.0000000
## 10:3-8:3     18.70000000  -43.0000452  8.040005e+01 1.0000000
## 12:3-8:3     46.00000000  -15.7000452  1.077000e+02 0.6263449
## 14:3-8:3     66.10000000    4.3999548  1.278000e+02 0.0175165
```

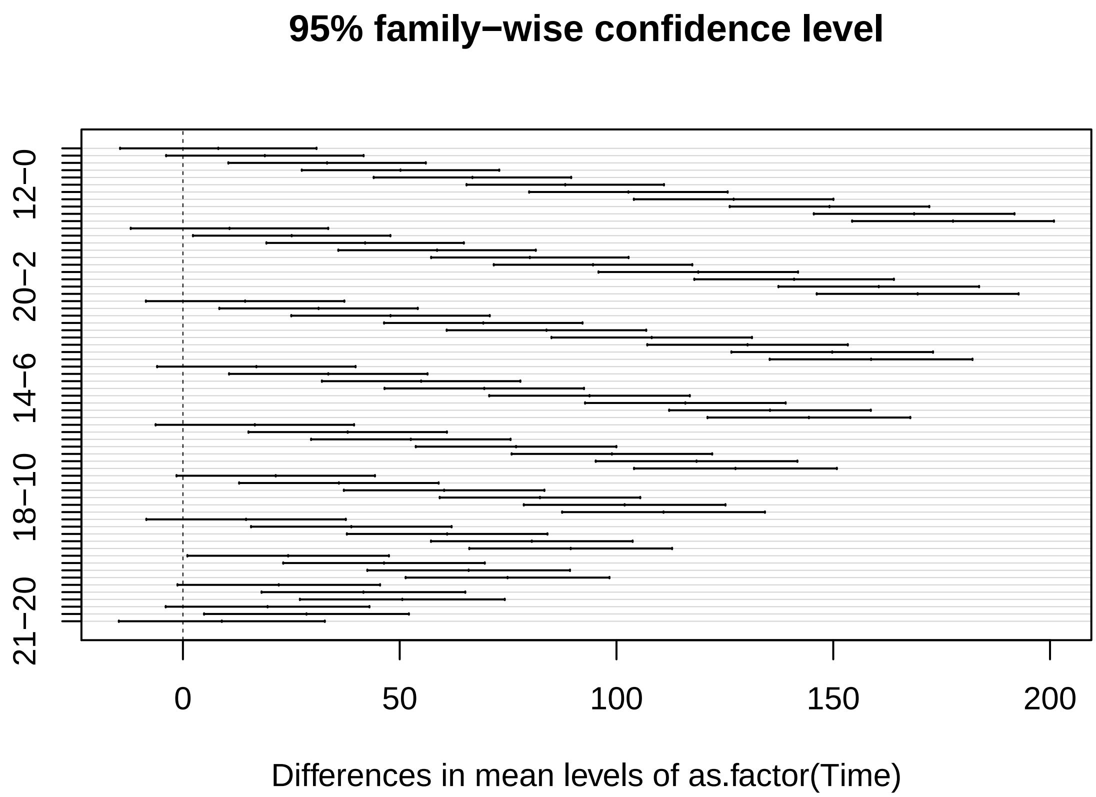
```
## 16:3-8:3    99.00000000   37.2999548  1.607000e+02 0.0000003
## 18:3-8:3   134.70000000   72.9999548  1.964000e+02 0.0000000
## 20:3-8:3   160.50000000   98.7999548  2.222000e+02 0.0000000
## 21:3-8:3   171.90000000  110.1999548  2.336000e+02 0.0000000
## 0:4-8:3    -57.40000000 -119.1000452  4.300045e+00 0.1227956
## 2:4-8:3    -46.60000000 -108.3000452  1.510005e+01 0.5929612
## 4:4-8:3    -33.90000000  -95.6000452  2.780005e+01 0.9909687
## 6:4-8:3    -14.50000000  -76.2000452  4.720005e+01 1.0000000
## 8:4-8:3      7.20000000  -54.5000452  6.890005e+01 1.0000000
## 10:4-8:3    27.60000000  -34.1000452  8.930005e+01 0.9998930
## 12:4-8:3    53.00000000   -8.7000452  1.147000e+02 0.2651486
## 14:4-8:3    63.40000000    1.6999548  1.251000e+02 0.0338294
## 16:4-8:3    83.60000000   21.8999548  1.453000e+02 0.0000971
## 18:4-8:3   104.50000000   42.7999548  1.662000e+02 0.0000000
## 20:4-8:3   135.48888889   72.0981185  1.988797e+02 0.0000000
## 21:4-8:3   140.15555556   76.7647852  2.035463e+02 0.0000000
## 12:3-10:3   27.30000000  -34.4000452  8.900005e+01 0.9999188
## 14:3-10:3   47.40000000  -14.3000452  1.091000e+02 0.5481161
## 16:3-10:3   80.30000000   18.5999548  1.420000e+02 0.0002883
## 18:3-10:3  116.00000000   54.2999548  1.777000e+02 0.0000000
## 20:3-10:3  141.80000000   80.0999548  2.035000e+02 0.0000000
## 21:3-10:3  153.20000000   91.4999548  2.149000e+02 0.0000000
## 0:4-10:3   -76.10000000 -137.8000452 -1.439995e+01 0.0010755
## 2:4-10:3   -65.30000000 -127.0000452 -3.599955e+00 0.0213875
## 4:4-10:3   -52.60000000 -114.3000452  9.100045e+00 0.2820235
## 6:4-10:3   -33.20000000  -94.9000452  2.850005e+01 0.9938282
## 8:4-10:3   -11.50000000  -73.2000452  5.020005e+01 1.0000000
## 10:4-10:3    8.90000000  -52.8000452  7.060005e+01 1.0000000
## 12:4-10:3   34.30000000  -27.4000452  9.600005e+01 0.9888926
## 14:4-10:3   44.70000000  -17.0000452  1.064000e+02 0.6966350
## 16:4-10:3   64.90000000    3.1999548  1.266000e+02 0.0235987
## 18:4-10:3   85.80000000   24.0999548  1.475000e+02 0.0000459
## 20:4-10:3  116.78888889   53.3981185  1.801797e+02 0.0000000
## 21:4-10:3  121.45555556   58.0647852  1.848463e+02 0.0000000
## 14:3-12:3   20.10000000  -41.6000452  8.180005e+01 1.0000000
## 16:3-12:3   53.00000000   -8.7000452  1.147000e+02 0.2651486
## 18:3-12:3   88.70000000   26.9999548  1.504000e+02 0.0000166
## 20:3-12:3  114.50000000   52.7999548  1.762000e+02 0.0000000
## 21:3-12:3  125.90000000   64.1999548  1.876000e+02 0.0000000
## 0:4-12:3  -103.40000000 -165.1000452 -4.169995e+01 0.0000001
## 2:4-12:3   -92.60000000 -154.3000452 -3.089995e+01 0.0000040
## 4:4-12:3   -79.90000000 -141.6000452 -1.819995e+01 0.0003279
## 6:4-12:3   -60.50000000 -122.2000452  1.200045e+00 0.0651133
## 8:4-12:3   -38.80000000 -100.5000452  2.290005e+01 0.9285324
## 10:4-12:3  -18.40000000  -80.1000452  4.330005e+01 1.0000000
## 12:4-12:3    7.00000000  -54.7000452  6.870005e+01 1.0000000
## 14:4-12:3   17.40000000  -44.3000452  7.910005e+01 1.0000000
## 16:4-12:3   37.60000000  -24.1000452  9.930005e+01 0.9530484
## 18:4-12:3   58.50000000   -3.2000452  1.202000e+02 0.0988494
## 20:4-12:3   89.48888889   26.0981185  1.528797e+02 0.0000292
## 21:4-12:3   94.15555556   30.7647852  1.575463e+02 0.0000057
## 16:3-14:3   32.90000000  -28.8000452  9.460005e+01 0.9947966
## 18:3-14:3   68.60000000    6.8999548  1.303000e+02 0.0091635
## 20:3-14:3   94.40000000   32.6999548  1.561000e+02 0.0000020
```

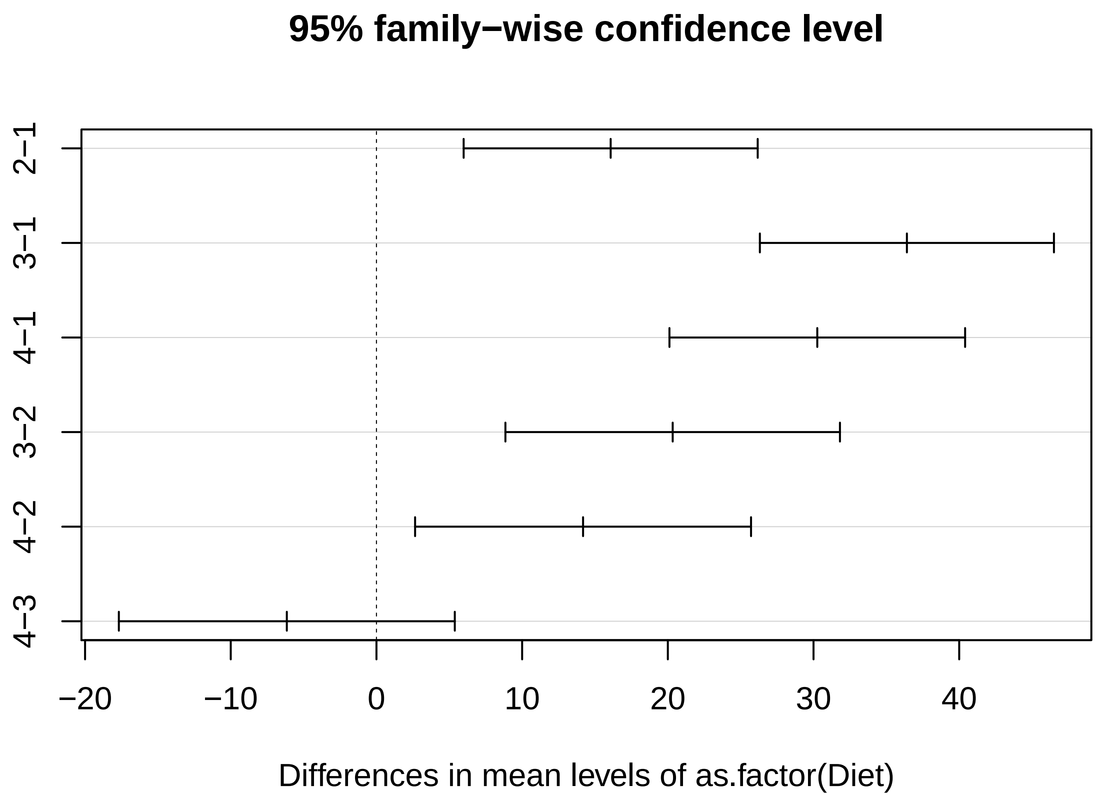
```
## 21:3-14:3  105.80000000   44.0999548  1.675000e+02 0.0000000
## 0:4-14:3  -123.50000000 -185.2000452 -6.179995e+01 0.0000000
## 2:4-14:3  -112.70000000 -174.4000452 -5.099995e+01 0.0000000
## 4:4-14:3  -100.00000000 -161.7000452 -3.829995e+01 0.0000002
## 6:4-14:3   -80.60000000 -142.3000452 -1.889995e+01 0.0002616
## 8:4-14:3   -58.90000000 -120.6000452  2.800045e+00 0.0911443
## 10:4-14:3  -38.50000000 -100.2000452  2.320005e+01 0.9353569
## 12:4-14:3  -13.10000000  -74.8000452  4.860005e+01 1.0000000
## 14:4-14:3   -2.70000000  -64.4000452  5.900005e+01 1.0000000
## 16:4-14:3   17.50000000  -44.2000452  7.920005e+01 1.0000000
## 18:4-14:3   38.40000000  -23.3000452  1.001000e+02 0.9375260
## 20:4-14:3   69.38888889    5.9981185  1.327797e+02 0.0121188
## 21:4-14:3   74.05555556   10.6647852  1.374463e+02 0.0035124
## 18:3-16:3   35.70000000  -26.0000452  9.740005e+01 0.9783616
## 20:3-16:3   61.50000000   -0.2000452  1.232000e+02 0.0522863
## 21:3-16:3   72.90000000   11.1999548  1.346000e+02 0.0027770
## 0:4-16:3  -156.40000000 -218.1000452 -9.469995e+01 0.0000000
## 2:4-16:3  -145.60000000 -207.3000452 -8.389995e+01 0.0000000
## 4:4-16:3  -132.90000000 -194.6000452 -7.119995e+01 0.0000000
## 6:4-16:3  -113.50000000 -175.2000452 -5.179995e+01 0.0000000
## 8:4-16:3   -91.80000000 -153.5000452 -3.009995e+01 0.0000054
## 10:4-16:3  -71.40000000 -133.1000452 -9.699955e+00 0.0042577
## 12:4-16:3  -46.00000000 -107.7000452  1.570005e+01 0.6263449
## 14:4-16:3  -35.60000000  -97.3000452  2.610005e+01 0.9793095
## 16:4-16:3  -15.40000000  -77.1000452  4.630005e+01 1.0000000
## 18:4-16:3    5.50000000  -56.2000452  6.720005e+01 1.0000000
## 20:4-16:3   36.48888889  -26.9018815  9.987966e+01 0.9800830
## 21:4-16:3   41.15555556  -22.2352148  1.045463e+02 0.8945500
## 20:3-18:3   25.80000000  -35.9000452  8.750005e+01 0.9999817
## 21:3-18:3   37.20000000  -24.5000452  9.890005e+01 0.9596483
## 0:4-18:3  -192.10000000 -253.8000452 -1.304000e+02 0.0000000
## 2:4-18:3  -181.30000000 -243.0000452 -1.196000e+02 0.0000000
## 4:4-18:3  -168.60000000 -230.3000452 -1.069000e+02 0.0000000
## 6:4-18:3  -149.20000000 -210.9000452 -8.749995e+01 0.0000000
## 8:4-18:3  -127.50000000 -189.2000452 -6.579995e+01 0.0000000
## 10:4-18:3 -107.10000000 -168.8000452 -4.539995e+01 0.0000000
## 12:4-18:3  -81.70000000 -143.4000452 -1.999995e+01 0.0001827
## 14:4-18:3  -71.30000000 -133.0000452 -9.599955e+00 0.0043790
## 16:4-18:3  -51.10000000 -112.8000452  1.060005e+01 0.3508240
## 18:4-18:3  -30.20000000  -91.9000452  3.150005e+01 0.9991029
## 20:4-18:3    0.78888889  -62.6018815  6.417966e+01 1.0000000
## 21:4-18:3    5.45555556  -57.9352148  6.884633e+01 1.0000000
## 21:3-20:3   11.40000000  -50.3000452  7.310005e+01 1.0000000
## 0:4-20:3  -217.90000000 -279.6000452 -1.562000e+02 0.0000000
## 2:4-20:3  -207.10000000 -268.8000452 -1.454000e+02 0.0000000
## 4:4-20:3  -194.40000000 -256.1000452 -1.327000e+02 0.0000000
## 6:4-20:3  -175.00000000 -236.7000452 -1.133000e+02 0.0000000
## 8:4-20:3  -153.30000000 -215.0000452 -9.159995e+01 0.0000000
## 10:4-20:3 -132.90000000 -194.6000452 -7.119995e+01 0.0000000
## 12:4-20:3 -107.50000000 -169.2000452 -4.579995e+01 0.0000000
## 14:4-20:3  -97.10000000 -158.8000452 -3.539995e+01 0.0000007
## 16:4-20:3  -76.90000000 -138.6000452 -1.519995e+01 0.0008421
## 18:4-20:3  -56.00000000 -117.7000452  5.700045e+00 0.1596387
## 20:4-20:3  -25.01111111  -88.4018815  3.837966e+01 0.9999964
```

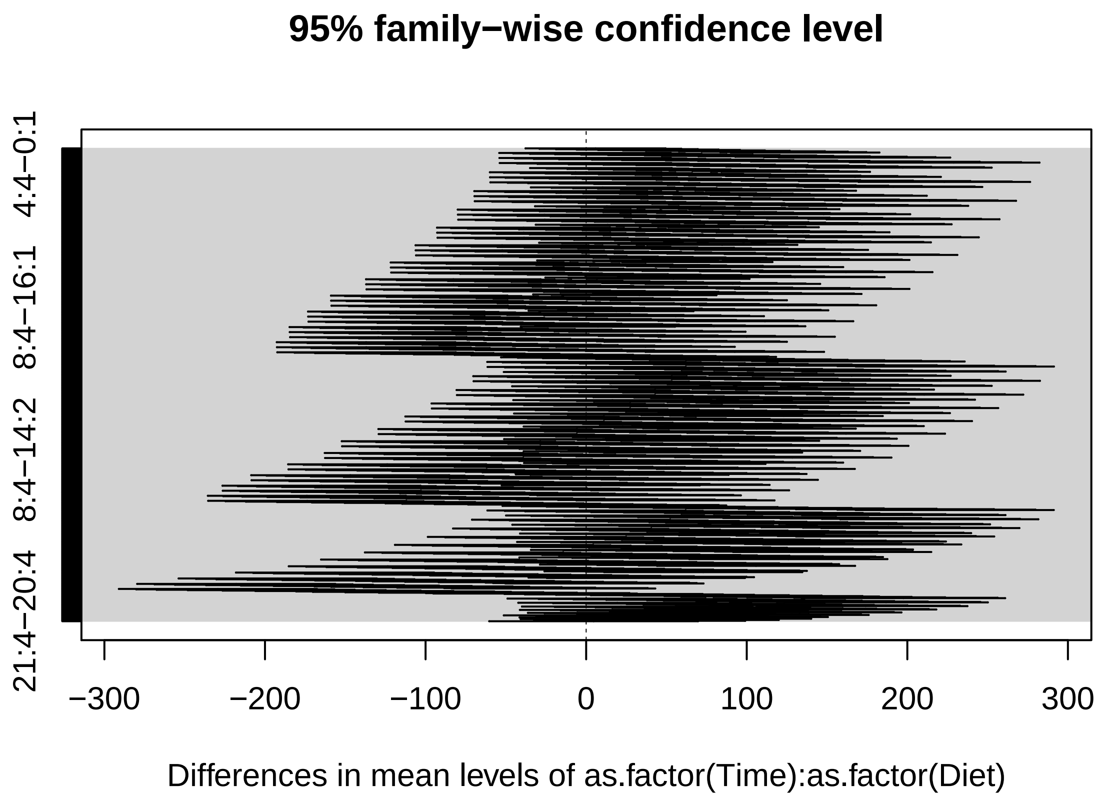
```
## 21:4-20:3  -20.34444444  -83.7352148  4.304633e+01 1.0000000
## 0:4-21:3  -229.30000000 -291.0000452 -1.676000e+02 0.0000000
## 2:4-21:3  -218.50000000 -280.2000452 -1.568000e+02 0.0000000
## 4:4-21:3  -205.80000000 -267.5000452 -1.441000e+02 0.0000000
## 6:4-21:3  -186.40000000 -248.1000452 -1.247000e+02 0.0000000
## 8:4-21:3  -164.70000000 -226.4000452 -1.030000e+02 0.0000000
## 10:4-21:3 -144.30000000 -206.0000452 -8.259995e+01 0.0000000
## 12:4-21:3 -118.90000000 -180.6000452 -5.719995e+01 0.0000000
## 14:4-21:3 -108.50000000 -170.2000452 -4.679995e+01 0.0000000
## 16:4-21:3  -88.30000000 -150.0000452 -2.659995e+01 0.0000191
## 18:4-21:3  -67.40000000 -129.1000452 -5.699955e+00 0.0125619
## 20:4-21:3  -36.41111111  -99.8018815  2.697966e+01 0.9807578
## 21:4-21:3  -31.74444444  -95.1352148  3.164633e+01 0.9985286
## 2:4-0:4     10.80000000  -50.9000452  7.250005e+01 1.0000000
## 4:4-0:4     23.50000000  -38.2000452  8.520005e+01 0.9999987
## 6:4-0:4     42.90000000  -18.8000452  1.046000e+02 0.7854926
## 8:4-0:4     64.60000000    2.8999548  1.263000e+02 0.0253896
## 10:4-0:4    85.00000000   23.2999548  1.467000e+02 0.0000604
## 12:4-0:4   110.40000000   48.6999548  1.721000e+02 0.0000000
## 14:4-0:4   120.80000000   59.0999548  1.825000e+02 0.0000000
## 16:4-0:4   141.00000000   79.2999548  2.027000e+02 0.0000000
## 18:4-0:4   161.90000000  100.1999548  2.236000e+02 0.0000000
## 20:4-0:4   192.88888889  129.4981185  2.562797e+02 0.0000000
## 21:4-0:4   197.55555556  134.1647852  2.609463e+02 0.0000000
## 4:4-2:4     12.70000000  -49.0000452  7.440005e+01 1.0000000
## 6:4-2:4     32.10000000  -29.6000452  9.380005e+01 0.9967741
## 8:4-2:4     53.80000000   -7.9000452  1.155000e+02 0.2333631
## 10:4-2:4    74.20000000   12.4999548  1.359000e+02 0.0019001
## 12:4-2:4    99.60000000   37.8999548  1.613000e+02 0.0000003
## 14:4-2:4   110.00000000   48.2999548  1.717000e+02 0.0000000
## 16:4-2:4   130.20000000   68.4999548  1.919000e+02 0.0000000
## 18:4-2:4   151.10000000   89.3999548  2.128000e+02 0.0000000
## 20:4-2:4   182.08888889  118.6981185  2.454797e+02 0.0000000
## 21:4-2:4   186.75555556  123.3647852  2.501463e+02 0.0000000
## 6:4-4:4     19.40000000  -42.3000452  8.110005e+01 1.0000000
## 8:4-4:4     41.10000000  -20.6000452  1.028000e+02 0.8596505
## 10:4-4:4    61.50000000   -0.2000452  1.232000e+02 0.0522863
## 12:4-4:4    86.90000000   25.1999548  1.486000e+02 0.0000313
## 14:4-4:4    97.30000000   35.5999548  1.590000e+02 0.0000007
## 16:4-4:4   117.50000000   55.7999548  1.792000e+02 0.0000000
## 18:4-4:4   138.40000000   76.6999548  2.001000e+02 0.0000000
## 20:4-4:4   169.38888889  105.9981185  2.327797e+02 0.0000000
## 21:4-4:4   174.05555556  110.6647852  2.374463e+02 0.0000000
## 8:4-6:4     21.70000000  -40.0000452  8.340005e+01 0.9999999
## 10:4-6:4    42.10000000  -19.6000452  1.038000e+02 0.8205140
## 12:4-6:4    67.50000000    5.7999548  1.292000e+02 0.0122398
## 14:4-6:4    77.90000000   16.1999548  1.396000e+02 0.0006176
## 16:4-6:4    98.10000000   36.3999548  1.598000e+02 0.0000005
## 18:4-6:4   119.00000000   57.2999548  1.807000e+02 0.0000000
## 20:4-6:4   149.98888889   86.5981185  2.133797e+02 0.0000000
## 21:4-6:4   154.65555556   91.2647852  2.180463e+02 0.0000000
## 10:4-8:4    20.40000000  -41.3000452  8.210005e+01 1.0000000
## 12:4-8:4    45.80000000  -15.9000452  1.075000e+02 0.6373750
## 14:4-8:4    56.20000000   -5.5000452  1.179000e+02 0.1539137
```

