Proceedings

Testing Inflation through the CMB

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Abstract: Inflation opens the door to the possibility that quantum fluctuations were the seed of structure formation. These fluctuations became density fluctuations which we study today on the Cosmic Microwave Background (CMB). In addition, inflation produced gravitational waves that also left their signature on the CMB. In this report, there is a brief review of inflation and how it connects with the CMB anisotropies, leading us to the conclusion that CMB can be use to a better understanding of inflationary models.

Keywords: Inflation; Cosmic Microwave Background; Quantum Fluctuations

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1. Introduction

Around the late 1970s and the early 80s, in order to understand some problems that arose from observations and theoretical predictions, it was develop a new theory called inflation. Inflation is a theory that predict a brief period of exponential growth of the Universe right after the big bang. This exponential growth allied with quantum fluctuations were the ingredients needed to understand the structure formation.

Inflation is a remarkable theory and a essential part of Universe's history. We can not make observations of the very early times, the far we can see on the Universe's history correspond to the recombination epoch. During that time, the photons started to travel freely throughout the Universe and today this remnant radiation reaches us as the Cosmic Microwave Background (CMB). However, this is not disappointing since inflation makes several predictions that can be observed on the CMB. The study of CMB anisotropies is vital to make constraints on these kind of models. After all, as predicted by inflationary models, the anisotropies on the CMB were made by the quantum fluctuations.

We can divide the CMB fluctuations in two types: scalar fluctuations and tensor fluctuations. Scalar fluctuations are the ones that contribute the most for the temperature anisotropies of the CMB. When one talks about scalar perturbations are in fact referring to density perturbations. On the other hand, tensor perturbations are the perturbations created by gravitational waves. Tensor perturbations leave their signature on the polarization of the CMB.

In this report we probe the link between inflation and the CMB. In Section 2 we explore inflation a little deep, by understanding the motivations behind it, its physics and how quantum fluctuations could lead to structure formation. In Section 3 we get how inflation and CMB can relate with each other by looking at CMB anisotropies. Finally, in Section 4 we conclude with a summary of the report.

2. Inflation

Inflation was a brief period of time, that lasted about 10^{-33} seconds, where the Universe expanded by an enormous factor, a factor around 10^{26}. In other words, we can define inflation as a period of the Universe's evolution in which the scale factor is accelerating. Therefore,

$$Inflation \Leftrightarrow \ddot{a} > 0. \tag{1}$$

With this in mind, one can conclude that during the inflationary epoch, the Universe was dominated by a fluid component that is describe by the following equation of state,

$$w < -\frac{1}{3}. \tag{2}$$

We do not know what kind of fluid this is.

From 1 we can deduce another interesting condition:

$$\ddot{a} > 0 \Leftrightarrow -\frac{\ddot{a}}{\dot{a}^2} < 0 \Leftrightarrow \frac{d}{dt}(\dot{a}^{-1}) < 0 \Leftrightarrow \frac{d}{dt}(aH)^{-1} < 0 \Leftrightarrow \frac{d}{dt}(cH^{-1}/a) < 0, \tag{3}$$

where cH^{-1} is the Hubble radius R_H. This means that inflation was a period where the comoving Hubble radius was decreasing, i.e., it was shrinking relatively to the physical space. At the end of inflation, particles outside the Hubble sphere can no longer communicate, but notice that before inflation these particles were inside the Hubble sphere, therefore they shared the same initial conditions.

2.1. Motivations for Inflation

Inflation was developed for the purpose of solving several problems that arose from the Hot Big Bang model. In the next subsections, it is explained why they are problems and how inflation can solve them.

2.1.1. Flatness Problem

We can know the curvature of the Universe by knowing the density parameter, Ω, which is given by:

$$\Omega = \frac{\rho}{\rho_c}. \tag{4}$$

From this result, we end up with three possible curvatures: if $\Omega = 1$ the Universe has a flat geometry; if $\Omega < 1$ the Universe has a open geometry, like a saddle; if $\Omega > 1$ the Universe has a closed geometry, like hypersphere in three dimensions.

