Proceedings

Looking for the Cosmic Neutrino Background

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Abstract: There are three types of cosmic background: the Cosmic Gravitational Background (CGB), the Cosmic Neutrino Background (CNB) and the Cosmic Microwave Background (CMB). Out of all these, the easiest to detect by far has been the CMB, but it can only tell us so much. Since the event associated with the CMB, the Recombination, happened about 300 000 years after the Big Bang, a look at even earlier in the Universe to understand how it works is essential. Since most of the CGB is still currently enshrouded in mystery, as gravitational waves are still being researched, the next best option is, of course, the CNB. These relic neutrinos decoupled from the primordial fluid at about 1 second after the Big Bang and, if we are able to capture them, we can learn so much more about the earlier stages of the Universe. Detecting the CNB, however, isn't an easy task. In this paper, I search for several ways that it can be detected, ranging from direct methods, like neutrino elastic scattering and neutrino β-decaying nuclei, to indirect methods, like cosmic rays that cause neutrino scattering, to even more fringe methods, like relying on gravitational lensing or even going beyond the Standard Model of particle physics.

Keywords: Cosmic Neutrino Background; Future Experiments; Standard Model; Gravitational Lensing

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1. Introduction

The Cosmic Microwave Background (CMB) has been infamous since its discovery in 1964 by Arno Penzias and Robert Wilson, for it has allowed us a glimpse into the Universe at its youngest known stage, and the amount of information acquired from it has been tremendous in aiding our understanding of what surrounds us. However, the CMB isn't the only mechanism that is able to tell us more about the young Universe, for there exists two other types of cosmic background. One of them is the Cosmic Gravitational Background (CGB) but, in this paper, I am going to discuss about one of the first types of matter to decopule from the primordial fluid, the neutrinos, that left some sort of relics wandering around the Universe in the form of a Cosmic Neutrino Background (CNB). Firstly, a discussion on the mechanism of this event will be explained where, afterwards, three main methods of detecting the CNB will be discussed, followed by some other alternative options to detect relic neutrinos.

2. Theoretical Framework

As humans, we don't know that much about the earliest phases of the Universe. However we can do what we do best, and that is theorise. The most important and famous theory about the beginning of the Universe is the Hot Big Bang Theory, and it is in this framework that we're going to discuss the Cosmic Neutrino Background (CNB). In this theory, the early Universe is this primordial "soup" that, as it cools down, decouples several particles until it reaches the composition of particles we know today. It's also important to say that we assume that the Universe is homogeneous and isotropic in this scale. It is relevant to note that the theoretical framework has been extracted from the lecture notes *Cosmology Part III - Mathematical Tripos* [1].

Understanding the history of the early universe proves to be a very difficult job, even in the simpler of situations. However, defining that Γ is the rate of particles interactions and H as the rate of expansion of the Universe (or the Hubble "constant"), then with the definition of

$$\Gamma = n\sigma v$$

where n is the number density of particles, σ the interaction cross sections and v the average velocity of the particle in question, we can define that

$$t_c \equiv \frac{1}{\Gamma}\,;\, t_H \equiv \frac{1}{H}$$

where t_c is the time scale of the particle while they're still attached to the primordial fluid and t_H is the time of the Universe. When $t_c \sim t_H$, then the particle decouples from the thermal bath, as the interaction rate is slower than the expansion of the Universe.

A brief overview of the history of the Universe can be seen on Table 1.

Table 1. A brief history of the Universe. Table adapted from *Cosmology Part III - Mathematical Tripos*, p. 46.

