Proceedings

An overview on Bouncing Cosmology

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Received: date; Accepted: date; Published: date

Abstract: Although the Standard Big Bang Cosmology is the most accepted concept about the beginning and evolution of the Universe, it has problems: the flatness problem, the horizon problem and the inhomogeneity problem. Here I present an overview of bouncing cosmologies as alternatives to inflation and as sources for the cosmological perturbations which are observed today. I focus on the motivation for considering bouncing cosmologies, how they solve the Standard Cosmology problems and the challenges they face. I explain the "wedge diagram", an intuitive way to illustrate how cosmological models with a non-singular bounce generically solve fundamental problems in cosmology. The same approach is used to compare with other cosmological scenarios.

Keywords: bouncing cosmology; inflation; matter bounce cosmology; ekpyrotic cosmology.

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1. Introduction

Even though the evolution of our Universe from redshift z $\sim$ 1000 to today is very well studied and modeled, what happened before that and after is still a great mystery. All we have are theoretical speculations of what could have happened. Based on those speculations, theoreticians created models that need to match the observations and explain the complex structure that we see today. Observations show that the Universe has been expanding and cooling for about 13.8 Gyrs and that the complex structure observed today emerged from a nearly uniform Universe when it was about 1/1100 of its current radius, as demonstrated by the Cosmic Microwave Background (CMB) maps of the last scattering surface.

Despite the fact that inflation solves many problems of the Standard Big Bang cosmology and makes predictions that have been successfully verified, it still faces a number of challenges and some predictions are not specific to inflation so, for those reasons, the search for alternative scenarios was motivated.

The bouncing scenario comes as a very ambitious and different idea than what has been dominating cosmology from the past 50 years or so. This theory has been developing recently and still needs further work but, so far, it hasn't reached any obstacle that it can't overcome. In a bouncing cosmology, there are two main parts to focus on: the **contraction phase** and the **bounce**. I will start by describing the contraction phase where the equations for an expanding Universe can also be applied since Einstein equations continue to be valid both for the expanding and contracting Universe. The difference arises at the bounce since it requires new physics to explain it, but it can be described, at least to leading order, by classical equations of motion.

2. Contraction phase

As already known, the scale factor, a(t), is a dimensionless quantity that describes how much a patch of space changes in size due to expansion (or contraction) and the Hubble parameter, H, defined as $H = \dot{a}/a$ tell us how much the expansion or contraction changes with time. If H is positive the Universe is expanding and if it's negative, the Universe is contracting. The Universe can be, in good approximation, assumed as spatially flat (k = 0) today since current observational evidence shows that the spatial curvature is negligible and a(t) can be normalized as unity at the present, $a(t_0) = 1$.

When analyzing expanding Universes, we rely on the Friedmann equations and those same equations, as I already mentioned, can be applied for a contracting Universe. The Friedmann equation, which tells us how the Hubble parameter depends on what the Universe contains, can be written as:

$$3H^2 = \frac{\rho_m}{a^3} + \frac{\rho_r}{a^4} + \frac{\rho_\phi}{a^{2\epsilon}} - \frac{3k}{a^2} \quad (1)$$

Where the first term is with respect to matter, the second term is for radiation and the fourth term is from the curvature. The third term is generic and stands for other energy component which have an equation of state $\epsilon = 3/2(1+w)$ where w measures the ratio of the pressure to the energy density (w = p/ρ).

In the Friedmann equation the dependence on the scale factor changes for different forms of energy. That difference in the exponent of a(t) depends precisely on ϵ so, in general, it's just $H^2 \sim a^{-2\epsilon}$. In the Standard Big Bang model, for the first 50.000 years the Universe is radiation dominated so p/ρ is equal to $1/3$ which means that ϵ is equal to 2 so $H^2 \sim a^{-4}$. For the remaining 13.8 Gyrs it is dust dominated corresponding to $\epsilon = 3/2$ because matter is pressureless (p = 0) so $H^2 \sim a^{-3}$. This leads to the dominance of different types of energy at different times, depending on the value of a(t) and its dependence on it.