The density parameter today, Ω_0, is very close to one. For this to happen, the density parameter should also be equal to one right after the big bang. Meaning, that the density of the early Universe was almost equal to it's critical density and this is the *flatness problem*.

Why does this constitute a problem? Well, because this conditions are extremely unlikely. Notice that any slightly deviation from this equality would cause a totally different fate for the Universe. If the density would be less than the critical density ($\Omega > 1$), the Universe would have expanded so rapidly and therefore matter could not form structure. Also, if the density was bigger than the critical density ($\Omega < 1$) the Universe would eventually collapse due to gravity. Since nothing of this happened, the Universe always had to have a density parameter equal to one with an incredible precision.

How does inflation solve this problem? Since inflation was a period of rapid expansion, the Universe grow in a way that its curvature, at least locally, seems to be zero, i.e., the Universe has a flat geometry. We can understand this with a simple analogy: Earth is a sphere, so it has a curvature, but we can not notice that, therefore, locally Earth seems flat.

2.1.2. Horizon Problem

Observations of the CMB show that its temperature is practically the same in all directions. This is a little odd since, if we think about two opposite regions in the sky, they have never been in contact. From the big bang until the time when that light was emitted, there was not enough time for those two regions to have a common past. This is called the *horizon problem*.

The only solution to this problem is implying that this two regions have a common past. Inflation solve this perfectly. Even if they were not in contact at the time when CMB's radiation was release, before inflation these regions were in contact, so they end up with almost the same temperature.

2.1.3. Unwanted Relics

Grand unified theories (GUT) describe the union of the strong, weak and electromagnetic force into a single one. At early times, this single force is broken into three and this leads to unwanted relics, such as magnetic monopoles, etc. However, this particles were never detected.

So, where are the missing particles? Inflation gives us the solution. Since the Universe expanded so rapidly by an enormous factor, the abundances of these particles decreased, and that is why we can not detect them.

2.1.4. Homogeneity and Isotropy

The Hot Big Bang model fails when it tries to explain why the Universe is homogeneous and isotropic on large scales. It predicts that the early Universe was made of several causally disconnected regions of space. Therefore, it is needed a model that can explain the homogeneity and isotropy on large scales and this is well done by inflationary models. This is because regions that we thought that were causally disconnected, after all, they have a common past, thus, they shared the same initial conditions.

It is important to notice that, although at large scales the Universe is homogeneous and isotropic, at small scales, it has inhomogeneities. This inhomogeneities must have been part of the initial conditions. Inflation may have created these inhomogeneities through quantum fluctuations. Posteriorly, these fluctuations end up forming structure in the Universe.

2.2. Physics of Inflation

In a FLRW spacetime, cosmic acceleration happens when:

$$\epsilon \equiv -\frac{\dot{H}}{H^2} = -\frac{dlnH}{dN} < 1, \tag{5}$$

where ϵ is the slowly-varying Hubble parameter and $dN \equiv dlna = Hdt$, which measures the number of e-folds (N) of inflationary expansion. In order to solve the Hot Big Bang problems usually we need $N \sim 40 - 70$.

Since ϵ needs to be small for a sufficient large number of e-folds, it is useful to introduce a second parameter that measures the how ϵ changes with time:

$$\eta \equiv \frac{dln\epsilon}{dN} = \frac{\dot{\epsilon}}{H\epsilon}. \tag{6}$$

So, for inflation to occur we need to have $|\eta| < 1$.

Inflation was driven by a scalar field called *inflaton* $\phi(t, x)$. Notice that if we consider a FLRW spacetime, the background value of the inflation does not depend on space, i.e., $\phi = \phi(t)$. For each value of ϕ there is a potential energy density $V(\phi)$ (Fig. 1).