```
## 16:4-8:4    76.40000000  14.6999548  1.381000e+02 0.0009815
## 18:4-8:4    97.30000000  35.5999548  1.590000e+02 0.0000007
## 20:4-8:4   128.28888889  64.8981185  1.916797e+02 0.0000000
## 21:4-8:4   132.95555556  69.5647852  1.963463e+02 0.0000000
## 12:4-10:4   25.40000000 -36.3000452  8.710005e+01 0.9999881
## 14:4-10:4   35.80000000 -25.9000452  9.750005e+01 0.9773798
## 16:4-10:4   56.00000000  -5.7000452  1.177000e+02 0.1596387
## 18:4-10:4   76.90000000  15.1999548  1.386000e+02 0.0008421
## 20:4-10:4  107.88888889  44.4981185  1.712797e+02 0.0000000
## 21:4-10:4  112.55555556  49.1647852  1.759463e+02 0.0000000
## 14:4-12:4   10.40000000 -51.3000452  7.210005e+01 1.0000000
## 16:4-12:4   30.60000000 -31.1000452  9.230005e+01 0.9988043
## 18:4-12:4   51.50000000 -10.2000452  1.132000e+02 0.3316631
## 20:4-12:4   82.48888889  19.0981185  1.458797e+02 0.0002893
## 21:4-12:4   87.15555556  23.7647852  1.505463e+02 0.0000640
## 16:4-14:4   20.20000000 -41.5000452  8.190005e+01 1.0000000
## 18:4-14:4   41.10000000 -20.6000452  1.028000e+02 0.8596505
## 20:4-14:4   72.08888889   8.6981185  1.354797e+02 0.0059992
## 21:4-14:4   76.75555556  13.3647852  1.401463e+02 0.0016345
## 18:4-16:4   20.90000000 -40.8000452  8.260005e+01 1.0000000
## 20:4-16:4   51.88888889 -11.5018815  1.152797e+02 0.3803289
## 21:4-16:4   56.55555556  -6.8352148  1.199463e+02 0.1891210
## 20:4-18:4   30.98888889 -32.4018815  9.437966e+01 0.9991272
## 21:4-18:4   35.65555556 -27.7352148  9.904633e+01 0.9864104
## 21:4-20:4    4.66666667 -60.3708915  6.970422e+01 1.0000000
```