Event	time t	redshift z	temperature T
Inflation	10^{-34} s (?)	-	-
Baryogenesis	?	?	?
EW phase transition	20 ps	10^{15}	100 GeV
QCD phase transition	20 μs	10^{12}	150 MeV
Dark matter freeze-out	?	?	?
Neutrino decoupling	**1 s**	$\mathbf{6 \times 10^9}$	**1 MeV**
Electron-positron annihilation	6 s	2×10^9	500 keV
Big Bang nucleosynthesis	3 min	4×10^8	100 keV
Matter-radiation equality	60 kyr	3400	0.75 eV
Recombination	260-380 kyr	1100-1400	0.26-0.33 eV
Photon decoupling	380 kyr	1000-1200	0.23-0.28 eV
Reionization	100-400 Myr	11-300	2.6-7.0 meV
Dark energy-matter equality	9 Gyr	0.4	0.33 meV
Present	13.8 Gyr	0	0.24 meV

Neutrinos were coupled to the primordial fluid due to processes like

$$\nu_e + \overline{\nu}_e \Leftrightarrow e^+ + e^-$$

$$e^- + \overline{\nu}_e \Leftrightarrow e^- + \overline{\nu}_e$$

The interaction cross section for the neutrino interactions is estimated to be

$$\sigma \sim G_F^2 T^2$$

where G_F is the Fermi constant, given by 1.17×10^5 GeV^{-2} and T the temperature of the Universe. When $t_c = t_H$, and given that $H = T^2/M_{pl}$ (where M_{pl} is the Planck mass), we have that:

$$\frac{\Gamma}{H} = \left(\frac{T}{1MeV}\right)^3$$

which means neutrinos decoupled when the temperature of the Universe was about 1 MeV, or about 1 second after the Big Bang.

Because of their low mass, neutrinos can be approximated to a relativistic Fermi-Dirac distribution (as they are fermions), which gives us that their particle density is

$$n_\nu \propto a^{-3} \int d^3q \frac{1}{exp(q/aT_\nu) + 1}$$

where a represents the scale factor of the Universe.

This means that neutrinos' temperature evolves as $T_\nu \propto a^{-1}$, which would mean it would evolve in the same way as photons, leading to the assumption that $T_\nu = T_\gamma$. However, this is false, because the electron-positron annihilation transfers energy to photons but not to the decoupled neutrinos. This means that photons cool down more slowly than neutrinos, with a difference given by

$$T_\nu = \left(\frac{4}{11}\right)^{1/3} T_\gamma$$

With this in mind, the temperature of what must be a Cosmic Neutrino Background (CNB) at the present has a slightly lower temperature than the Cosmic Microwave Background (CMB), which is in according with the previous equation, giving us that $T_\nu = 1.95$ K, compared to the photon temperature $T_\gamma = 2.73$ K. With this, the present number density of neutrinos is given by

$$n_\nu = \frac{3}{4} N_{eff} \times \frac{4}{11} n_\gamma$$

where N_{eff} here represents the effective number of neutrinos. If there wasn't any electron-positron annihilation, $N_{eff} = 3$, however, because the decoupling of the neutrinos wasn't quite complete when the annihilation began, some of the energy and entropy leaked to the neutrinos, raising the value to $N_{eff} = 3.046$.

With this in mind, the number density of neutrinos per flavour is, thus, 112 neutrinos cm^{-3}. If neutrinos were massless, their energy density would be given by

$$\rho_\nu = \frac{7}{8} N_{eff} \left(\frac{4}{11}\right)^{1/3} \rho_\gamma \;\Rightarrow\; \Omega_\nu h^2 \approx 1.7 \times 10^{-5} \; (m_\nu = 0)$$

However, experiments have proven that neutrinos do indeed have mass, with the limits of $\sum m_{\nu,i} > 60$ meV and $\sum m_{\nu,i} < 1$ eV. With this information the energy density changes to

$$\rho_\nu = \sum m_{\nu,i} n_{\nu,i} \;\Rightarrow\; \Omega_\nu h^2 \approx \frac{\sum m_{\nu,i}}{94\,\text{eV}} \; (m_\nu > 0)$$

Although the mass limitations imply that neutrinos contribute more than 25 times the energy density of the photons, overall they're still a subdominant component, with $0.001 < \Omega_\nu < 0.02$.

