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Primordial Black Holes - origin and evolution

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Abstract: The existence of primordial black holes (PBHs) is of utmost relevance in Cosmology because they can be the seeds of supermassive black holes (SMBHs) that we observe today in the centre of massive galaxies and they are very good dark matter candidates. In this present report, I will present a model that describes how PBHs can be formed and how they evolve with time during the early phases of the Universe by considering gravitational collapse of bound states of stable supermassive elementary particles called gravitinos during the radiation era.

Keywords: Primordial Black Holes; Dark Matter; Gravitinos; Cosmology

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1. Introduction

The existence of PBHs was first proposed by Zel'dovich and Novikov in 1966 [1] where they found that the existence of bodies with radii $R_G < 2GM/c^2$ at the early stages of expansion of the Universe leads to the accretion of radiation by them. Later on, in 1971, Hawking [2] studied their origin and how they were formed.

It is thought that PBHs can be the predecessors of the observed SMBHs in the centre of galaxies. In 2015, Clesse and García-Bellido [3] proposed that, assuming that the accretion is uninterrupted at the Eddington limit, then there would be a need of black holes seeds with masses of the order of $10^3 M_\odot$ at redshifts $z \approx 15$ to justify SMBHs before 500 Myr. They also proposed that PBHs can merge to form intermediate mass black holes (IMBHs) and that the seeds can then merge and accrete to form SMBHs.

Another hypothesis is that PBHs can be good candidates for dark matter. In 2010, Frampton et al. [4] discuss the idea that PBHs can be unique candidates for dark matter based on their long longevity and the fact that there isn't a need to introduce symmetries for this. In 2018, Espinosa et al. [5] show that an instability at the scale 10^{11} GeV induced by the Standard Model Higgs potential could be dark matter in the form of PBHs.

In this report, I will present a model by Meissner and Nicolai [6] that describes how PBHs can be formed and their origin based on the collapse of bound states of supermassive elementary particles called gravitinos during the radiation era and how they evolve during the matter dominated era.

2. Gravitino

In trying to describe their model, first the gravitino needs to be introduced. The gravitino, $\widetilde{G}$, is the gauge fermion supersymmetric partner of the hypothesized graviton. In a previous paper, Meissner and Nicolai [7] show the potential relevance for them to be candidates for dark matter. In particular, their proposal implies a number of features for the dark matter gravitinos, for instance, that their mass is assumed to be of the order of M_{Pl}, that gravitinos with this mass would be stable and that they do not interact with the CMB, independently of if they are in a bound state or not.

In another paper [8], they argue that stable supermassive gravitinos proposed in [7] can serve as seeds for giant primordial black holes during the radiation period. In this paper, they conjecture the existence of these massive particles, stable against decay into Standard Model matter, which can form bound states during this period which, in turn, can collapse to form black holes.

Meissner and Nicolai propose that these seeds are made out of tightly bound states of fractionally charged gravitinos during the radiation period that form under the universally attractive combination of electric and gravitational forces that are present between gravitinos and the gravitino's antiparticle, the anti-gravitinos. When groups of bound states of gravitinos merge and collapse due to the gravitational force, the result is a mini black hole that, when the radiation temperature is higher than the Hawking temperature, can "survive" Hawking evaporation.

Furthermore, they also show that the resulting black holes, when the transition to the matter dominated era occurs, evolve according to an exact solution of Einstein's equations to emerge as macroscopic black holes.

3. Origin and formation of PBHs

Now that we know how superheavy gravitinos can be seeds for mini black holes, we can now discuss how these black holes evolve during the matter dominated phase. As it is reported by Meissner and Nicolai (2020) [8], the expected gravitino mass, M_g, is hypothesized to be between $M_{BPL} < M_g < M_{Pl}$, where M_{BPL} is the mass for which the electrostatic repulsion between two gravitinos or anti-gravitinos of equal charge is the same as their gravitational attraction and M_{Pl} is the reduced Planck Mass. For purposes of numerical estimations, they assumed $M_{BPL} \sim 0.01\, M_{Pl}$ and so

$$0.01\, M_{Pl} < M_g < M_{Pl} \tag{1}$$

This constraint ensures that even if the gravitinos are of equal electric charges, the force between them remains attractive.