From equation (1) we get $H^{-1} \sim a^{\epsilon}$ where H^{-1} is called the Hubble radius (if we consider c = 1) which is a key concept in General Relativity. In an expanding Big Bang Universe (where $\epsilon > 1$), it is equal up to a factor of order one to the particle horizon size, which is the maximum distance light or particles could travel for a given time and, hence, the maximum distance an observer can see at time t. Ignoring the proportionality factor of O(1) we can treat the particle horizon size as equal to the Hubble horizon. Alternatively, it can be seen as to what parts of the Universe are causally connected at a given time.

From this expression the time dependence for a(t) can determined, $a \sim |t|^{1/\epsilon}$ where the modulus appears to allow it to be used both for contraction and expansion. When $\epsilon < 1$, there is a "fast" expansion/contraction because the scale factor grows more rapidly than time. When $\epsilon > 1$, it's a "slow" expansion/contraction.

The acceleration of the Universe ($\ddot{a}$) can be described by the following equation:

$$\frac{\ddot{a}}{a} = \frac{1}{2}\rho(1-\epsilon)$$

where it's shown to be proportional to (1- ϵ) so if $\epsilon < 1$ => $\ddot{a} > 0$ which means an accelerated expansion/contraction. If $\epsilon > 1$ => $\ddot{a} < 0$ so the expansion/contraction of the Universe is decelerating.

If we were to analyze the solutions of these equations it would be complex and time consuming so, Anna Ijjas and Paul J. Steinhardt (2018) came up with a simpler idea of representing this cosmology in a visual way: the "wedge diagram" (or *cunha*, in Portuguese).

3. Wedge diagram explained

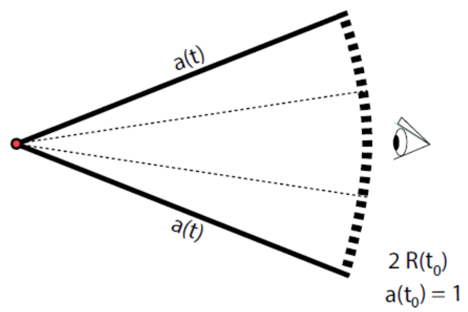

Figure 1. Wedge diagram for the Big Bang model. The evolution of a patch of interest starts at the singularity (vertex on the left) and evolves into the present (t = t_0). The thin dotted lines represent the evolution of a smaller patch of space. Credits: Anna Ijjas and Paul J. Steinhardt (2018).

The two edges of the wedge describe the evolution of a given patch of space that grows linearly with a(t) between the Big Bang (the vertex on the left where a = 0) and today (the outermost arc where the scale factor is equal to $a(t_0)$). The arcs connecting the sides represent the physical sizes of some patch of interest and, for today, $a(t_0)$ = 1 so the arc representing the patch that we see today is represented in figure (1) by the dashed solid black arc. For a smaller patch in space (for the same time) we get a smaller wedge as shown by the thin dotted lines and for a larger patch, even larger than what we can observe, we would get a wider wedge, not represented in the figure.

If we want to know what a patch looked like at earlier times, we just connect the sides for the a(t) at that time as shown in the next figure.

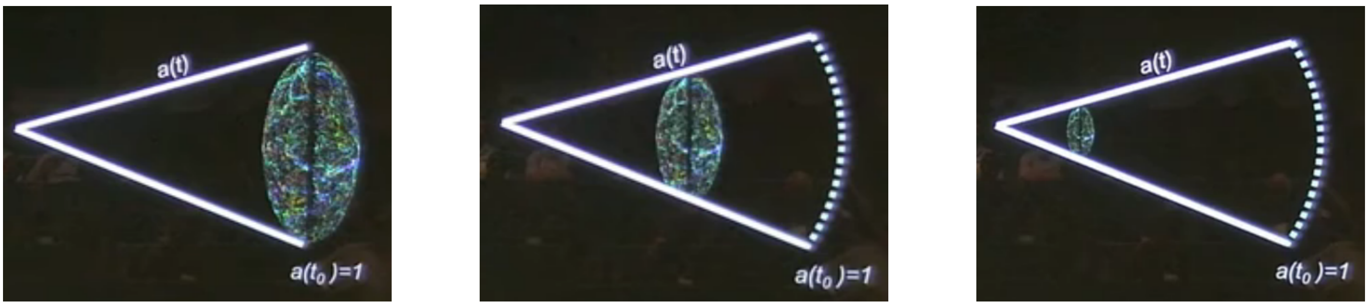

Figure 2. **[Left]** Observed patch for the present ($a(t_0) = 1$). **[Center]** Observed patch for $a(t) = a(t_0)/2$. **[Right]** Observed patch for $a(t) = a(t_0)/4$. Credits: Paul Steinhardt's (talk).