3. Primary Methods of Detection

Detecting the Cosmic Neutrino Background (CNB) is proving to be an endeavour in the scientific community however, if we're able to find a consistent way to detect these relic neutrinos, then we will be able to obtain direct information of the Universe at 1 second old. This is the main reason why finding the CNB is so important - knowing experimental information about the Universe when it was this young will make or break the theoretical models present, giving us further knowledge on its development.

An important thing to note is that, since neutrinos have mass, they can be trapped in gravitational potentials of massive objects (like galaxies) and cluster in certain areas, forming an overdensity of particles. With different models, this overdensity gives different results. For example:

$$\frac{n}{\langle n \rangle} = \begin{cases} 12\,(m_\nu = 0.6\,\text{eV})\,;\, 1.4\,(m_\nu = 0.15\,\text{eV}) \rightarrow \text{For the NFWhalo model} \\ 20\,(m_\nu = 0.6\,\text{eV})\,;\, 1.6\,(m_\nu = 0.15\,\text{eV}) \rightarrow \text{For the MWnow model} \end{cases}$$

where n represents the local number density of neutrinos and $\langle n \rangle$ represents the number density of neutrinos predicted by the Big Bang theory.

With this section I intend to mention the three primary methods of detection, based on the paper by Yanagisawa (2014) [2]. The 3 methods are, as follows:

- Direct detection of CNB elastic scattering with target nuclei through momentum transfer;
- Direct detection by capture of β-decaying nuclei;
- Indirect detection by finding spectral distortion through CNB interactions with ultra-high energy neutrinos or protons/nuclei from unknown sources.

3.1. Neutrino Elastic Scattering

The phenomena of coherent elastic scattering happens when the de Broglie wavelength of the CNB is much larger than the inter-atomic spacing of the material that is being targeted. The de Broglie wavelength of the relic neutrinos is given by

$$\begin{cases} \frac{h}{p_\nu} \sim \frac{h}{4\,T_\nu} \approx 2.4\,\text{mm (for relativistic and unclustered neutrinos)} \\ 1.2\,\text{mm} \cdot \frac{1\,\text{eV}}{m_\nu\,(\text{eV})}\ \text{(for non-relativistic and clustered neutrinos)} \end{cases}$$

When a neutrino with a certain momentum p suffers from coherent elastic scattering on a certain target and emerges with a different momentum p', one can use the concept of neutrino optics and define an index of refraction as $n = p'/p$, where $n - 1 = G_F$, and G_F is the Fermi constant.

Two possible sub-methods exist in this method and both depend on the order of the Fermi constant: it can be on the order of G_F or G_F^2.

3.1.1. Order G_F effects

With the concept of neutrino optics, refraction and reflection were suggested to detect the CNB, however, the forces at play and energy exchange are cancelled out by the order of G_F when the target is in a uniform neutrino density. This means that the only viable effect to detect the CNB uses a method that splits the energy state of non-relativistic electrons in the target material that is immersed in the CNB, for which polarized electrons are needed - this effect is proportional to $n_\nu - n_{\overline{\nu}}$. Considering that there is a large asymmetry that favours the neutrinos and not its anti-particle counterpart, then the relation becomes, simply $n_\nu - n_{\overline{\nu}} \approx n_\nu$.

With this in mind, we're considering several types of neutrinos to model the CNB detection: Relativistic (R) or Non-Relativistic (NR), Clustered (C) or Non-Clustered (NC) and Dirac (D) or Majorana (M) neutrinos. The energy splits between them can be seen in Table 2.

Table 2. Table with the energy splits for each type of neutrino present in the CNB. g_A is a factor that is $1/2$ for ν_e and $-1/2$ for $\nu_{\mu,\tau}$. β_{Earth} is the velocity of the Earth relative to the CNB normalized to the speed of light - a model gives us that $\beta_{Earth} = 1.4 \times 10^{-3}$ and $\beta_{Earth} = 2.2 \times 10^{-3}$ for neutrino overdensities of 20 and 1.4, respectively.