```
plot(TukeyHSD(anova2Wdieta))
```


95% family−wise confidence level
12−0
20−2
14−6
18−10
21−20
0
50
100
150
200
Differences in mean levels of as.factor(Time)


95% family−wise confidence level
2−1
3−1
4−1
3−2
4−2
4−3
−20
−10
0
10
20
30
40
Differences in mean levels of as.factor(Diet)



2h

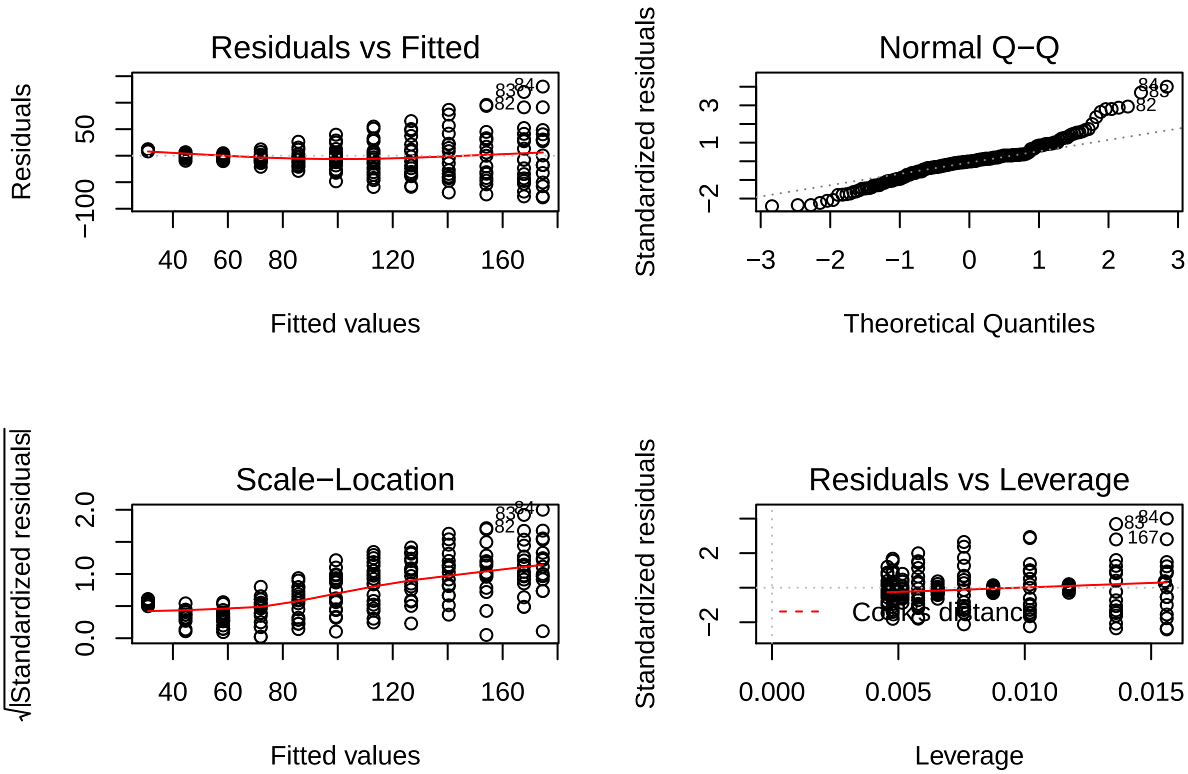
We implement all the 4 regressions, one for each diet, and represent them below

```
#h
dadosdieta1<-subset(dados22,dados22[,4]==1)
dadosdieta2<-subset(dados22,dados22[,4]==2)
dadosdieta3<-subset(dados22,dados22[,4]==3)
dadosdieta4<-subset(dados22,dados22[,4]==4)
regdieta1<-lm(weight~Time,dadosdieta1)
summary(regdieta1)
```

```
##
## Call:
## lm(formula = weight ~ Time, data = dadosdieta1)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -78.609 -15.677  -0.324  11.069 130.391
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept)  30.9310     4.0948   7.554 1.15e-12 ***
## Time          6.8418     0.3286  20.822  < 2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

```
##
## Residual standard error: 32.85 on 218 degrees of freedom
## Multiple R-squared:  0.6654, Adjusted R-squared:  0.6639
## F-statistic: 433.5 on 1 and 218 DF,  p-value: < 2.2e-16
```

```
par(mfrow=c(2,2))
plot(regdieta1)
```

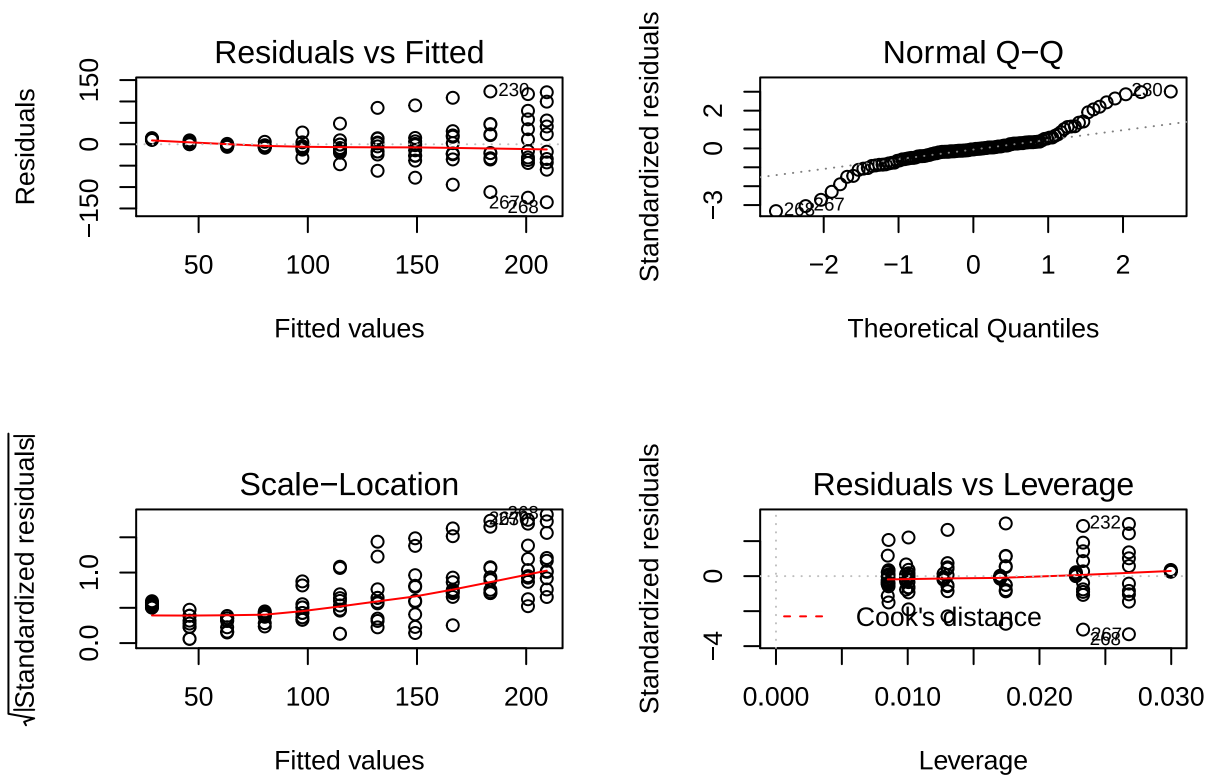



```
regdieta2<-lm(weight~Time,dadosdieta2)
summary(regdieta2)
```

```
##
## Call:
## lm(formula = weight ~ Time, data = dadosdieta2)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -135.425  -16.780   -2.116   11.616  123.402
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept)  28.6336     7.1597   3.999 0.000111 ***
## Time          8.6091     0.5572  15.449  < 2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 41.36 on 118 degrees of freedom
```

```
## Multiple R-squared:  0.6692, Adjusted R-squared:  0.6664
## F-statistic: 238.7 on 1 and 118 DF,  p-value: < 2.2e-16
```

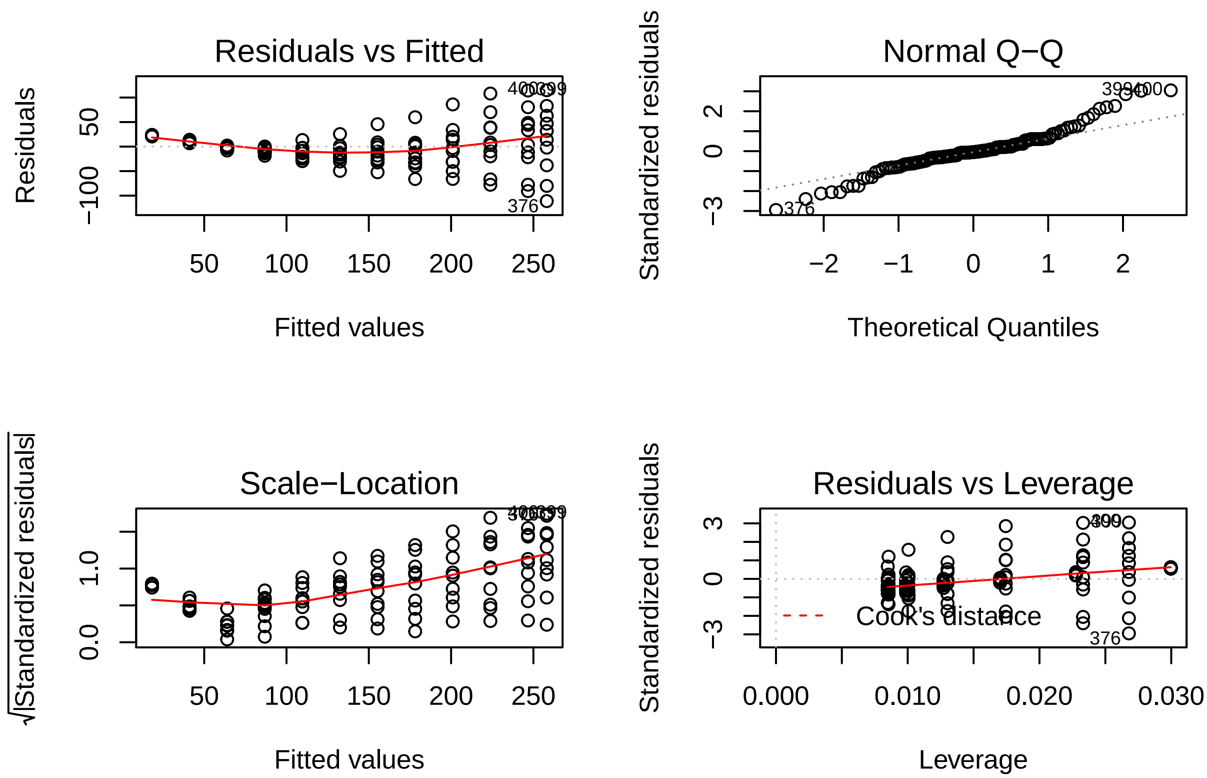
```r
plot(regdieta2)
```




```r
regdieta3<-lm(weight~Time,dadosdieta3)
summary(regdieta3)
```

```
##
## Call:
## lm(formula = weight ~ Time, data = dadosdieta3)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -111.131  -19.056   -1.865   15.484  114.869
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept)   18.250      6.617   2.758  0.00674 **
## Time          11.423      0.515  22.181  < 2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 38.22 on 118 degrees of freedom
## Multiple R-squared:  0.8066, Adjusted R-squared:  0.8049
## F-statistic:   492 on 1 and 118 DF,  p-value: < 2.2e-16
```