As estimated in [8], the number of gravitinos or anti-gravitinos, N, needed in a bound state for there to be a minimum mass for the formation of a mini black hole in the early radiation phase is

$$N \gtrsim 10^{12} \tag{2}$$

This result leads to a minimum mass of a black hole that forms from gravitational collapse of such a bound state of about (assuming $M_g \sim 10^{-9}$ kg and setting $c = 1$)

$$M_{seed} \sim N\, M_g \sim 10^{12}\, M_g \sim 10^3 \text{ kg} \Rightarrow G\, M_{seed} \sim 10^{-24} \text{ m} \tag{3}$$

It would be expected for a black hole of such small mass to evaporate very rapidly due to Hawking radiation. The time it takes for a black hole of mass m to evaporate is given by

$$\tau_{evap}(m) = t_{Pl} \left(\frac{m}{M_{Pl}} \right)^3 \tag{4}$$

While this result is valid in empty space, during the early radiation phase, the presence of extremely hot and dense radiation capable of "feeding" the black hole is a process that needs to be accounted for when we are discussing the evaporation of a black hole during this phase. The absorption of this radiation is, therefore, capable of stabilizing the black hole against Hawking decay such that, even for small black holes, mass accretion during the radiation era with initially extremely high temperatures can overwhelm Hawking evaporation. Given a black hole of mass m, the criterion for accretion to overcome the Hawking radiation rate is

$$T_{rad}(t) > T_{Hawking}(m) = \frac{\hbar}{8\pi G m} \tag{5}$$

The break-even point happens when $T_{rad}(t_0) > T_{Hawking}(m)$, where $t_0 = t_0(m)$. For $t > t_0$, a black hole with mass m will eventually decay. With this equality in mind, the relevant mass at time t can be deduced, yielding

$$m^4(t) \simeq \frac{M_{Pl}^3}{t_{Pl}} \frac{1}{G^2 \rho_{rad}(t)} = \frac{32\pi M_{Pl}^3}{3G t_{Pl}} t^2 \tag{6}$$

If we read this equation from left to right, it tells us the latest time for a mini black hole with mass m to remain stable against Hawking decay during the radiation phase. It's the case for $t < t_0 \equiv t(m) \propto m^2$, after which the black hole will decay. Inversely, a mini black hole with initial mass greater than $m(t)$, for a given time t, will be able to grow and overcome Hawking decay.

Now, we take the initial value for the mass to be $\sim M_{seed}$ and assume that the formation of such mini black hole is possible in the time interval

$$t_{min} = 10^8 t_{Pl} \simeq 10^{-34} \text{ s} < t < t_{max} \simeq 10^{-18} \text{ s} \tag{7}$$

During this time interval, a black hole with initial mass $M_{seed} \sim 10^3$ kg (as seen before) is able to survive and grow by means of accreting radiation. The lower bound of the interval was chosen as to avoid the quantum gravity regime and a possible inflationary phase and the upper bound of the interval is determined by setting $t_{max} \equiv t(M_{seed})$.

The evolution of these mini black holes during the radiation phase (up until $t \sim t_{eq} \sim 47000$ yr, time when matter starts to dominate) is governed by the solution derived in [8]. After this time, these mini black holes become macroscopic.

Considering equation 7, the ranges of mass is, therefore

$$10^{-12} M_{\odot} \lesssim m(t_{eq}) \lesssim 10^{-3} M_{\odot} \tag{8}$$

However, the solution derived in [8] is only valid for the radiation era and not to the matter dominated phase. In order to study the evolution of these black holes during the phase, a new solution needs to be implemented. For this new solution, conformal coordinates, with conformal time η, will be used instead of t, the cosmic time coordinate used previously. The usage of conformal coordinates is justified because spacetime's causal structure is often easier to analyze and the use of conformal time allows the showcase of a simple closed form solution that encompasses the radiative and the matter dominated phase.