Energy splits	Equations
$\Delta E_R^D = \Delta E_R^M$	$2\sqrt{2} g_A G_F \lvert\beta_{Earth}\rvert n_\nu$
$\Delta E_{NC,NR}^M = 2\Delta E_{NC,NR}^D$	$1.7\sqrt{m_\nu/(1.7\times10^{-4}\xi\ \text{eV})}\ \Delta E_R^D$
$\Delta E_{C,NR}^D$	$\sqrt{2} g_A G_F \lvert\beta_{Earth}\rvert n_\nu$
$\Delta E_{C,NR}^M$	≈ 0

The acceleration caused by the CNB is given by

$$a = 10^{-27} f \cdot \frac{\gamma}{10} \cdot \frac{100}{A} \cdot \frac{1\,\text{cm}}{R} \cdot \frac{\beta_{Earth}}{10^{-3}}\ \text{cm/s}^2$$

where f is the neutrino overdensity, A is the mass number of the target, R its radius and γ is a geometric factor.

Current measurable accelerations reach the order of 10^{-13} cm/s^2, but with possible improvements to our technology we could possibly reach 10^{-23} cm/s^2. However, even with the optimal values of $f = 10, \gamma = 10, A = 100$ and $R = 1$ cm, the acceleration would be 10^{-26} cm/s^2, which means there still needs to be an increase in sensitivity of a factor of about 1000 to detect the CNB.

3.1.2. Order G_F^2 effects

When our planet moves through the sea of relic neutrinos, a target on Earth experiences, through the phenomenon of elastic scattering, momentum transfer from neutrinos. On our rest frame, that transfer is given by (considering Relativistic (R) or Non-Relativistic (NR) and Clustered (C) or Non-Clustered (NC) neutrinos)

$$\langle \Delta p \rangle = \begin{cases} \beta_{Earth}\,(E_\nu/c) & \text{(R neutrinos)} \\ \beta_{Earth}\,(4T_\nu/c) & \text{(NR and NC neutrinos)} \\ \beta_{Earth} c m_\nu & \text{(NR and C neutrinos)} \end{cases}$$

With this momentum, the acceleration can be given by

$$a = \Phi_\nu\,(N_{\mathrm{Av}}/A)\,\sigma_{\nu-A}\langle \Delta p \rangle$$

where Φ_ν is the CNB flux, N_{Av} is the Avogadro number, A is the mass number of the nucelus that is affected by the momentum transfer and $\sigma_{\nu-A}$ is the neutrino-nucleus cross section. This cross section is given by $\sigma_{\nu-A} \approx G_F^2 m_\nu^2/\pi \approx 10^{-56}(m_\nu/\mathrm{eV})^2\ \mathrm{cm}^2$ for NR neutrinos and $\sigma_{\nu-A} \approx G_F^2 E_\nu^2/\pi \approx 5 \times 10^{-63}(T_\nu/1.9\,\mathrm{K})\ \mathrm{cm}^2$ for R neutrinos. As it is clear to see, this is where the second order effects of the Fermi constant come into play.

With the coherent elastic scattering condition satisfied, this adds factor of A^2 to the cross section which, after some calculations, gives us that the acceleration is, for each type of neutrino:

$$a = \begin{cases} 2 \times 10^{-34} f\,\rho(\mathrm{g\ cm^{-3)}}\ \mathrm{cm/s^2} & \text{(Dirac/Majorana R neutrinos)} \\ 3 \times 10^{-28} f\,(m_\nu(\mathrm{eV}))^2\,(T_\nu/1.9\,\mathrm{K})^{-2}\,\rho(\mathrm{g\ cm^{-3)}}\ \mathrm{cm/s^2} & \text{(Dirac NR and NC neutrinos)} \\ 10^{-27} f\,\rho(\mathrm{g\ cm^{-3)}}\ \mathrm{cm/s^2} & \text{(Dirac NR and C neutrinos)} \end{cases}$$

for the NR Majorana neutrinos, the corresponding cross sections are reduced by $\beta_\nu^2 \approx 10^{-6}$. However, as in the previous section, with the optimal conditions of $\rho \sim 100, a \approx 10^{-25}\ \mathrm{cm/s^2}$, the sensitivity of $10^{-23}\ \mathrm{cm/s^2}$ is still too small to detect the relic neutrinos.