It is, however, important to recall that there was evolution between the different times, but this diagram is only meant to describe the sizes and not the fine-structure details.

4. The problems

One must note that all these wedges have a starting point at the tip of the wedge, which for the Standard Cosmology would be the Big-Bang, but since it's still very poorly understood, it can be seen as what it really is: a **singularity problem**. This problem arises because, when trying to apply the Einstein equations for a(t) very close to zero, the equations stop working (it goes to infinity) and it is a problem that happens in every cosmology that is based on the Big-Bang. Therefore, looking for cosmologies that *don't* rely on the Big-Bang seemed like an option to pursuit.

From the Friedmann equations, as already mentioned, the horizon size is proportional to a^ϵ. In Standard Big-Bang models, the value of ϵ depends on the phase considered since different types of energy (that have different ϵ) dominate at different times. For the radiation dominated phase, $\epsilon = 2$, then there is a quick transition and ϵ starts to be 3/2 in a matter dominated phase. However, in this simple visualization, the exact value is not important but rather the fact that it is above 1. In this case, the patch size grows with a(t) but the horizon size grows with $a(t)^\epsilon$ and if we consider both to be equal at the present and extrapolate back in time, the patch size is always larger than the horizon size (pink shaded area) as seen in top part of figure (3).

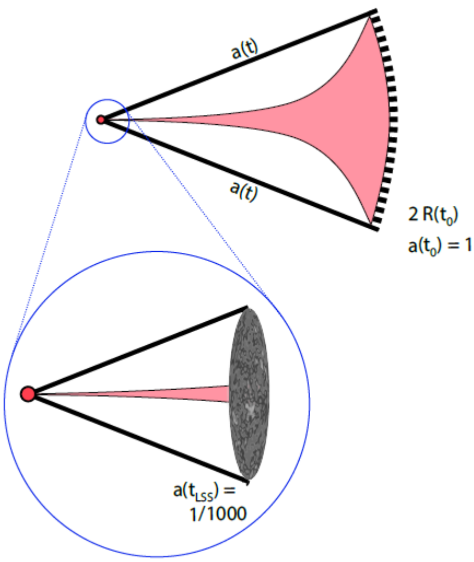

Figure 3. Wedge diagram for the Standard Big Bang model, comparing the patch size (the full wedge) to the horizon size (pink shaded region). The magnified section represents the wedge where $a(t) \sim 1/1000$, highlighting the period between the Big Bang and the last scattering surface. Credits: Anna Ijjas and Paul J. Steinhardt (2018).

In particular, zooming to the time when a(t) $\sim 1/1100$ it is clearer that the arc that represents the last scattering surface (CMB) is much larger than the horizon size. Nevertheless, the CMB measurements show that the density and temperature were nearly uniform across the entire patch which constitutes the **horizon problem** since it's not clear how to explain the uniformity of the CMB over lengths scales greater than the horizon size.

There is an additional problem because CMB measurements also reveal a spectrum of small amplitude density fluctuations that are nearly scale-invariant. This inhomogeneity includes hot and cold spots that are much bigger than the horizon size at the time of last scattering (**Super-Horizon Problem**) and finding a physical mechanism capable of generating this nearly scale-invariant spectrum, constitutes the **homogeneity problem**.