Using conformal time η and for a spatially flat Universe with a vanishing Λ, the Friedmann equations read

$$\dot{a}^2 = \frac{8\pi G}{3}\rho a^4 \qquad , \qquad a\ddot{a} - \dot{a}^2 = -\frac{4\pi G}{3}(\rho + 3p)a^4 \tag{9}$$

where

$$\dot{a} \equiv da/d\eta \qquad , \qquad dt = a(\eta)d\eta \tag{10}$$

The required solution is

$$a(\eta) = A\eta + B^2\eta^2 \Rightarrow t = \frac{1}{2}A\eta^2 + \frac{1}{3}B^2\eta^2 \tag{11}$$

The resulting density and pressure, from equation 9, are

$$8\pi G\rho(\eta) = \frac{3A^2}{a^4(\eta)} + \frac{12B^2}{a^3(\eta)} \qquad , \qquad 8\pi G p(\eta) = \frac{A^2}{a^4(\eta)} \tag{12}$$

where, for our Universe (starting from nucleosynthesis)

$$A = 2.1 \times 10^{-20}\ \mathrm{s}^{-1} \qquad , \qquad B = 6.2 \times 10^{-19}\ \mathrm{s}^{-1} \tag{13}$$

Both A and B can be calculated from known data up to re-scaling $n \to \lambda n$, $A \to \lambda^{-2}A$, $B \to \lambda^{-3/2}B$ and $a \to \lambda^{-1}a$. For the case of a, this scale is fixed by setting $a(t_0) = 1$ conventionally, where we consider $t_0 \simeq 13.8 \times 10^9$ yr. At matter-radiation equilibrium, we have, following from [9],

$$a(\eta_{eq}) \simeq \frac{1}{3400} \qquad , \qquad t_{eq} \simeq 1.5 \times 10^{12}\ \mathrm{s} \tag{14}$$

And, at the last scattering, also following from [9]

$$a(\eta_{LS}) \simeq \frac{1}{1090} \qquad , \qquad t_{eq} \simeq 1.2 \times 10^{13}\ \mathrm{s} \tag{15}$$

These numbers give rise to the ones in equation 13. Generalizing the solution of [8] by substituting equation 11 into the metric *ansatz*, we get

$$ds^2 = a(\eta)^2 \left[-\widetilde{C}(r)d\eta^2 + \frac{dr^2}{\widetilde{C}(r)} + r^2 d\Omega^2 \right] \tag{16}$$

In the above equation, the unknown function $\widetilde{C}(r)$ is uniquely fixed by imposing that in the case of pure radiation ($B = 0$), the trace of the Einstein tensor that results from 16 must vanish, i.e., $T^{\mu}{}_{\mu} = 0 \Rightarrow (r^2\widetilde{C})'' \stackrel{!}{=} 2$ [8] and in the case of pure matter ($A = 0$), the pressure must vanish, i.e., $p = 0 \Rightarrow (r^2\widetilde{C})' \stackrel{!}{=} 1$. By imposing these two requirements, the solution is then

$$\widetilde{C}(r) \equiv C(r) := 1 - \frac{2G\mathfrak{m}}{r} \tag{17}$$

The important feature is that the metric that was derived (equation 16) allows us to evolve a black hole through the radiative and matter dominated periods while having a smooth transition between the two.