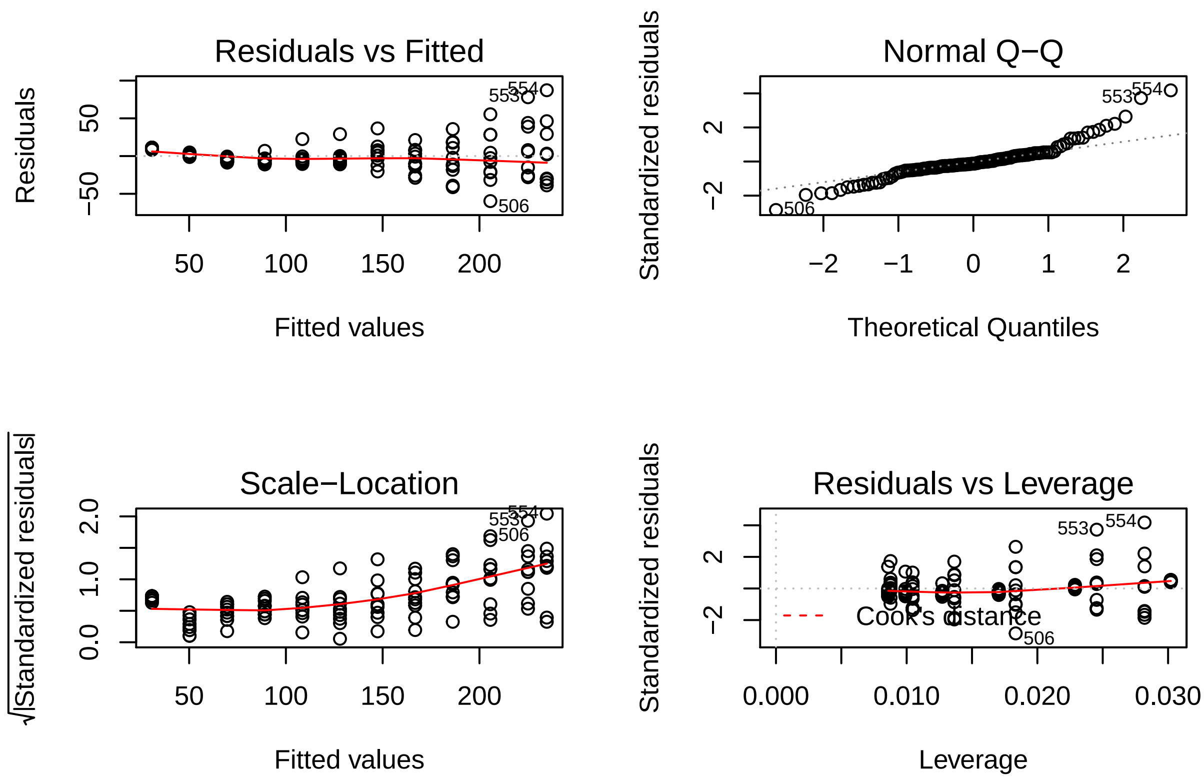
```r
plot(regdieta3)
```




```r
regdieta4<-lm(weight~Time,dadosdieta4)
summary(regdieta4)
```

```
##
## Call:
## lm(formula = weight ~ Time, data = dadosdieta4)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -59.651  -8.971  -2.650   7.849  87.206
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept)  30.7921     3.6817   8.363 1.55e-13 ***
## Time          9.7144     0.2904  33.454  < 2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 21.18 on 116 degrees of freedom
## Multiple R-squared:  0.9061, Adjusted R-squared:  0.9053
## F-statistic:  1119 on 1 and 116 DF,  p-value: < 2.2e-16
```

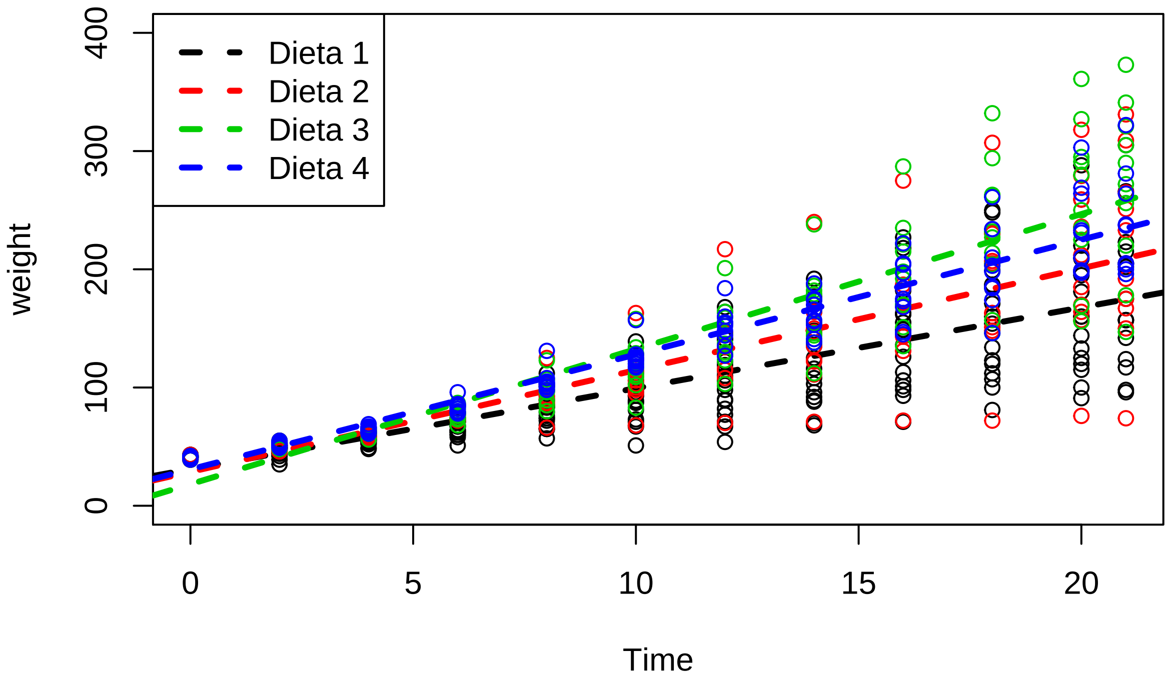
```
plot(regdieta4)
```



What seems obvious is that the regression model is not adequate, the main problem being the obvious problem with the variances of the residuals, which are not constant, but proportional to time. This is expected, as the variance in weights increases as time goes by, and individual variation across chicks kicks in. This is another indication that we would need a model that incorporates the repeated measurements nature of the data - a mixed model.

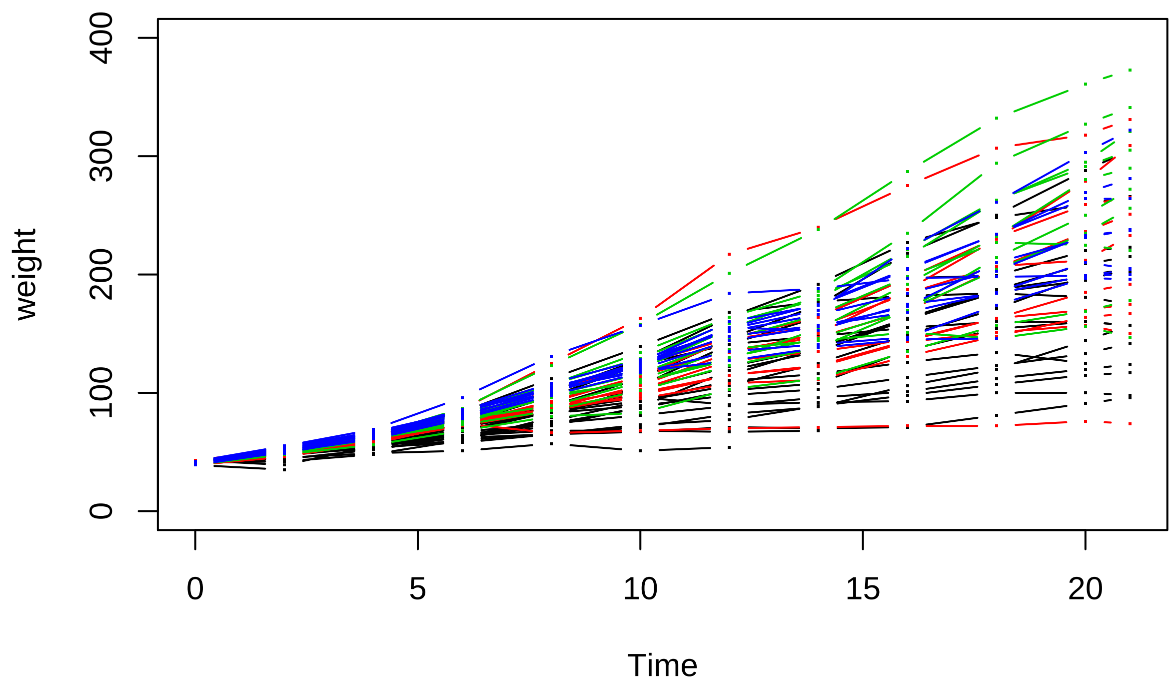
Nonetheless, we can put all the regressions together in a simple plot, and it becomes clear that diets 3 and 4 are the best, while 1 is the worst and 2 is somewhere in between the others.

```
par(mfrow=c(1,1))
with(dadosdieta1,plot(weight~Time,ylim=c(0,400)))
abline(regdieta1,lwd=3,lty=2,col=1)
with(dadosdieta2,points(weight~Time,col=2))
abline(regdieta2,lwd=3,lty=2,col=2)
with(dadosdieta3,points(weight~Time,col=3))
abline(regdieta3,lwd=3,lty=2,col=3)
with(dadosdieta4,points(weight~Time,col=4))
abline(regdieta4,lwd=3,lty=2,col=4)
legend("topleft",lty=2,lwd=3,col=1:4,legend=paste0("Dieta ",1:4))
```

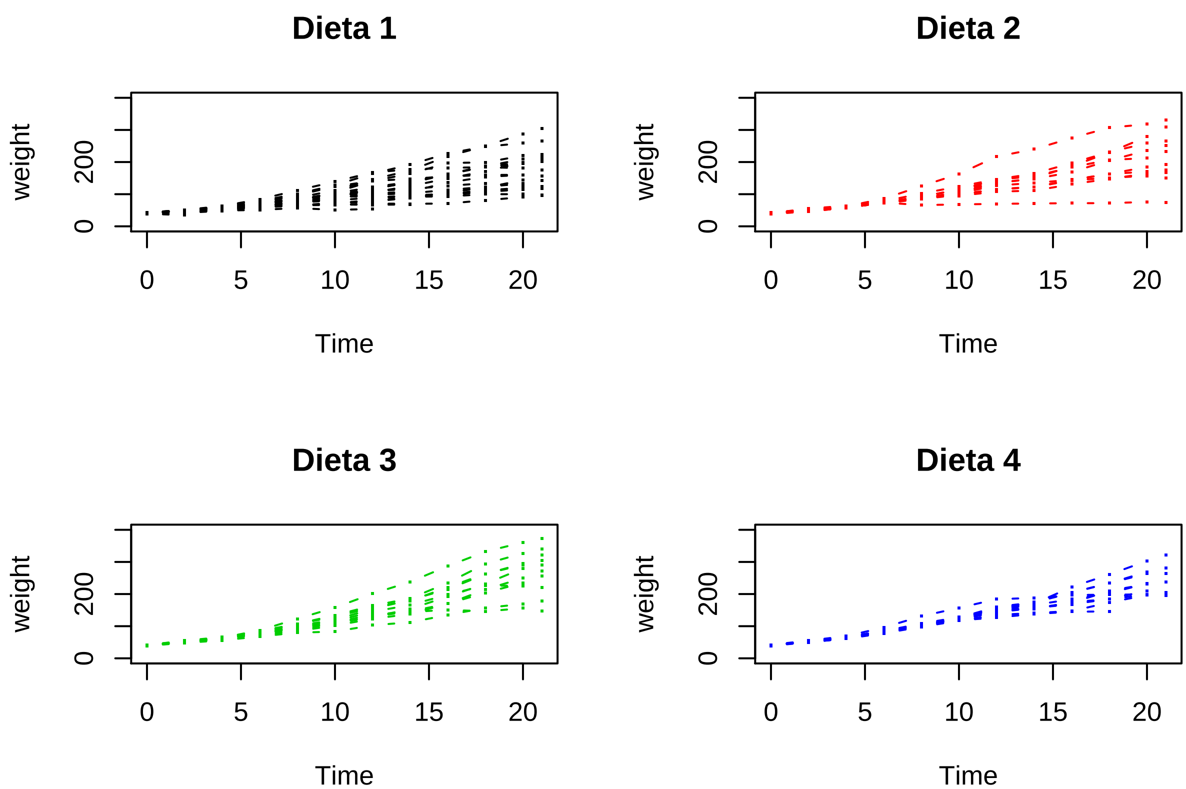


Note that all the individual chicks weights could be plotted, but that becomes a big mess. Nonetheless, that reminds us that observations are related over time and that we ignored that non-independence in all the above

```
with(dados22[dados22$Chick==1,],plot(weight~Time,type="b",pch=".",col=Diet,ylim=c(0,400)))
for(i in 2:50){
  with(dados22[dados22$Chick==i,],lines(weight~Time,type="b",pch=".",col=Diet,ylim=c(0,400)))
}
```




```
#all, with a plot per diet
par(mfrow=c(2,2))
d=1
with(dados22[dados22$Chick==1,],plot(weight~Time,type="b",pch=".",col=Diet,
main=paste0("Dieta ",d),ylim=c(0,400)))
for(i in 2:50){
  if (dados22$Diet[dados22$Chick==i][1]==d){
  with(dados22[dados22$Chick==i,],lines(weight~Time,type="b",pch=".",col=Diet))
    } else {
      d=d+1
    with(dados22[dados22$Chick==i,],plot(weight~Time,type="b",pch=".",col=Diet,
main=paste0("Dieta ",d),ylim=c(0,400)))
  }
}
```