The third problem is the **Flatness Problem**, and it comes from the curvature term in equation (1). In an expanding Universe, the curvature term decreases but in a contracting Universe it grows and starts to become significant. Starting from equation (1) and dividing all terms by $3H^2$:

$$1 = \frac{\rho_m}{3H^2a^3} + \frac{\rho_r}{3H^2a^4} + \frac{\rho_\phi}{3H^2a^{2\epsilon}} - \frac{3k}{3H^2a^2} \tag{2}$$

$$1 = \Omega_m + \Omega_r + \Omega_\phi + \Omega_k;\ with\ \Omega_k \equiv -\frac{3k}{3H^2a^2} \tag{3}$$

$$\Omega_k \propto \left(\frac{H^{-1}}{a}\right)^2 \propto (Hubble\ size/patch\ size)^2 \tag{4}$$

In this context, the wedge diagram is very informative since it allows us to estimate the ratio of the Hubble size to the patch size:

$$(Hubble\ size/patch\ size)^2 \propto \left(\frac{a(t)^\epsilon}{a(t)}\right) \tag{5}$$

For an expanding Universe, a(t) grows so Ω_k also grows with time (depending on the power of ϵ but let's continue to consider $\epsilon > 1$). This is a problem because, if one computes the exact value, it would return an enormous value which meant we should clearly see a curved Universe but that's not the case.

These are the main problems that each cosmology faces. It is clear that some of the problems come from the fact of having a "starting-point" or singularity so, an "out of the box" idea is therefore to get rid of the starting point and consider a "big bounce" instead. This "big bounce" can be described by Einstein equations both for the contracting and expanding phase, except for the bounce that I will discuss later.

5. Does bouncing cosmology solve the problems?

Let's see if changing from a Big Bang to a "big bounce" can solve some of the problems we've encountered and consider the wedge diagram for a generic classical (non-singular) bouncing cosmology.

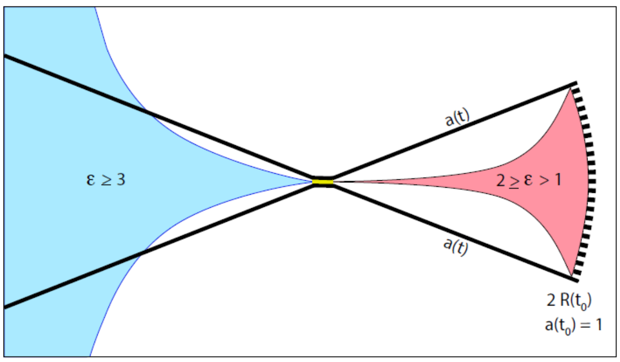

Figure 4. Wedge diagram for non-singular bouncing cosmology. The figure illustrates the period of contraction (lhs) that is followed by a bounce (small yellow shaded area, middle) and the current period of expansion (rhs). Credits: Anna Ijjas and Paul J. Steinhardt (2018).

By adding a contracting phase and bounce, the causal structure is fundamentally changed. The black line, representing the evolution of a(t), is now decreasing in the left-hand side (lhs) as the Universe is contracting. The horizon size continues to scale as $a(t)^{\epsilon}$ with the difference that now a(t) is decreasing so the horizon size (blue area) also decreases. As illustrated in the image, the **Horizon Problem is solved** since there is a huge amount of time before the bounce (far left) where the horizon is larger than the patch size so there is enough time for it to be in causal contact. That means the patch that will eventually evolve into the Universe we observe today is not only causally connected at some point before the bounce (i.e., within a particle horizon), but it remains within the Hubble horizon and causally connected arbitrarily far into the past.

At closer look, we can also point out that the diagram is **not symmetric**, and the reason comes from the value of ϵ in the Friedmann equation. As the contraction is taking place, a(t) is getting smaller so terms with larger ϵ will dominate and, if there is some form of scalar field, it will dominate near the bounce because, after solving the Einstein equations for the scalar field, it is seen that $\epsilon = 3$ (for a case without a potential). If it had a potential the value would be higher than 3 making it dominate even more. After the bounce, if the scalar field is unstable and decays, it turns to radiation leading to the radiation dominated phase but, in that phase, ϵ has a different value than before and that's why the horizon size (a^{ϵ}) has a different shape.