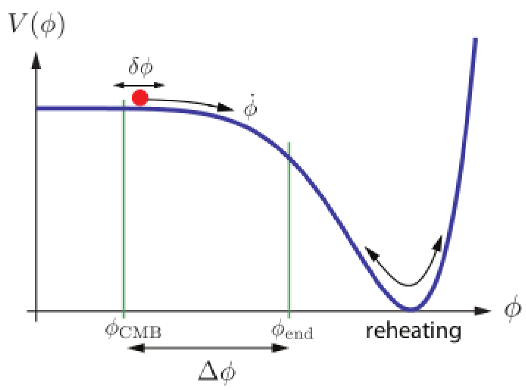

Figure 1. Example of an inflaton potential. Inflation happens when the kinetic energy of the field is small compare to the potential energy. At ϕ_{end}, the kinetic energy grow enough to be comparable to the potential energy, therefore, inflation ends. Credits: [1]

The energy density of the Universe is given by:

$$\rho = \frac{1}{2}\dot{\phi}^2 + V(\phi), \tag{7}$$

where $\frac{1}{2}\dot{\phi}^2$ and $V(\phi)$ are the kinetic energy and the potential energy of the inflaton field, respectively. On the other hand, pressure is given by the difference of these two energies, so:

$$p = \frac{1}{2}\dot{\phi}^2 - V(\phi). \tag{8}$$

For a slowly evolving field, the potential energy is larger than the kinetic energy, $V(\phi) \gg \frac{1}{2}\dot{\phi}^2$ and the pressure ends up with a negative value. In this conditions the Universe's expansion accelerates and the scale factor grows exponentially. This mechanism is known as the slow-roll inflation.

2.3. Quantum Fluctuations

Quantum fluctuations at early times are the very beginning of structure formation. Therefore, these fluctuations are the source of the primordial power spectra $P_s(k)$ and $P_t(k)$ of scalar and tensor perturbations, respectively, which are define in the next section.

To analyze the fluctuations it is convenient to split the inflaton field into an homogeneous background $\overline{\phi}(t)$ and a spatially varying perturbation $\delta\phi(t,x)$, so we get

$$\phi(t,x) = \overline{\phi}(t) + \delta\phi(t,x). \tag{9}$$

The inflaton takes slightly different values at different points in space, therefore, inflation ends at slightly different times in different regions of space. This means, that different regions inflate by a different amount, which leads to density perturbations and eventually to the temperature anisotropies in the CMB. Thus, the fluctuations in the inflaton field $\delta\phi$ are the origin of the energy density fluctuations $\delta\rho$ after inflation. The efficiency of the conversion from inflaton fluctuations to energy density fluctuations depends on the slope of the inflaton potential V' and the size of the quantum fluctuations in ϕ depend on the scale of the potential V [1].

3. Testing Inflation with the CMB

Quantum mechanical fluctuations during inflation evolve and become macroscopic density fluctuations that can be studied through the CMB. This density inhomogeneities can be describe as:

$$\delta\rho(t,x) \equiv \rho(t,x) - \overline{\rho}(t), \tag{10}$$

where $\delta\rho(t,x)$ are the density fluctuations and $\overline{\rho}(t)$ is the homogeneous background density. Regarding to $\delta\rho(t,x)$ function, one is only interested in a statistical analysis, therefore, one can study the correlations between the density fluctuations at two different points in space. So, we can know the probability of a high density region at a given distance from another high density region. In Fourier space we get the power spectrum,

$$P_s(k) = A_s(k/k_*)^{n_s-1} \tag{11}$$

which corresponds to a two-point correlation in real space. Where A_s corresponds to an amplitude and n_s is the scalar spectral index. These two quantities are measured at a reference scale k_*. Inflationary models predict the shape of the function $P_s(k)$, which can then be compared with observations of the CMB. This is due to the fact that at early times these density fluctuations leave their signature in the CMB as temperature anisotropies.