Exercício 3

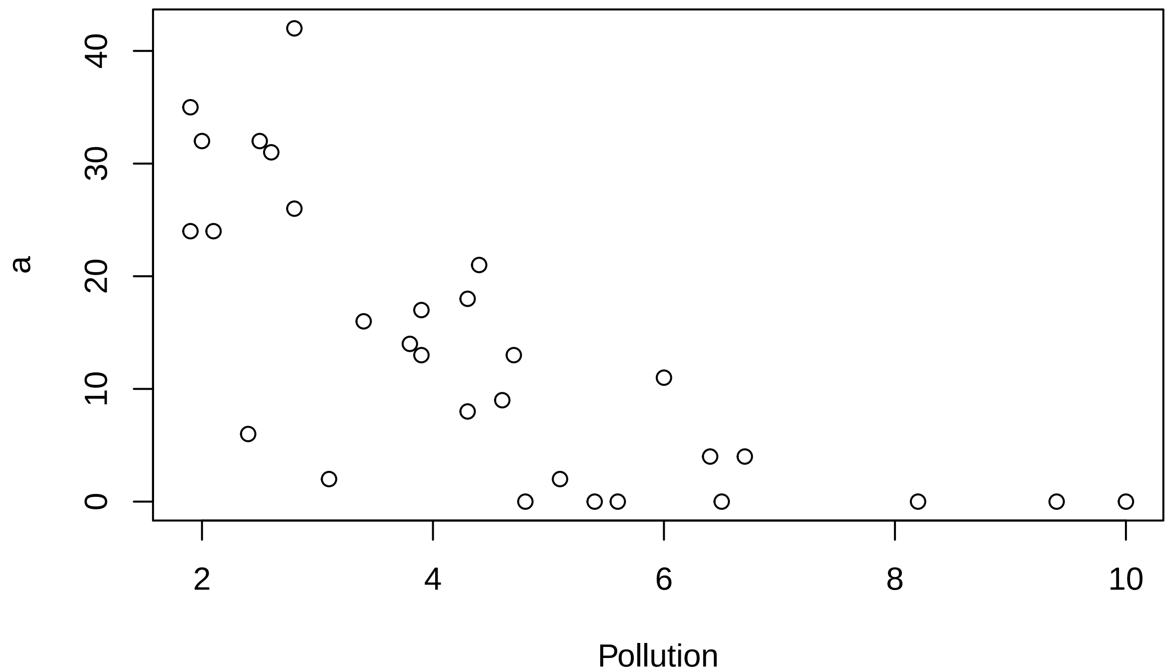
Com base nos dados dados21.txt, crie um objecto `dados1` apenas com os dados da espécie `a`, e crie um modelo muito simples que dê a estimativa da abundância de indivíduos da espécie `a` em função de 4 níveis de poluição (`Pollution`) de um sistema. Comente aspectos positivos e negativos deste modelo.

First we create object `dados1`, removing the data from species `b`,`c`,`d`,`e`.

```
dados1=dados21[,-c(3:6)]
```

We can look at the abundance of `a` as a function of `Pollution`

```
with(dados1,plot(Pollution,a))
```



Now, we need to set up 4 levels of `Pollution`. Without a better way of doing it, we simply divide the range of the data into 4 equal bins of `Pollution` (looking at the data, that might be about less than 4, 4 to 6, 6 to 8, more than 8). We could do this manually

There is actually a convenient function that allows us to separate a continuous variable into a factor with levels

```
dados1$polclass=cut(dados1$Pollution, breaks=c(0,4,6,8,12))
```

and now we could simply use function `tapply` to get the same means per levels of pollution

```
mbyPolclass=tapply(X=dados1$a,INDEX=dados1$polclass,FUN=mean)
```

This is essentially a regression model, a very simple one. It equates to the simplest regression model, a response variable as a function of one factor. Or in other words, it is the equivalent to an ANOVA!

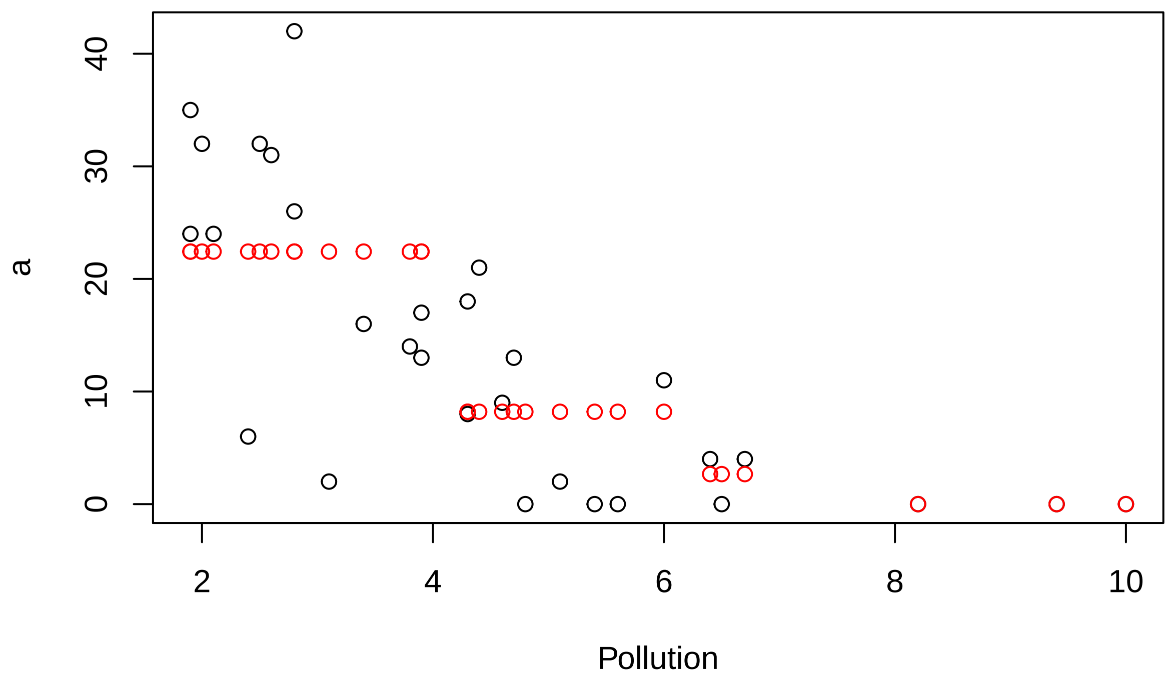
```
lma=lm(a~0+dados1$polclass,data=dados1)
summary(lma)
```

```
##
## Call:
## lm(formula = a ~ 0 + dados1$polclass, data = dados1)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -20.429  -6.371   0.400   4.493  19.571
##
## Coefficients:
##                       Estimate Std. Error t value Pr(>|t|)
```

```
## dados1$polclass(0,4]    22.429      2.500   8.970 1.94e-09 ***
## dados1$polclass(4,6]     8.200      2.958   2.772   0.0102 *
## dados1$polclass(6,8]     2.667      5.401   0.494   0.6257
## dados1$polclass(8,12]    0.000      5.401   0.000   1.0000
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 9.356 on 26 degrees of freedom
## Multiple R-squared:  0.7727, Adjusted R-squared:  0.7377
## F-statistic:  22.1 on 4 and 26 DF,  p-value: 4.774e-08
```

Is this a good representation of the data? Probably not!

```r
with(dados1,plot(Pollution,a))
points(dados1$Pollution,predict(lma),col="red")
```



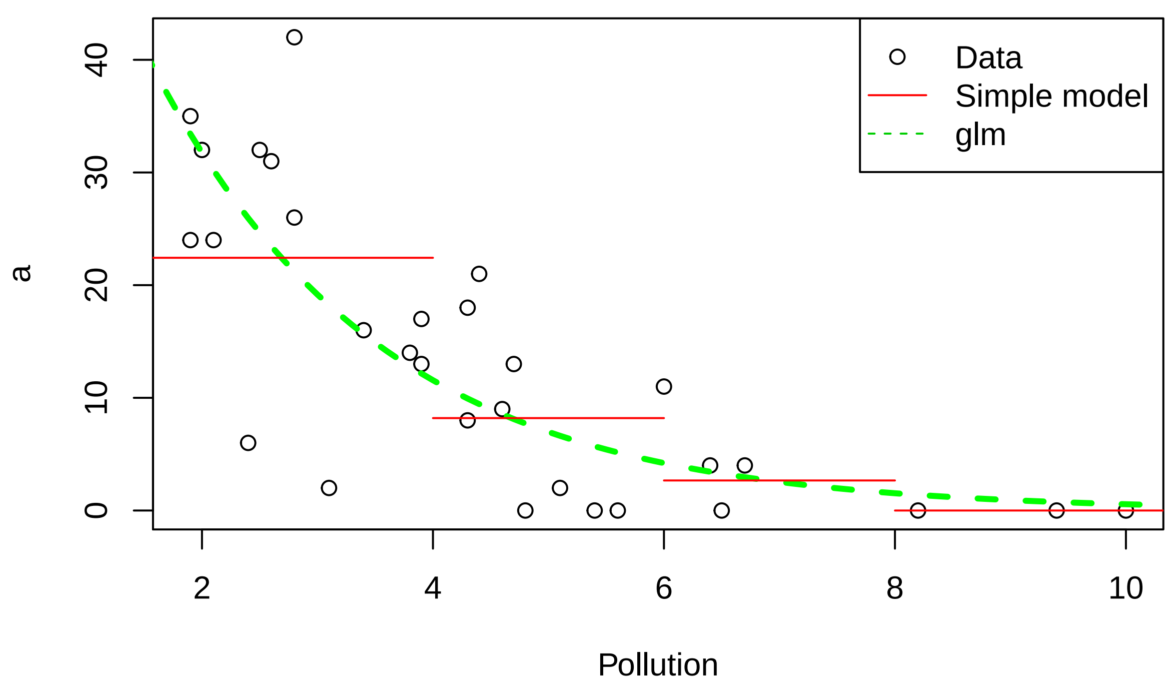
A GLM might do better!

```r
glma=glm(a~Pollution,data=dados1,family=poisson)
summary(glma)
```

```
##
## Call:
## glm(formula = a ~ Pollution, family = poisson, data = dados1)
##
## Deviance Residuals:
##     Min       1Q   Median       3Q      Max
## -4.8582  -1.7026   0.1216   1.2326   3.9779
```

```
##
## Coefficients:
##             Estimate Std. Error z value Pr(>|z|)
## (Intercept)  4.46907    0.13132   34.03   <2e-16 ***
## Pollution   -0.50534    0.03903  -12.95   <2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## (Dispersion parameter for poisson family taken to be 1)
##
##     Null deviance: 391.95  on 29  degrees of freedom
## Residual deviance: 153.36  on 28  degrees of freedom
## AIC: 259.29
##
## Number of Fisher Scoring iterations: 5
```

```
with(dados1,plot(Pollution,a))
xPols=seq(1,11,by=0.1)
apredsglm=predict(glma,newdata=data.frame(Pollution=xPols),type="response")
lines(xPols,apredsglm,col="green",lty=2,lwd=3)
#points(dados1$Pollution,predict(lma),col="red")
segments(x0=c(0,4,6,8,12)[-5],y0=mbyPolclass,x1=c(0,4,6,8,12)[-1],y1=mbyPolclass,col="red")
#add legend
legend("topright",col=1:3,legend=c("Data","Simple model","glm"),lty=c(NA,1,2),pch=c(1,NA,NA))
```



One might think that the GLM model provides a more sensible fit, but in fact when we compare the fit of the two models, using a tool like AIC, the simpler model with the means is preferred (this was surprising to me, also because the "simpler" model is not simpler at all, in the sense that it uses more parameters, and AIC penalizes for model complexity, i.e. for the number of parameters)

```
AIC(glma,lma)
```

```
##      df      AIC
## glma  2 259.2884
## lma   5 225.0016
```

One disadvantage of said model is that the classes were chosen arbitrarily, so depending on what classes one chooses, one could end with different results!