3.2. Capture by β-decaying nuclei

First suggested by Weinberg in 1962, the use of neutrino capture by β-decaying nuclei (NCB) to detect the CNB uses the following reaction

$$\nu_e/\overline{\nu}_e + N \Rightarrow e^+/e^- + N'$$

where the electron neutrino (or anti-neutrino) captured by a nucleus N undergoes beta (or positron) decay to a daughter nucleus N'. This reaction obviously has to follow the energy conservation equation, $M(N) - M(N') = Q_\beta > 0$, where $M(N)$ and $M(N')$ represent the masses of the original and daughter nuclei, respectively, and Q_β is the kinetic energy of the electron/positron. As we know neutrinos have mass, then the electron kinetic energy in this reaction is calculated to give

$$E_e = Q_\beta + E_\nu \geq Q_\beta + m_\nu$$

which neglects the nucleus recoil energy from electrons during the decay. This means that there's a gap of $2m_\nu$ around the kinetic energy of the electron between the β-decay and the neutrino capture.

Among the nuclei that suffer from this reaction, ^{3}H and ^{187}Re have large ratios between β-decay events and neutrino capture β-decay events, with 3.0×10^5 and 5.9×10^7, respectively. The cross section of these events (multiplied the neutrino velocity) is given by

$$\sigma_{NCB} \cdot v_\nu = 2\pi^2 \frac{\ln 2}{A\,t_{1/2}}$$

where A is in function only of the energy of the neutrino and $t_{1/2}$ is the half-life of the nucleus. For tritium and ^{187}Re, this cross section, with the equation of $\sigma_{NCB} \cdot (v_\nu/c)$, is given by $7.8 \times 10^{-45}\ \mathrm{cm}^2$ and $4.3 \times 10^{-52}\ \mathrm{cm}^2$, respectively. ^{3}H seems like the best choice, even if it has a smaller ratio.

With 100 g of tritium as the target, the event rate of NCB per year is calculated to be:

$$\text{With } m_\nu = 0.6\,\text{eV} \rightarrow \begin{cases} 7.5 \text{ events per year} & \text{(For the Fermi-Dirac distribution)} \\ 90 \text{ events per year} & \text{(For the NFWhalo model)} \\ 150 \text{ events per year} & \text{(For the MWnow model)} \end{cases}$$

$$\text{With } m_\nu = 0.3\,\text{eV} \rightarrow \begin{cases} 7.5 \text{ events per year} & \text{(For the Fermi-Dirac distribution)} \\ 23 \text{ events per year} & \text{(For the NFWhalo model)} \\ 23 \text{ events per year} & \text{(For the MWnow model)} \end{cases}$$

$$\text{With } m_\nu = 0.15\,\text{eV} \rightarrow \begin{cases} 7.5 \text{ events per year} & \text{(For the Fermi-Dirac distribution)} \\ 10 \text{ events per year} & \text{(For the NFWhalo model)} \\ 12 \text{ events per year} & \text{(For the MWnow model)} \end{cases}$$

Several experiments were proposed to capture these relic neutrinos. First, there is the KATRIN (Karlsruhe Tritium Neutrino Experiment) experiment, who only uses 50 μg of ^{3}H, which is insufficient to capture a significant number of these neutrinos. Since increasing the mass doesn't exactly work for this experiment, other methods must be taken into account. As such, the ambitious Project 8 which, besides intending to find and absolute mass for neutrinos [3], uses cold atomic tritium instead of KATRIN's molecular tritium, which would improve the chances of finding relic neutrinos. Lastly, there's the PTOLEMY (Princeton Tritium Observatory for Light-Early Universe Massive-neutrino Yield) experiment, whose main purpose is to find the CNB. This experiment, for now, is still at a developmental stage.