The appearance of $\mathfrak{m}$ in equation 17 is because this variable does not represent the physical mass of the black hole, as was the case for $m(t)$. We can see that in the following

$$\frac{G\mathfrak{m}}{r} \to \frac{G\mathfrak{m}a(\eta)}{ra(\eta)} \equiv \frac{G\mathfrak{m}a(\eta)}{r_{phys}} \Rightarrow m(\eta) = \mathfrak{m}a(\eta) \tag{18}$$

Now, combining equations 3, 7 and the relation above and using $\eta_{min} = 10^{-7}$ s, $\eta_{max} = 10$ s, $G\mathfrak{m}_{min} = GM_{seed}/a_{max}$ and $G\mathfrak{m}_{max} = GM_{seed}/a_{min}$, we obtain

$$G\mathfrak{m}_{min} \sim 5 \times 10^{-6}\,\mathrm{m} \qquad , \qquad G\mathfrak{m}_{max} = 5 \times 10^{2}\,\mathrm{m} \tag{19}$$

Considering now the non-vanishing components of the Einstein tensor, for the metric derived in equation 16 with $C(r)$ of equation 17, they are given by

$$\begin{aligned} 8\pi G T_{\eta\eta} &= \frac{3\dot{a}^2}{a^2} = \frac{3(A + 2B^2\eta)^2}{(A\eta + B^2\eta^2)^2} \\ 8\pi G T_{r\eta} &= \frac{2G\mathfrak{m}}{r^2 C(r)}\frac{\dot{a}}{a} = \frac{2G\mathfrak{m}}{r^2 C(r)}\frac{A + 2B^2\eta}{A\eta + B^2\eta^2} \\ 8\pi G T_{rr} &= \frac{\dot{a}^2 - 2a\ddot{a}}{a^2 C^2(r)} = \frac{1}{C^2(r)}\frac{A^2}{(A\eta + B^2\eta^2)^2} \end{aligned} \tag{20}$$

With the equations from [8]

$$T_{\theta\theta} = C(r) r^2 T_{rr} \qquad , \qquad T_{\phi\phi} = \sin^2\theta T_{\theta\theta} \tag{21}$$

In order for us to understand the physical meaning of the last equation of the set of equations in 20, we need to interpret first the right hand side of it. Rewriting the energy momentum tensor (ignoring matter self-interactions and higher derivatives in u_μ) in the form

$$T_{\mu\nu} = p g_{\mu\nu} + (p + \rho) u_\mu u_\nu - u_\mu q_\nu - u_\nu q_\mu \tag{22}$$

In order for the density and pressure to match between equation 22 and the last equation of the set of equations in 20, we need to include a factor of $1/C(r)$ in the equations present in 12 as so to account for the curvature, resulting in

$$\begin{aligned} 8\pi G\rho(\eta) &= \frac{1}{C(r)}\left(\frac{3A^2}{a^4(\eta)} + \frac{12B^2}{a^3(\eta)}\right) \\ 8\pi G p(\eta) &= \frac{1}{C(r)}\frac{A^2}{a^4(\eta)} \end{aligned} \tag{23}$$

with $a(\eta)$ as the one in equation 11. Disregarding the trivial solution, the 4−velocity vector is

$$u_\mu = -\frac{a(\eta)}{C^{1/2}(r)}(C(r)\cosh\xi, \sinh\xi, 0, 0) \tag{24}$$

and the heat flow vector is

$$8\pi G q_\mu = -\frac{2G\mathfrak{m}\dot{a}(\eta)}{r^2 C^{3/2}(r) a^2(\eta)}(C(r)\sinh\xi, \cosh\xi, 0, 0) \tag{25}$$

Both these vectors obey the conditions $u^\mu u_\mu = -1$ and $u^\mu q_\mu = 0$. The parameter ξ, which depends on η and r and obeys the condition $\xi(\eta, r) > 0$, is determined from

$$\tanh \xi = \frac{G\mathfrak{m}\eta}{r^2} \left(1 - \frac{B^4\eta^2}{A^2 + 3AB^2\eta + 3B^4\eta^2} \right) \tag{26}$$

The signs in equations 24 and 25 are chosen such that when considering the contravariant components of the 4−velocity vectors, we have $u^\eta > 0$ and $u^r < 0$, that is, inward flow of matter. If we were to choose the opposite sign for the components of u_μ, the result would correspond to a shrinking white hole.