As an example, if the second class was defined by values larger than 5.9 rather than 6

```
dados1$polclass2=cut(dados1$Pollution, breaks=c(0,4,5.9,8,12))
mbyPolclass2=tapply(X=dados1$a,INDEX=dados1$polclass2,FUN=mean)
mbyPolclass
```

```
##     (0,4]     (4,6]     (6,8]    (8,12]
## 22.428571  8.200000  2.666667  0.000000
```

```
mbyPolclass2
```

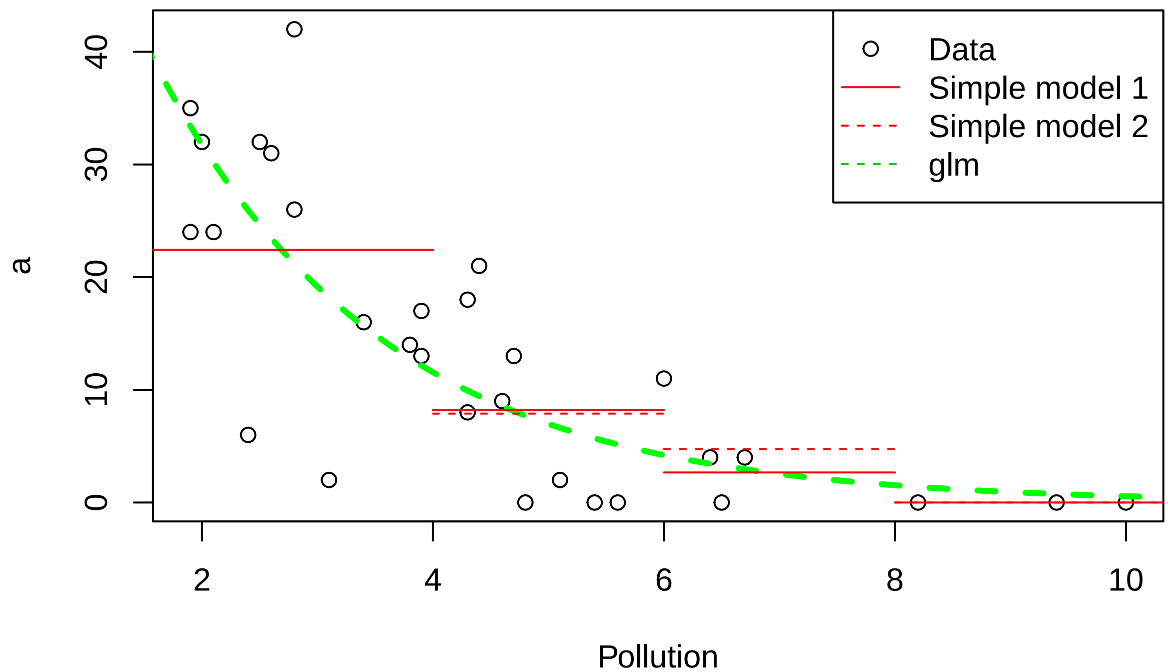
```
##     (0,4]   (4,5.9]   (5.9,8]    (8,12]
## 22.428571  7.888889  4.750000  0.000000
```

```
lma2=lm(a~0+dados1$polclass2,data=dados1)
summary(lma2)
```

```
##
## Call:
## lm(formula = a ~ 0 + dados1$polclass2, data = dados1)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -20.429  -6.294   0.000   5.965  19.571
##
## Coefficients:
##                          Estimate Std. Error t value Pr(>|t|)
## dados1$polclass2(0,4]      22.429      2.524   8.886 2.34e-09 ***
## dados1$polclass2(4,5.9]     7.889      3.148   2.506   0.0188 *
## dados1$polclass2(5.9,8]     4.750      4.722   1.006   0.3237
## dados1$polclass2(8,12]      0.000      5.453   0.000   1.0000
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 9.444 on 26 degrees of freedom
## Multiple R-squared:  0.7684, Adjusted R-squared:  0.7327
## F-statistic: 21.56 on 4 and 26 DF,  p-value: 6.071e-08
```

```
with(dados1,plot(Pollution,a))
xPols=seq(1,11,by=0.1)
apredsglm=predict(glma,newdata=data.frame(Pollution=xPols),type="response")
lines(xPols,apredsglm,col="green",lty=2,lwd=3)
#points(dados1$Pollution,predict(lma),col="red")
segments(x0=c(0,4,6,8,12)[-5],y0=mbyPolclass,x1=c(0,4,6,8,12)[-1],y1=mbyPolclass,col="red")
segments(x0=c(0,4,6,8,12)[-5],y0=mbyPolclass2,x1=c(0,4,6,8,12)[-1],y1=mbyPolclass2,col="red",lty=2)
#add legend
```

```r
legend("topright",col=c(1,2,2,3),
legend=c("Data","Simple model 1","Simple model 2","glm"),lty=c(NA,1,2,2),pch=c(1,NA,NA,NA))
```



Exercício 4

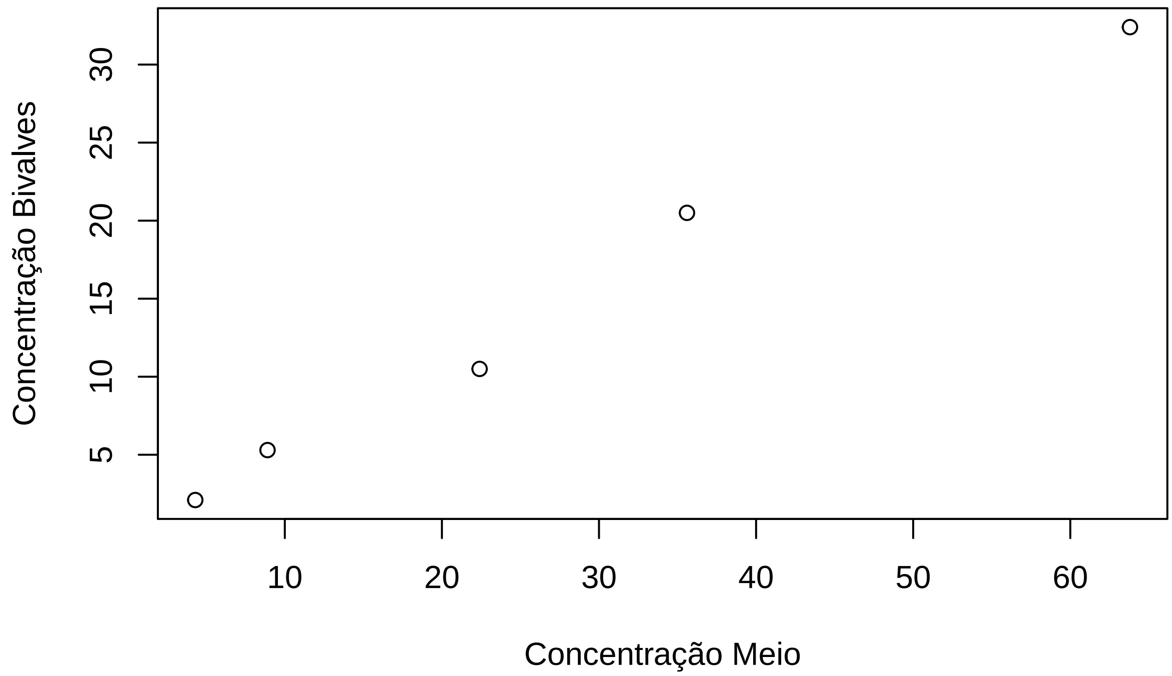
A concentração de um contaminante no tecido muscular de um bivalve é uma função directa da concentração no meio. A partir dos seguintes vectores correspondentes a pares de concentrações medidas no bivalve e no meio (bivalve, meio; (2.1, 4.3); (5.3, 8.9); (10.5, 22.4);(20.5,35.6); (32.4, 63.8), crie uma função que permita descrever este processo ecológico e gerar estimativas da concentração de contaminante no bivalve em função da concentração no meio.

a. Comente as limitações do modelo.
b. A partir de concentrações da ordem dos 80 no bivalve, o organismo já não tem capacidade para acumular mais. Introduza esta particularidade no modelo.
c. Desenvolva uma componente do modelo que traduza a desintoxicação numa estação de depuração.
d. Crie uma função que dê o tempo de permanência na depuração em função do nível de contaminação até se atingirem os níveis aceitáveis para consumo 5.5.

4a

Plot the data

```
biv=c(2.1,5.3,10.5,20.5,32.4)
mei=c(4.3,8.9,22.4,35.6,63.8)
plot(biv~mei,xlab="Concentração Meio",ylab="Concentração Bivalves")
```

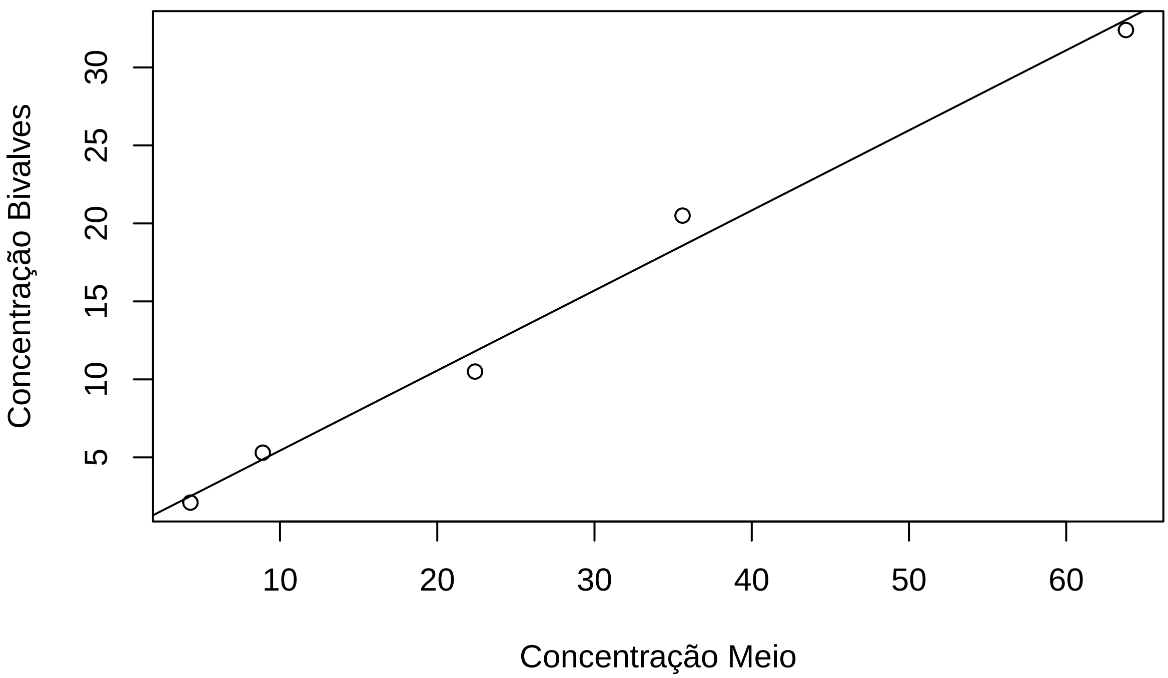


The simplest function that describes this is a linear model

```
lmbiv=lm(biv~mei)
summary(lmbiv)
```