3.3. Cosmic Rays that cause CNB Scattering

The interaction between cosmic ray photons and the CMB photons imposes an energy threshold beyond which cosmic ray photons don't survive - this is the Greisen-Zatsepin-Kuzmin (GZK) cutoff, with the value of $E_{GZK} = 5 \times 10^{19}$ eV. However, it is argued that neutrinos actually can survive this cutoff, and interactions between the ultra-high energy cosmic ray neutrinos with the CNB introduces a dip at a certain energy in the ultra-high energy neutrino flux. Although the AGASA (Akeno Giant Air Shower Array) experiment claimed to detect these dips in energy, other experiments like the HiRes (High Resolution Fly's Eye) or the Auger were unable to confirm that such a phenomenon happens.

Another interaction of cosmic rays with CNB to detect it is in terms of the inverse β-decay reaction (like $p + \overline{\nu}_e \rightarrow n + e^+$). This reaction experiences a change in the power index at about $10^{15.3}$ eV and at $10^{17.5}$ eV, the first observed by the CASA-BLANCA experiment and the second by the previously mentioned HiRes. With the previously mentioned interaction where we consider that the anti-neutrinos is part of the CNB, then the center of mass energy should be as follows:

$$E_{CM} \approx \sqrt{m_p^2 + 2E_p m_\nu} > m_p + m_n$$

where m_p is the mass of the proton, E_p its kinetic energy and m_n the mass of the neutron. If the energy of the change of power mentioned previously is, with the help of experimental data, at $(3 \pm 1) \times 10^{15}$ eV and its cause is this inverse β-decay reaction, then it would be compatible with the mass of the neutrino, meaning we'd have a way to indirectly detect the CNB.

3.4. Can we actually detect the CNB?

Out of all these approaches, the tritium based experiments seems to be the most promising: with the development of PTOLEMY, an experiment specifically designed to capture relic neutrinos, the future seems bright. KATRIN can even suffer some modifications, increasing the mass of tritium a bit and we can get an upper limit on the local CNB density [4]. However, suffice it to say, there still isn't a reliable way to detect the CNB in the same way we can detect the CMB, but ahead of us lies a world of possibilities.

4. Other Methods of Detection

In the previous section, we talked about the main methods of detection of the CNB. However, these aren't all of the possible ways to detect these relic neutrinos - this is what this section for. Whether it includes physics from beyond the Standard Model of particle physics or simply uses gravitational lensing, I'm going to discuss two alternate methods to detect the CNB.

4.1. CNB as a Ferromagnet

Proposed by Paolo et al (2014) [5], neutrinos interacting with each other can lead to observable phenomena, but, for this to be true, the interactions between particles can't come from the W or Z bosons, but instead from another boson altogether - the theorized boson X, which is beyond the Standard Model of particle physics.

Let us first consider the interaction between two neutrinos by the X boson

$$L = -g\bar{\nu}\gamma_\mu \nu X^\mu + \frac{1}{2}M_X^2 X^2$$

where M_X is the mass of the boson, g the effective coupling constant of this "secret" interaction and X_μ is the vector field that parameterizes this secret neutrino interaction. With this Lagrangian, we can then get the Hamiltonian, defining $J_\mu = \bar{\nu}\gamma_\mu \nu$ and

$$H = \frac{g^2}{M_X^2} J_\mu J^\mu$$

Stressing that spin-spin interactions are the dominant ones, because the transferred momentum to the vector boson is small, turns the interaction exclusively into a contact one. What we want with all this information is that, when there's a phase transition at a certain critical temperature T_c, the CNB acquires spontaneous magnetization. Defining once again $J = g^2/M_X^2\, n_\nu$, then the condition for this phase transition is given by

$$\frac{J}{T} \geq 1$$

This means that, if $J = T_c$, the phase condition reads as $T_c > T$.