By demanding that $\tanh \xi < 1$, we assure that ξ is both real and finite. We can also see that

$$\tanh \xi \sim \begin{cases} \frac{G\mathfrak{m}\eta}{r^2} & \text{for } B^2\eta \ll A \text{ (radiation)} \\ \frac{2}{3}\frac{G\mathfrak{m}\eta}{r^2} & \text{for } B^2\eta \gg A \text{ (matter)} \end{cases} \tag{27}$$

It is important to note that the representation in equation 22 is valid if all the quantities involved remain real and finite. For this, it is required that $r^2 > \mathcal{O}(1)G\mathfrak{m}\eta$, with a strictly positive $\mathcal{O}(1)$ prefactor. As r reaches the value for which we have $\tanh \xi = 1$, the components of u_μ and q_μ diverge, and the expansion of equation 22 breaks down. For an external observer, the average speed of matter that's falling towards the black hole reaches the speed of light and so everything happening inside this shell is then shielded from the outside. Since the $\mathcal{O}(1)$ factors are not important, it is defined

$$r_H(\eta) := a(\eta)\sqrt{G\mathfrak{m}\eta} \tag{28}$$

and the associated outward moving shell is interpreted as an effective horizon that lies above the real event horizon of the black hole. Taking into account the coordinate re-scalings mentioned before (after equation 13), r_H is invariant under them. Physically, the formation of inhomogeneities in the region $r_{phys} > r_H(\eta)$ makes this region run out of "fuel" and thus it is expected that matter inside the shell $r_{phys} \lesssim r_H(\eta)$ is sucked into the black hole. The presence of extra matter in this latter region therefore enhances the black hole's growth substantially beyond the linear growth that equation 18 implied with the presence of the scale factor.

As inhomogeneities start to occur, the metric 16 is no longer valid because the growth of the black hole and the growth of the scale factor, $a(\eta)$, become decoupled. After this time, the black hole grows in a more standard way by means of a much lower accretion. In order to estimate the mass of the black hole, an effective Schwarzschild radius is defined by taking the value of r_H at that time. With this, it is possible to equate the mass with the maximum energy that can fit inside this shell of radius r_H.

Because the inhomogeneities in the CMB are of the order of $\mathcal{O}(10^{-5})$, the time t at which to evaluate $r_H(t) \equiv r_H(\eta(t))$ is well after decoupling. Instead, the time $t_{inhom} \simeq 10^8$ yr $\simeq 3.2 \times 10^{15}$ s, which is the time at which the first stars are born, is the one taken. This time corresponds to a value of $\eta_{inhom} \simeq 2.7 \times 10^{17}$ s which, in turn, corresponds to a value of $a(\eta_{inhom}) \simeq 0.034$. By substituting equation 19 in equation 28 and using $r_S(M_\odot) = 3$ km, the range of possible black hole masses obtained at $t \sim 100$ Myr is

$$10^5 M_\odot \lesssim m_{BH} \lesssim 2 \times 10^9 M_\odot \tag{29}$$

which is consistent with the observations in [10]. For these values of mass to be achieved, the replacement made in equation 28 of $G\mathfrak{m}$ by $\sqrt{G\mathfrak{m}\eta}$ is of extreme importance.

4. Conclusions

In this report, a model was proposed to describe the origin and growth of PBHs during the radiation dominated era and the matter dominated era. Based on the work done by Meissner and Nicolai ([6], [7] and [8]), PBHs can be formed due to the gravitational collapse of lumps of bound states of stable supermassive elementary particles called gravitinos during the radiation era and their evolution through this era and the matter dominated era is described by an exact solution of Einstein's equations.

Following their previous papers, the reasoning applied to comprehend and describe the evolution of PBHs during the early phases of the Universe is one that should be considered carefully. Its foundations imply the existence of the gravitino which, to this day, has never been observed. This thought process leads to the "creation" of new physics. Although their results agree to observations, there may be some other mechanism that doesn't involve new physics to explain how these black holes come to be and how they evolve through time.

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