This means that this diagram is not the same as simply gluing together a Big Bang and its time reverse. As we've seen the equation of state ϵ is generically greater in the period before the bounce than in the period after the bounce; the wedge doesn't end on the left-hand side, in fact, in the case of a one-time bounce as considered here, the wedge extends far into the past and the "starting point" is replaced by a bounce in which the scale factor shrinks to a finite size, well above the Planck length, and then rebounds.

From this interpretation we see that the **flatness problem is solved** because a(t) is decreasing (so the curvature is decreasing); ϵ is bigger on the left hand side than on the right hand side (makes it decrease even more) and the contraction occurs for a longer time period, so it causes a **"super-flattening" effect** that is much more effective than, for instance, inflation.

In Standard models, if there is a scalar field in the beginning, as the Universe is expanding the quantum fluctuations in that field evolve into density perturbations and then curvature perturbations so, to avoid that, they require substantial fine-tuning that isn't very successful at describing earlier times which then ends up creating multiverses, as the fluctuations become uncontrolled. For a bouncing cosmology with a contracting phase the opposite occurs: the perturbations shrink, and the final result is a smooth Universe. But this is also a problem because the observations show hot and cold spots that are not predicted here with one scalar field. However, if we add another scalar field that doesn't contribute to the contraction, it will undergo quantum fluctuations that won't disappear after the bounce making it possible to explain the hot and cold spots in the CMB.

To understand this effect let's represent the horizon size by a circle and a perturbation wave that has a wavelength smaller than the horizon size at some initial time t_1 (left image in figure 5).

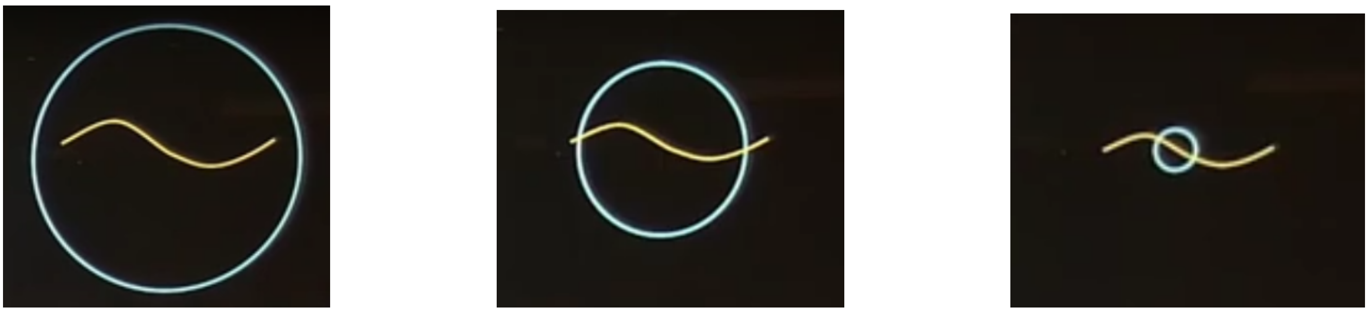

Figure 5. [Left] Representation of the horizon size (blue circle) and a perturbation wave (yellow) at some initial time. **[Center]** Perturbation wave becomes bigger than the horizon size. **[Right]** Perturbation wave becomes a super horizon fluctuation at later times. Credits: Paul Steinhardt (talk).

Since the horizon size is shrinking faster than the patch size, there will be a moment when the wavelength becomes bigger than the horizon size (figure 5 in the center) because the wave has shrunk but the horizon shrunk more. After some time, the wave is much bigger than the horizon and becomes a super horizon fluctuation (figure 5 on the right). Then some different physics in the bounce convert it to a true curvature fluctuation which makes this a possible explanation for the perturbations in the CMB.

As for the **problem of entropy**, it is only a problem if we assume that the Universe begins small (Planck size). Inflation explains it by the exponential expansion of space that leads to an exponential increase in the total energy density of matter. This energy density is converted into radiation during the reheating phase at the end of inflation, producing the large entropy that our current Universe contains. For the models that I will briefly discuss later, like the Matter Bounce and Ekpyrotic cosmologies, it is assumed that the Universe starts large and cold so there is no problem whatsoever in explaining the current entropy.