A relation between the redshift and the temperature can also be found, where

$$(1 + z^2) \geq \frac{T_{0\nu} M_X^2}{g^2 n_0}$$

where $T_{0\nu} = 1.9K$, the present temperature of the CNB, and n_0 is the present number density of neutrinos.

Assuming that at the recombination epoch, the neutrino-neutrino interaction rate doesn't exceed the cosmic expansion rate ($\Gamma_{\nu\nu\to\nu\nu} < H_{rec}$), then we have two scenarios for this situation:

1. **Boson X is nearly massless ($M_X \leq m_\nu$).** This changes the redshift equation to

$$(1 + z^2) \geq \frac{T_{0\nu} m_\nu^2}{g^2 n_0}$$

Imposing the neutrino decoupling redshift limit on this equation ($z < 10^9$), we have that

$$2.2 \times 10^{-13} \left(\frac{m_\nu}{10^{-4}\,\text{eV}}\right)^2 \leq g$$

Respecting the recombination epoch free-streaming neutrinos where $z = 1500$, the Hubble parameter is $H_{rec} = 4.5 \times 10^{-29}$ eV at this redshift and the neutrino-neutrino interaction rate is given by $\Gamma_{\nu\nu\to\nu\nu} = n_\nu \langle \sigma v \rangle$ (where $\langle \sigma v \rangle \sim g^4 n_\nu / \langle s \rangle$), then we have that

$$g < 2.3 \times 10^{-7}$$

Spontaneous magnetization of the CNB is possible between these two redshifts, with a nearly massless X boson and with the constant coupling between two limits

$$2.2 \times 10^{-13} \left(\frac{m_\nu}{10^{-4}\,\text{eV}}\right)^2 < g < 2.3 \times 10^{-7}$$

2. **Boson X is massive (**$M_X >> m_\nu$**).** In this case, we consider a Fermi-like constant with the value of $G_X = g^2/M_X^2$. Considering the critical temperature for a phase transition, the constant has the value of

$$G_X \geq 4.8 \times 10^{13} G_F,\; G_F \sim 10^{-23}\,\text{eV}$$

This gives us that the interaction rate is $\Gamma_{\nu\nu\to\nu\nu} = G_X^2 T^2 n_\nu$. Adding up both regions of interest, however, it's not possible for the CNB to suffer from spontaneous magnetization with a massive boson before the epoch of recombination, because the two required constraints don't compliment each other ($4.8 \times 10^{13} G_F < G_X < 6.4 \times 10^{10} G_F$) and because a vector boson in between these limits has been ruled out by astrophysical experiments.

Considering the case of the nearly massless boson, we have that the magnetic seed field by the CNB is given by

$$B_{CNB} = \mu_\nu n_\nu$$

where μ_ν is the neutrino magnetic moment, with the approximate value of $\mu_\nu \sim 10^{-19}\mu_B$. Using the redshift window defined above, we have that the magnetic seed field by the CNB is in between two values

$$2.3 \times 10^{-27}\,\text{G} \leq B_{CNB} \leq 6.8 \times 10^{-10}\,\text{G}$$

The upper limit of the correlation length of this magnetic field would be the Hubble radius at the time of the generation of this field, given by $H^{-1}(z)$. As such, the length of this field ranges between 3.4 npc and 100 kpc, for the neutrino decoupling and recombination epoch, respectively.

As magnetic fields of the values of about $B = 1\,\mu$G in galactic clusters corresponds to a primordial magnetic field of $B \sim 1$ nG at the redshift of $z \sim 1100$, the results seem to agree with the calculations made in this paper, which means they can be evidence of the spontaneous magnetization of the CNB via phase transition of interactions through the X boson. However, there is the possibility that something else is the source of this magnetic field and, until the boson in question is discovered, this remains purely theoretical.