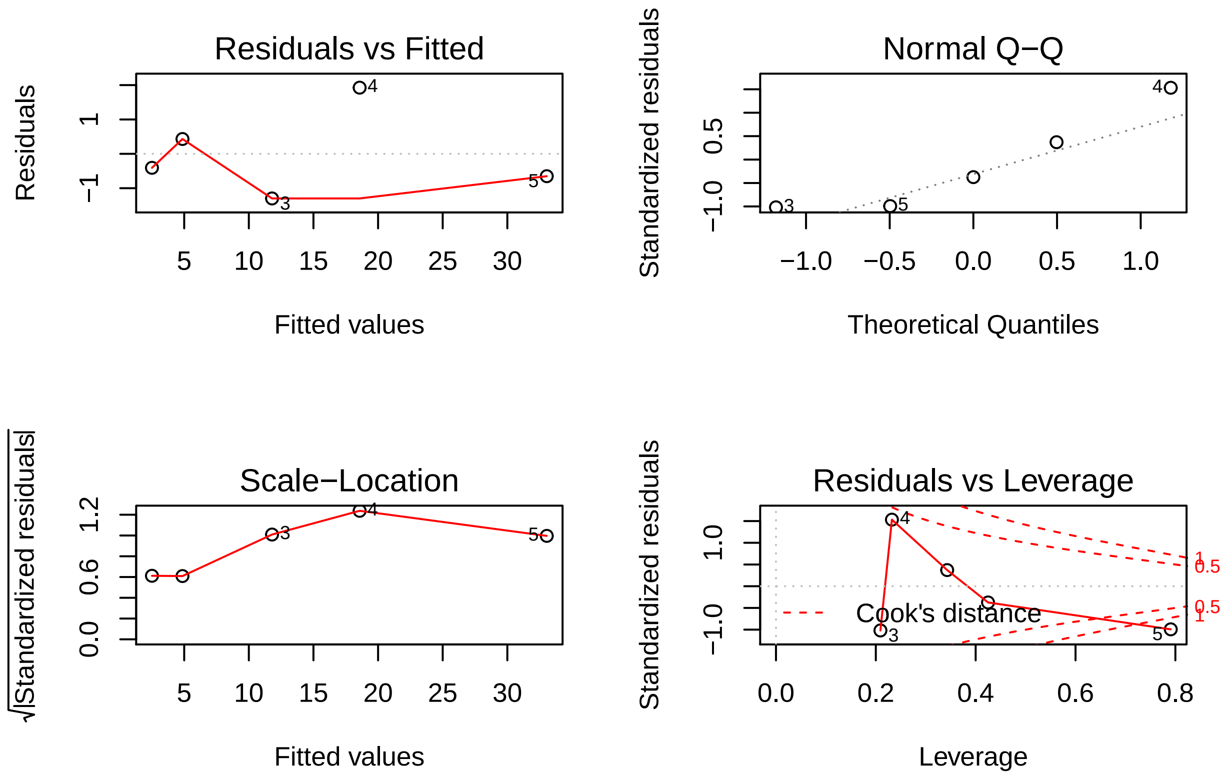
```
##
## Call:
## lm(formula = biv ~ mei)
##
## Residuals:
##       1       2       3       4       5
## -0.4071  0.4315 -1.2986  1.9252 -0.6511
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept)  0.29969    1.03208    0.29 0.790448
## mei          0.51334    0.02995   17.14 0.000433 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 1.434 on 3 degrees of freedom
## Multiple R-squared:  0.9899, Adjusted R-squared:  0.9865
## F-statistic: 293.8 on 1 and 3 DF,  p-value: 0.0004327
```

```r
plot(biv~mei,xlab="Concentração Meio",ylab="Concentração Bivalves")
abline(lmbiv)
```



We can see if this model is adequate or not (well, really, we could if we had enough data, with 5 data points, there's hardly much one can say!). Nonetheless, we can plot diagnostics plots:

```r
par(mfrow=c(2,2))
plot(lmbiv)
```



A model might then be

```
mypreds1=function(meio){coef(lmbiv)[1]+coef(lmbiv)[2]*meio}
```

The limitations of this model stems from the data; with just 5 points is hard to say if such a simple model is adequate or not. In biological phenomena it is unlikely that relationships are linear for the full possible range of the covariates, i.e., at sometime they might stabilize.

4b

We would simply add a constraint to the model we defined above

```
mypreds2=function(meio){
  mypred=mypreds1(meio)
  res=ifelse(mypred<80,mypred,80)
  return(res)
}
```

testing it

```
mypreds1(c(2,40,79,80,120,200))
```

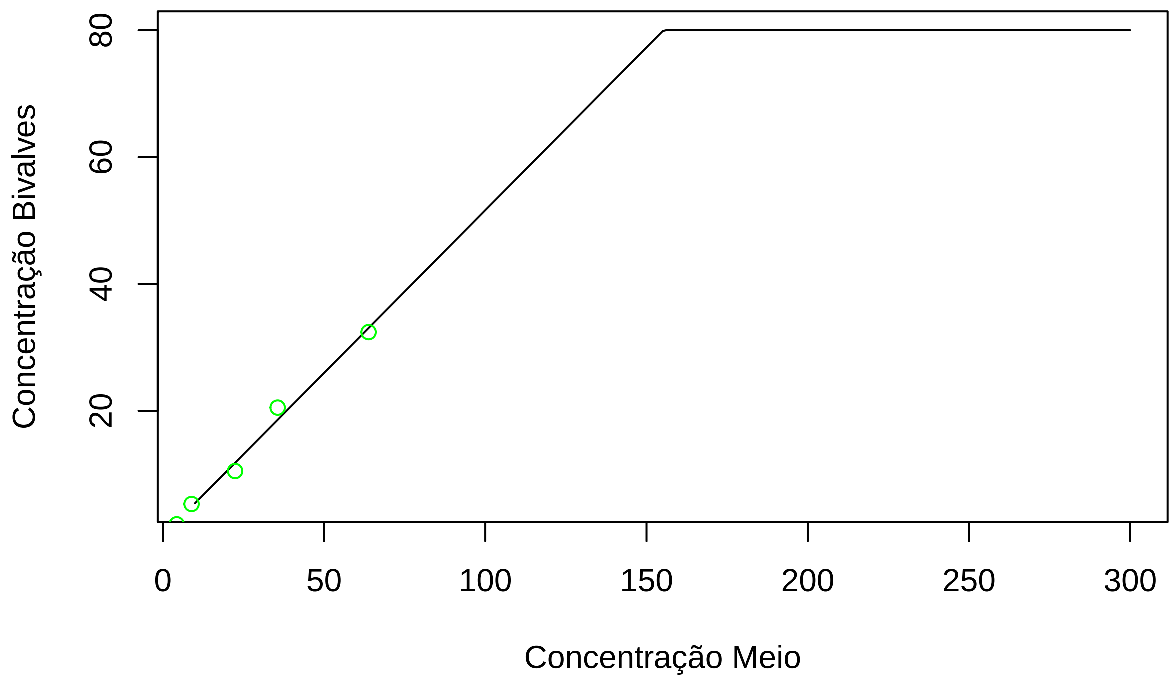
```
## [1]   1.326377  20.833484  40.853935  41.367280  61.901076 102.968669
```

```
mypreds2(c(2,40,79,80,120,200))
```

```
## [1]  1.326377 20.833484 40.853935 80.000000 80.000000 80.000000
```

We can actually plot the function

```
meios=seq(10,300,by=1)
preds2=mypreds2(meios)
plot(meios,preds2,xlab="Concentração Meio",ylab="Concentração Bivalves",type="l")
points(biv~mei,col="green")
```



4c

It depends on how we assume the cleaning process occurs. Is it a constant value? A value that depends on the initial value? A value that depends on time? These actually correspond to different conceptual "models" of how the system might behave.

We can try each of these

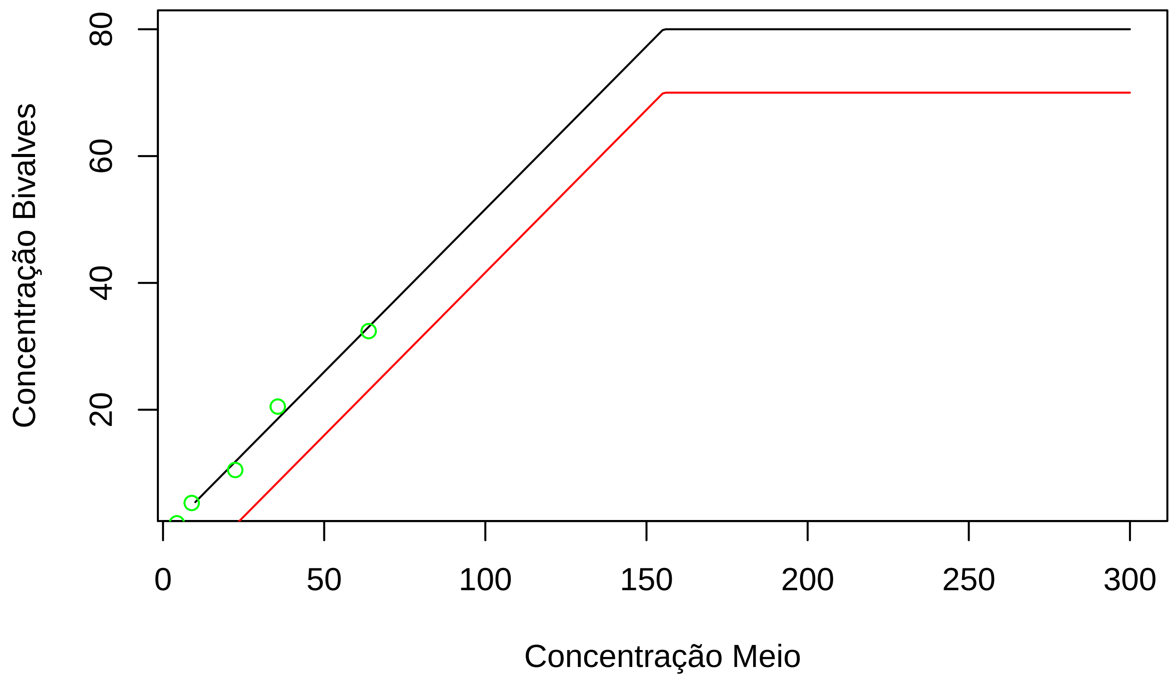
A constant cleaning rate

```
mypreds3=function(meio){
  mypred=mypreds1(meio)
  resI=ifelse(mypred<80,mypred,80)
  #always removes 10 of what it accumulated
  res=resI-10
  return(res)
}
```

We can plot the resulting function on top of the previous one

```
plot(meios,preds2,xlab="Concentração Meio",ylab="Concentração Bivalves",type="l")
points(biv~mei,col="green")
```

```r
preds3=mypreds3(meios)
lines(meios,preds3,col="red")
```

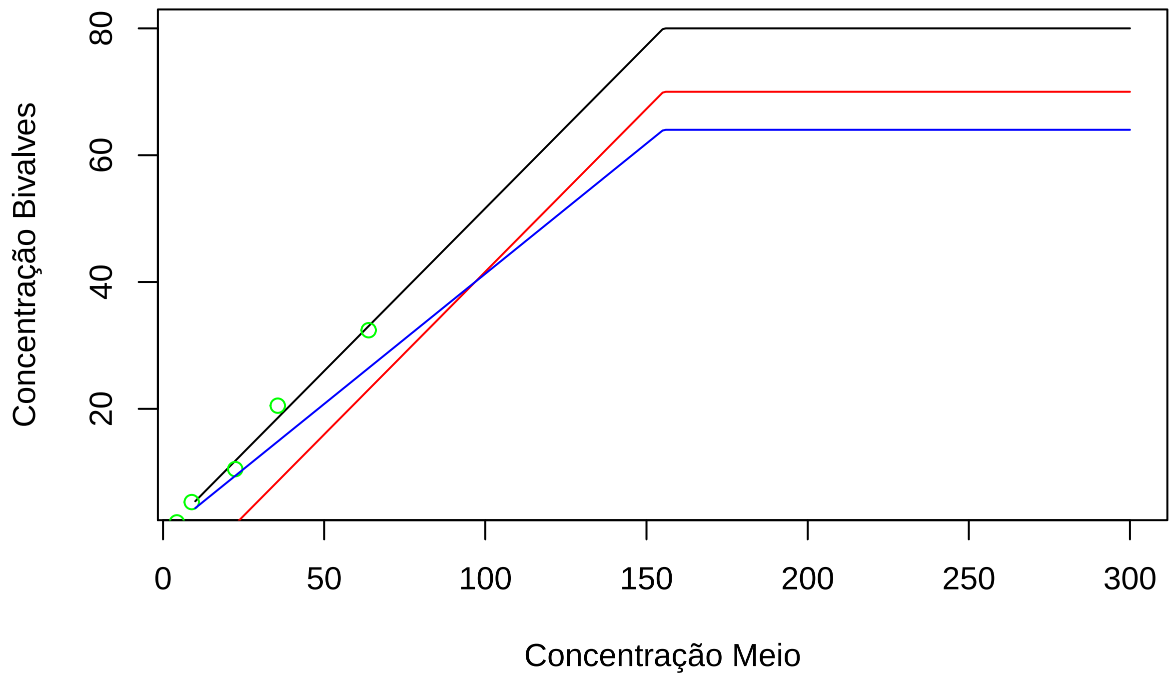


A value that depends on the existing value, e.g. cleans up 20% of the contamination

```r
mypreds4=function(meio){
  mypred=mypreds1(meio)
  resI=ifelse(mypred<80,mypred,80)
  #always removes 20% of the accoumulated
  res=resI*0.8
  return(res)
}
```

Plotting all of them

```r
plot(meios,preds2,xlab="Concentração Meio",ylab="Concentração Bivalves",type="l")
points(biv~mei,col="green")
preds4=mypreds4(meios)
lines(meios,preds3,col="red")
lines(meios,preds4,col="blue")
```