6. The bounce

Achieving a mechanism capable of making a smooth transition from the contracting to the expanding phase without disrupting the properties we have seen is more complicated. The evolution before and after the bounce can be classically described so It's expected that the bounce can also be classically determined, at least in leading order. This means that the Universe should bounce at a lower energy level than proposed by Big-Bang models therefore not needing to include quantum effects.

For simplicity, I have only considered the case of a single bounce separating the period of contraction from the current period of expansion, although cyclically bouncing models are also possible in principle. Those theories are particularly interesting because they do not just describe the early evolution of the Universe, but its entire history.

If there is a contracting Universe before our expanding Universe, then we have to be able to explain the transition from one phase to the other. If we consider General Relativity and matter that satisfies the null energy condition, then a singularity is inevitable. In order to avoid such singularity, the null energy condition must be violated and to achieve it, quantum fields, modified gravity or even a theory of quantum gravity could be used.

7. Observational Signatures

Alternative models for the early Universe must be able to explain all of the existing data and must be consistent with the current constraints. It is also very important that they make new predictions with which they can be differentiated from the current and widely accepted inflationary paradigm. Given that in the near future new telescopes will provide a wealth of data this is an ideal time to consider such predictions. In most cosmological models, matter is modelled in terms of a scalar field ϕ with a non-trivial background dynamics $\phi_0(t)$. Linear fluctuations of geometry and matter about the background can be classified according to how they transform under spatial rotations. There are scalar modes, vector modes and tensor modes (gravitational waves).

7.1. Power-spectrum

In every early Universe model, we need to be able to compute the power spectrum (P_r) of curvature perturbations (a combination of perturbations in the metric and of matter density) that is given by the expression $P_r \propto A_s(k/K_{pivot})^{n_s-1}$ where k is wave number, A_s is amplitude and n_s is spectral index that describes the slope of the spectrum.

Measurements of n_s put it as 0.965 ± 0.004 (Planck 2018) so not exactly one which means that P_r, that is a function of k, is almost but not completely independent of k. Theoreticians called it a "red-tilt" and it is very well measured. The value of the index $n_s = 1$ would correspond to a scale-invariant spectrum, i.e., a power spectrum which was independent of the scale k.

One way to obtain such scale-invariant power-spectrum theoretically is to consider the Sasaki-Mukhanov equation defining $\ddot{z}/z$ as $2/\tau^2$ where $z^2 \equiv 2\epsilon a^2$ and considering initial vacuum conditions. If that's so, the resulting power-spectrum is independent of k. But why impose such definition? If we assume a constant equation of state ($p/\rho = constant$) then $\ddot{z}/z = \ddot{a}/a$ that must be equal to $2/\tau^2$. If a(t) is described by a power-law $a(t) = a_0(-\tau)^n$ then:

$$\frac{\ddot{a}}{a} = \frac{n(n-1)}{\tau^2} = \frac{2}{\tau^2} \Rightarrow n = -1, 2 \tag{6}$$

This leads to two solutions: $a(\tau) = \frac{1}{H(-\tau)}$ and $a(\tau) = a_0(-\tau)^2$. The first solution leads to an exponential expansion $a(t) \propto e^{Ht}$ and corresponds to the inflation scenario. The second one leads to $a(t) \propto (-t)^{2/3}$ which is a matter-dominated contraction that is the idea behind a scenario that I will later discuss called the Matter-Bounce Scenario.

7.2. Running of the Scalar Spectrum

For simple single field inflationary models, the red tilt of the scalar spectrum is due to the fact that the Hubble expansion rate H is very slowly decreasing during the period of inflation, so the slope of the spectrum is smaller at larger values of the momentum which implies that the running of the spectrum ($\alpha = dn_s/dln(k)$) is negative.

In the matter bounce scenario, on the other hand, at large values of k, the spectrum converges to a scale-invariant one, so the running of the scalar spectrum is positive. Thus, a measurement of the running of the scalar spectrum would allow us to differentiate inflation models from the matter bounce scenario.

7.3. Tensor to Scalar Ratio

The tensor to scalar ratio r is defined by $r = P_t/P_r$ where P_t and P_r are the power spectra of the tensor and scalar modes, respectively.