Also, in addition to these fluctuations, inflation produced gravitational waves. We can compute the power spectrum for the gravitational waves, which is

$$P_t(k) = A_t(k/k_*)^{n_t}, \tag{12}$$

where n_t is the tensor spectral index. Although these primordial gravitational waves can not be detected directly, since they are too faint, they can also be studied through the CMB, because they left small imprints in its polarization.

3.1. Temperature Anisotropies

At the early times, after the big bang, the Universe was extremely hot and dense, so atoms could not form and all the matter formed a ionized plasma. But as the Universe expanded, temperature eventually started to dropped down and this allowed atoms to form. When this happened, photons were able to travel throughout the Universe. This period became known as epoch of recombination. CMB is a remnant radiation from the epoch of recombination, since before that photons could not travel great distances because their mean free path was very small, thus, the Universe was opaque.

The last scattering surface are the limit in our visible Universe, so, it dates the recombination epoch. When photons from the last scattering surface travel throughout the Universe they can encounter dense gravitational regions, as galaxy clusters. When they do, photons enter in a gravity well and gain energy, as they pass through the cluster and finally escape from the well they lose energy. The net energy should be zero, but as the photons pass through the cluster, the Universe keeps expand, therefore the clusters became less denser, and when the photons finally escape the gravity well do not lose the same amount of energy, so they end up with more energy than they had in the beginning. This effect is seen on the CMB and correspond to hotter regions (reddest regions) (see Figure 2). The opposite happens for voids, and at the end of the void, we get less energetic photons, therefore, colder regions on the CMB (bluest regions). This is known as the Sachs-Wolfe effect.

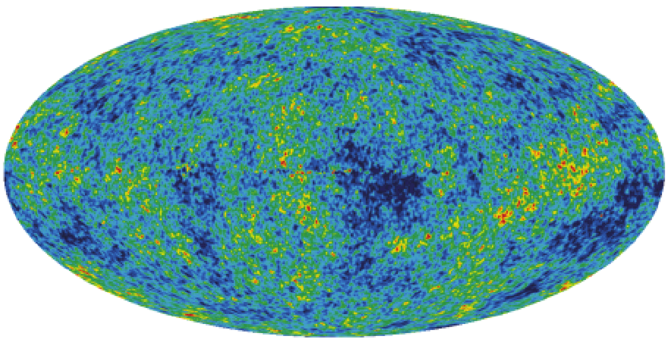

Figure 2. Temperature fluctuations in the CMB. Credits: [1]

CMB behaves almost like a homogeneous and isotropic black body with a average temperature of $T \approx 2.73K$. So, basically, when we analyze the anisotropies of CMB we are looking to the temperature fluctuations, ΔT, around the average temperature, $\Delta T/T$, described by the angular TT power spectrum, C_ℓ.

We can write down the expression for temperature fluctuations using a spherical harmonic expansion, therefore we get

$$\Delta T(\theta, \phi) = \sum_{l,m} a_{lm} Y_{lm}(\theta, \phi), \tag{13}$$

where (θ, ϕ) are position angles on the sky. The angular TT power spectrum is then given by,

$$C_\ell = \frac{1}{2\ell + 1} \sum_m |a_{lm}|^2. \tag{14}$$

Since the temperature fluctuations follow a Gaussian distribution, all the statistical information in the CMB map is contained in the angular TT power spectrum. The shape of the angular TT power spectrum depend on the cosmological parameters as well as on the initial conditions. In Figure 3 we can see how the power spectrum of CMB temperature anisotropies is in agreement with data.