A value that depends on a time spent cleaning, say "time", that can be from (say!) 10 minutes to 100 minutes

```
mypreds5=function(meio,tempo){
  mypred=mypreds2(meio)
  resI=ifelse(mypred<80,mypred,80)
  #always removes 10 of what it accomulated
  res=resI-resI*(0.8*tempo/100)
  return(res)
}
```

We can now test them all

```
mypreds3(c(50,60,70,80,90,100))
```

```
## [1] 15.96693 21.10038 26.23383 31.36728 36.50073 41.63418
```

```
mypreds4(c(50,60,70,80,90,100))
```

```
## [1] 20.77355 24.88031 28.98706 33.09382 37.20058 41.30734
```

```
mypreds5(c(50,60,70,80,90,100),tempo=40)
```

```
## [1] 17.65751 21.14826 24.63901 54.40000 54.40000 54.40000
```

```
mypreds5(c(50,60,70,80,90,100),tempo=50)
```

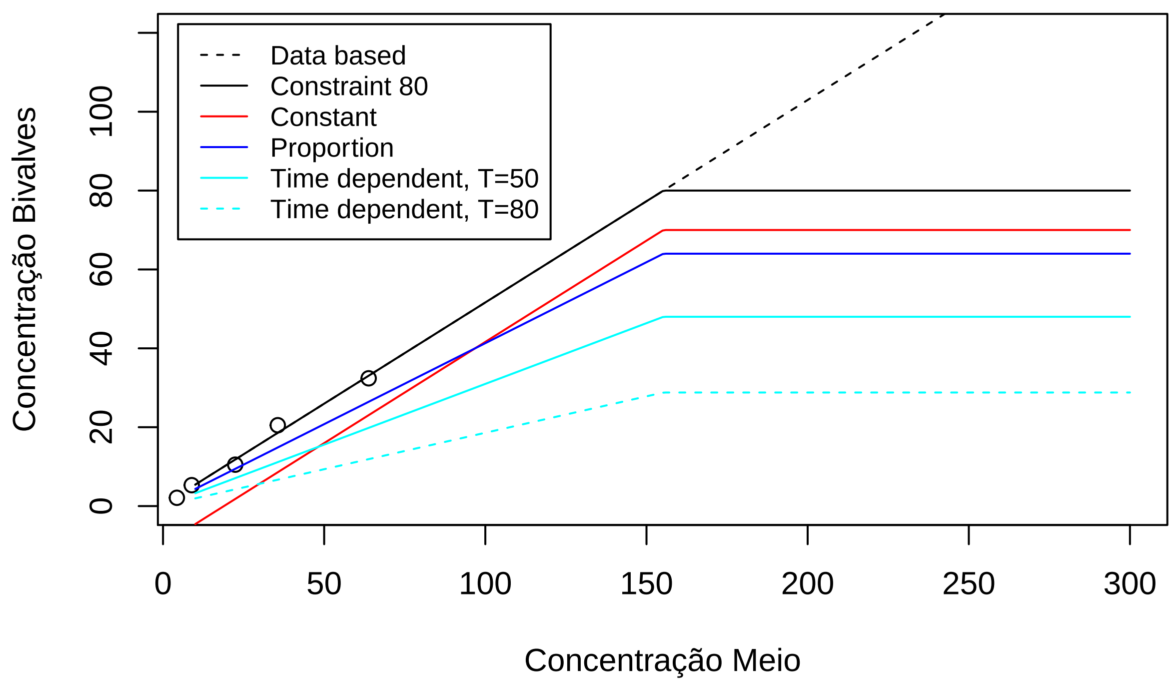
```
## [1] 15.58016 18.66023 21.74030 48.00000 48.00000 48.00000
```

```
mypreds5(c(50,60,70,80,90,100),tempo=90)
```

```
## [1]  7.270741  8.708107 10.145473 22.400000 22.400000 22.400000
```

Plotting all of them

```
plot(meios,mypreds1(meios),xlab="Concentração Meio",
ylab="Concentração Bivalves",type="l",lty=2,ylim=c(0,120))
lines(meios,preds2)
points(biv~mei,col=1)
preds5=mypreds5(meios,50)
preds6=mypreds5(meios,80)
lines(meios,preds3,col=2)
lines(meios,preds4,col=4)
lines(meios,preds5,col=5)
lines(meios,preds6,col=5,lty=2)
legend("topleft",lty=c(2,1,1,1,1,2),col=c(1,1,2,4,5,5),
legend=c("Data based","Constraint 80","Constant","Proportion",
"Time dependent, T=50","Time dependent, T=80"),inset=0.02,cex=0.8)
```



Please take a minute to think about the conceptual model that is behind this exercise and what might be its components. A couple of really nice contributions are shown in Figures 1 and 2 below, by Mariana Cravo (Obrigado Mariana!) and Joana Araújo (Obrigado Joana).

4d

This would depend on the function we selected above, and it will be of course a function of the initial concentration. It also assumes that the cleaning process process works until no contaminant is present, which might be an oversimplification.

Lets say it was

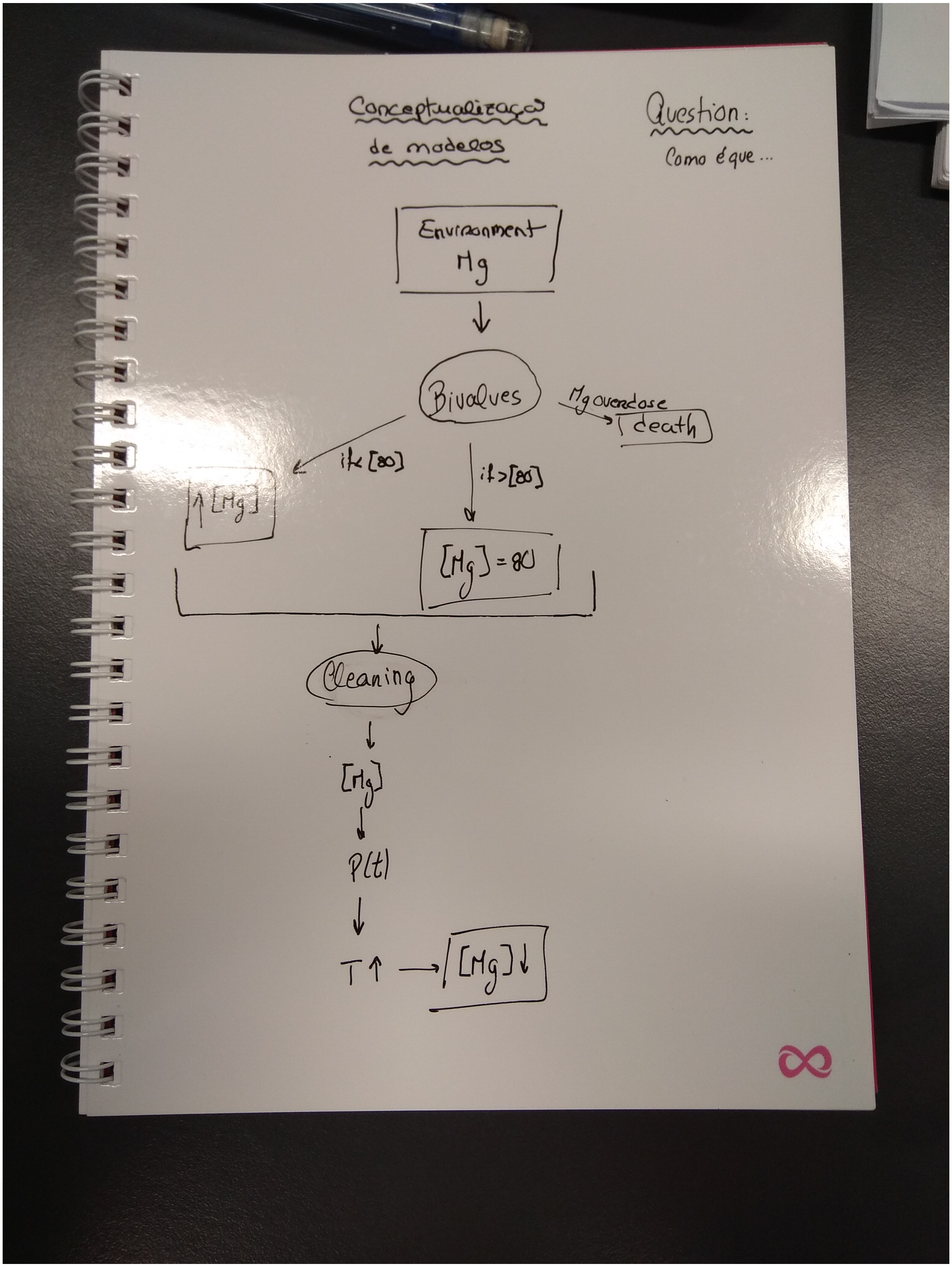


Figure 1: Modelo conceptual do problema, by Mariana Cravo

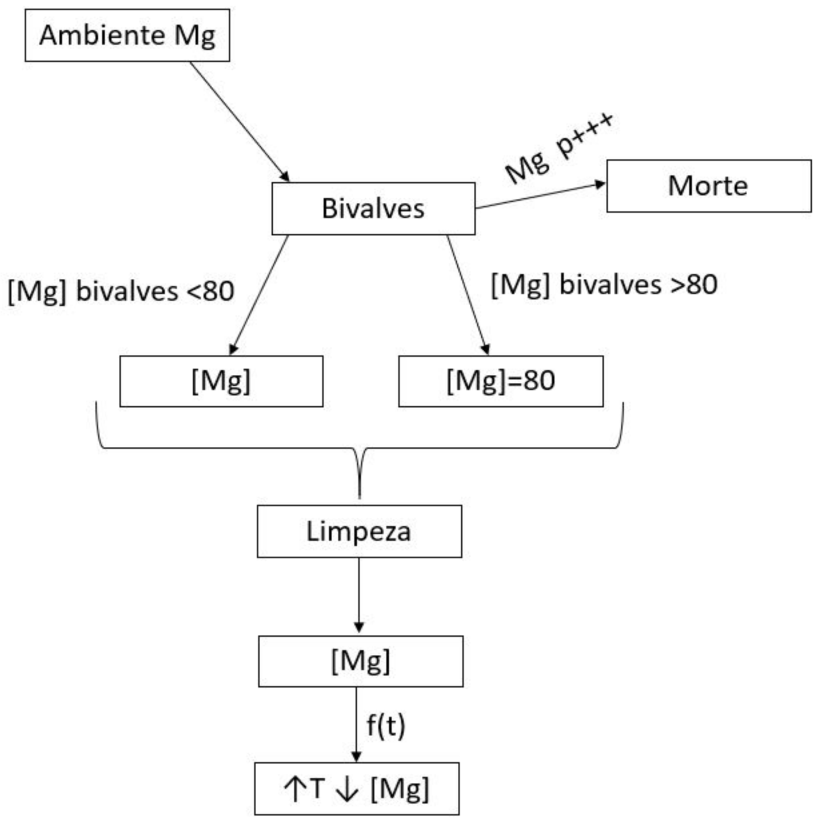


Figure 2: Modelo conceptual do problema, by Joana Araujo

```r
mypreds6=function(meio,tempo){
  mypred=mypreds2(meio)
   res=mypred-0.2/tempo
  return(res)
}
```

Then we can use the function for a range of initial concentrations, and say a time of 90 minutes

```r
mypreds6(c(50,60,70,80,90,100),tempo=90)
```

```
## [1]  7.966933 13.100382 18.233831 23.367280 28.500729 33.634178
```

How would one obtain the time till the limit is below what is the legal limit. We just need to invert the above function for time such that the result is 5.5. Therefore

```r
depurtime<-function(cini){
n=length(cini)
res=ifelse(cini>rep(5.5,n),(cini-5.5)/0.2,0)
return(res)
}
```

We can now test the function for several values of the initial concentration of contaminants

```r
depurtime(c(4.4,10,50,100))
```

```
## [1]   0.0  22.5 222.5 472.5
```

As expected, if the concentration is below the threshold, the time required is 0, and if it is above, the higher the contamination the higher the required time.

Note we could make a general function, that would work for any desired value to obtain for concentration

```r
depurtime2<-function(cini,lim=5.5){
n=length(cini)
t=ifelse(cini>rep(lim,n),(cini-lim)/0.2,0)
return(t)
}
```

Naturally, for a lower concentration level in the end, the time increases as the limit decreases, for a given initial concentration in the medium

```r
depurtime2(160,5.5)
```

```
## [1] 772.5
```

```r
depurtime2(160,3.5)
```

```
## [1] 782.5
```