In the matter bounce scenario, both the tensor and the scalar fluctuations have a scale-invariant spectrum, and the tensor to scalar ratio before the bounce phase is predicted to be of order one since the scalar and tensor modes obey the same equation of motion. However, during the bounce, it is possible that the scalar modes are enhanced relative to the tensor modes, so a large value of r is predicted afterwards. Hence, the value of r does not provide a good window to differentiate between inflation and the matter bounce.

The situation is completely different in the case of the Ekpyrotic scenario. In this case that will be discussed shortly, both the scalar spectrum and the tensor spectrum retain their original vacuum slope which means that the fluctuations are negligible on cosmological scales. In fact, any sizeable measurement of r would immediately rule out the Ekpyrotic scenario unless some yet-unknown mechanism is invoked.

The value of r is, however, very highly model dependent in bouncing cosmologies.

8. Different models of bouncing cosmology

There are different classes of bouncing models that predict different contracting phases. For a contracting phase that is the time reverse of the Standard cosmology expansion phase which corresponds to a symmetric bounce, as showed in section 7.1, it is called the Matter Bounce scenario. The origin of the fluctuations in bouncing cosmologies depends on the model. It is often taken to be quantum vacuum perturbations (Ekpyrotic and Matter Bounce scenarios) but for example, in String Gas cosmology, it is of thermal nature.

8.1. Matter bounce cosmology

This scenario, as I already showed earlier, allows for scale invariant curvature perturbations on super-Hubble scales but if we add a component to matter which corresponds to the current dark energy (e.g., a small cosmological constant), we get a slight red tilted spectrum.

Regarding the flatness problem, this cosmology is neutral because in the case of a symmetric bounce the impact of the spatial curvature term decreases during the period of contraction by the same amount that it increases during the expansion phase.This theory is favorable because it's easily modeled although it suffers from instabilities with respect to anisotropies.

8.2. Ekpyrotic Cosmology

This scenario comes from string theory where there are two 4D branes in 5D that have a negative exponential potential which causes the two boundary branes to approach each other. The period when the two branes are approaching each other is a contracting phase and the time when the two branes collide corresponds to a singular bounce.

This model predicts $n_s - 1 = 2$ which is not consistent with the measurements, however, some new proposals suggest the addition of a spectator scalar field χ evolving in a similar negative exponential potential that will couple with ϕ, the field that generates the Ekpyrotic contraction, at the background level thus leading to n_s -1 ≈ 0 as expected.

For the flatness problem the situation is better: the contribution of spatial curvature decreases faster during the period of contraction than it increases during the expanding phase resulting in a flat Universe.

Ekpyrotic cosmology washes out the anisotropies because there is an additional term in the Friedmann equation ($\rho_{ek}/a^{3(1+w)}$) that dominate at higher energies (where a(t) is small), therefore, not needing to fine tune the model.

8.3. String gas cosmology

String gas cosmology is another model of Bouncing cosmology and it also comes from string theory and is, in fact, based on fundamental principles of superstring theory. It can make some predictions that match the current observations, but it needs further work because string theory is still poorly understood. For more information on its status, see “String Gas Cosmology” by Robert H. Brandenberger (2008).

9. Conclusions

Bouncing cosmologies still have a long way to go before they can be considered as sound as the inflationary paradigm, but the aim of bouncing cosmologies is more ambitious since it strives (among other goals) to solve the singularity problem of our current cosmological models. To do this, one has to go beyond General Relativity and use matter that doesn’t obey the usual energy conditions.

There are various scenarios in which a scale-invariant spectrum of cosmological perturbations emerges that can explain the current data. We have discussed the Matter Bounce and Ekpyrotic scenarios, and very briefly mentioned String Gas cosmology. However, none of the bouncing cosmologies considered here are at the present time fully understood. Of course, nothing prevents that the actual history of our Universe contains an inflationary phase preceded by a bounce. In fact, most mechanisms for constructing a cosmological bounce allow for the inclusion of an inflationary phase after the bounce but it is unclear if we will ever be able to find discriminating measurements.

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