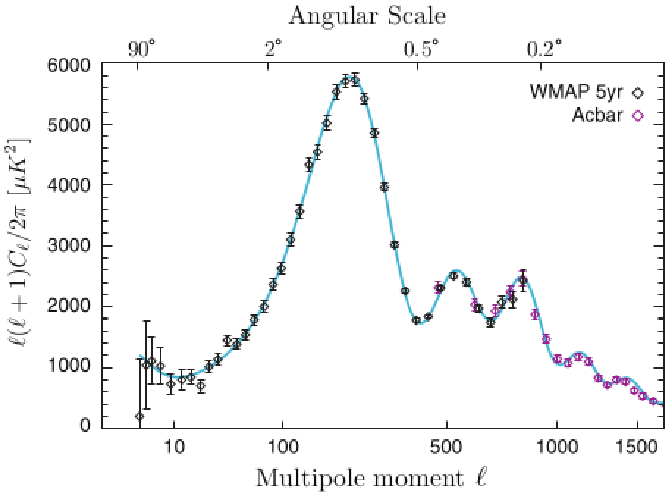

Figure 3. Power spectrum of CMB temperature fluctuations. The blue solid line is the theoretical prediction of ΛCDM model. Credits: [1]

3.2. Polarization Anisotropies

The signature of primordial gravitational waves can be seen in the polarization anisotropies of the CMB. Since the temperature anisotropies are scalar perturbation, they can not distinguish the contributions of scalar and tensor perturbations. That is why polarization anisotropies are so important. Polarization anisotropies can be decomposed into two orthogonal modes: E-mode and B-mode.

The main contribution to CMB polarization anisotropies comes from scalar perturbations, i.e., density perturbations. But these perturbations only generate the E-mode polarization. B-mode polarizations are only produced by gravitational waves (tensor perturbations). Notice that gravitational waves also produce E-mode polarizations, but since B-mode polarizations are generated by gravitational waves alone, we are interested in the latter.

One important thing about primordial B-mode anisotropies is that they are at least one order of magnitude smaller than E-mode polarization. In addiction it is very hard to distinguish between primordial B-modes and B-modes created by another astrophysical sources, like polarized dust in our galaxy.

Although inflationary models predict that gravitational waves will create B-modes at angular scales of a degree or larger, they do not predict their amplitude because it depends on the energy scale of inflation, which varies between models. For that reason we parametrize the amplitude of gravitational waves with the tensor-to-scalar ratio,

$$r \equiv \frac{P_t}{P_s}. \tag{15}$$

Observations of the B-modes are sensitive to this ratio.

The tensor-to-scalar ratio is extremely important, in fact, inflationary models can be classified according to their predictions of the scalar spectral index n_s and the tensor-to-scalar ratio r.

3.3. Power Spectra

In the end, we end up with four types of power spectrum: the autocorrelation of the temperature fluctuations, TT; the autocorrelation of the E-modes, EE; the autocorrelation of the B-modes, BB; and the cross-correlation between the temperature fluctuations and E-modes, TE. The correlations TB and EB vanish for symmetry reasons.

All of these power spectra depend on the cosmological parameters, each one in a different way. For example, we can see in Figure 4, the dependence of the TT power spectrum on the cosmological parameters. This means, that a combined measurement of all the power spectra really improves the constraints on the cosmological parameters.