4.2. Observing the CNB through Gravitational Lensing

This section's proposal of the observation of the CNB via gravitational lensing was all taken from the paper by Yao-Yu Lin & Holder (2020) [6].

Gravitational lensing is a consequence of massive bodies (usually galaxies) that distort light around them. This causes a break in statistical isotropy, so the correlations between the CNB will have spatial variations, as what happens in the CMB. Assuming a simple case where the lens is a singular isothermal sphere (SIS), then the Einstein radius for photons, which is the ring that forms from the the lensing, is given by

$$\theta_{E,\gamma}^{\text{SIS}} = \frac{4\pi\sigma_\gamma^2}{c^2}\frac{D_{LS}}{D_S}$$

where σ_γ is the velocity dispersion of the lens galaxy, D_S the comoving distance between the observer and the source and D_{LS} is the distance between the lens and the source.

However, as discussed previously, neutrinos aren't massless and, therefore, do not behave as photons. As such, we have to consider that they don't move at the speed of light c, but instead in a time varying-velocity. The distribution of neutrino momentum can be given by the Fermi-Dirac distribution that was established in the first seconds after the Big Bang, when cosmic neutrinos were still ultra-relativistic. After some calculations, we can determine the neutrino velocity in function of the scale factor and their initial velocity

$$v(a) = \frac{v_0}{\sqrt{a^2 + \frac{v_0^2}{c^2}(1 + a^2)}}$$

The angle of deflection of a neutrino passing through a gravitational lens is, therefore, given by

$$\alpha(R) = \frac{4GM(R)}{Rc^2} \frac{c^2 + v_{\text{lens}^2}}{2v_{\text{lens}}^2}$$

where $v_{\text{lens}}(a)$ is the velocity of the neutrino passing through the lens.

Using the Born approximation (i.e. the angle of deflection is large enough that it can be measured in degrees rather than in radians) and the thin lens approximation, we can say that the Einstein radius for neutrinos on a SIS is

$$\theta_E^{\text{SIS}} = \frac{4\pi\sigma_\gamma}{c^2} \left(\frac{c^2 + v_{\text{lens}}^2}{2v_{\text{lens}}^2} \right) \frac{D_{LS}(v_0, D_L)}{D_S(v_0)}$$

where D_L is the comoving distance between the observer and the lens.

Because neutrinos have three mass eigenstates, this means that a fixed momentum can correspond to three possible velocities, leading to a superposition of of three different lensed neutrino maps, each corresponding to a different source of the particles. If the masses and mixings are known, then a time evolution of each individual lense can be constructed. Three Einstein rings could be seen, corresponding to each mass eigenstate, for the unlensed lines going through the center of the halo.

If the gravitational lens is evolving, then the neutrinos passing through it can show us its time evolution, due to its different velocities.

If it's able to be detected, then the gravitational lensing of the CNB can be an extremely rich source of information, telling us the gravitational evolution of the Universe within our casual horizon.

5. Conclusion

The detection of the Cosmic Neutrino Background is still a challenge but, if detected, it will be able to show us a look at the Universe at a very early stage, considering the neutrino decoupling from the primordial fluid. Three main methods of detection are currently being pondered to detect the CNB: one using the phenomenon of neutrino elastic scattering, other considering the neutrino capture beta-decaying nuclei and a last one considering cosmic rays that cause the CNB to scatter. Out of all these, the beta-decaying nuclei seems the most promising solution, considering that the PTOLEMY experiment is being specifically designed to capture these relic neutrinos.

However, going beyond the Standard Model of particle physics and even giving another look at gravitational lensing, we can consider other, more fringe, methods to discover the CNB and what information we can acquire from it.

I leave this work with a hopeful eye towards the future - the PTOLEMY experiment and perhaps the subject matter beyond the Standard Model are the start of something new in the study of the primordial Universe. We've reached this far as a species and we can go beyond. As the Latin proverb says, *"Per aspera, ad astra"* or, in English, "Through hardships, to the stars."

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