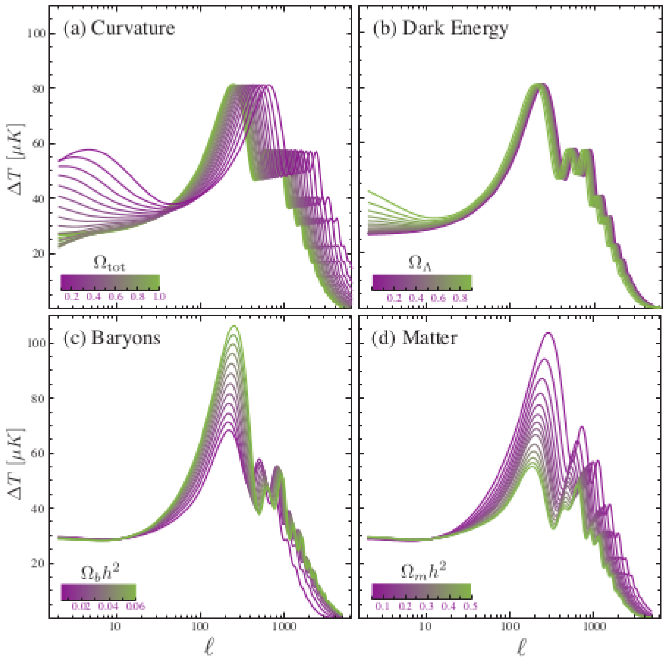

Figure 4. Fluctuations in the CMB as a function of the cosmological parameters. (a) The variation of the the total density, shifts the position of the peaks of the spectrum. Therefore we can know the Universe's curvature by observing the CMB. (b) Dark energy contributions affect the power spetrum on large scales. This is related with the Sachs-Wolfe effect. (c) The baryon density affects the relative peak heights. (d) Increasing the matter content uniformly damps power on all subhorizon scales. Credits: [1]

3.4. Predictions for the CMB

It is now clear that, the primordial fluctuations predicted by inflation are related to the fluctuations that we see on the CMB. In fact, there are a few characteristics that inflationary models predict for the CMB perturbations.

One of those characteristics is a *flat geometry*. Therefore, the observable Universe should not have a spacial curvature. We came to this conclusion by measuring the position of the first peak on the CMB power spectrum along with measurements of the Hubble constant.

Another characteristic for the CMB perturbations that these models predict is *Gaussianity*. It is not easy explain why CMB is Gaussian without considering a inflationary model. Notice that, although, most of the primordial fluctuations follow a Gaussian distribution, which is predicted by slow-roll inflation, non-Gaussianity is also present and it is an important part in order to understand the inflationary dynamics.

Also, these models predict *scale-invariance*. This means that, to a first approximation, the power spectrum is scale-invariant, which correspond to say that $n_s = 1$ and $n_t = 0$ in equations 11 and 12, respectively. However, inflationary models also allow small deviations from scale-invariance. In fact, with measurements of this deviations we can explore the shape of the inflaton potential.

Another characteristic predicted is *adiabaticity*. Therefore, the temperature and the matter density fluctuations should satisfy the condition $\delta T/T = \frac{1}{3}\delta\rho/\rho$, which is the same correlation for the adiabatic compression of a gas. Meaning that, regions with high density also have an high temperature.

Inflationary models also predict *super-horizon fluctuations*. Considering the CMB, the horizon problem tell us that, angular scales larger than 2° correspond to regions that never have been in contact. Therefore, those regions are in each other super-horizon. However, there exist correlations between the anisotropies of those regions, which are explained by inflationary models.

At last, another prediction of inflationary models are the *primordial gravitational waves*. Has explained before, they generate temperature and polarization anisotropies.

4. Conclusions

As we saw, inflation solve some cosmological problems, e.g., the flatness problem, the horizon problem. It also predict that the production of quantum fluctuations in the very early times lead to the Universe we see today. Inflation stretched the quantum fluctuations out of the horizon and when they eventually reentered the horizon they became the seed of structure formation. This is due to the fact, that quantum fluctuations became density fluctuations, and of course, we can study this density fluctuations through the CMB.

Beyond that, inflation predicts gravitational waves, which also left their imprint on the CMB. We study them through B-mode polarization, but it is extremely hard since it is difficult to distinguish if the B-mode polarization that we detect is from the early Universe or from another astrophysical sources.

In the end, CMB is a magnificent tool to understand the Universe. Using its power spectra we can make constraints on its geometry as well as on its cosmological parameters, thus, we can know the abundances of dark energy, dark matter, etc.

It is remarkable the way that inflation left its imprint on the CMB. How a brief period of accelerated expansion had a huge impact on the Universe throughout its existence. Therefore, in order to understand the early times we should study more careful the CMB, since it is the farther observation that we can make in time.

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