Proceedings

Rotation Curves of Spiral Galaxies as an Observational Evidence of Dark Matter

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Abstract: Short introduction about the first considerations on dark matter on the galactic and extra galactic level between the 1920's and 1940's. Insight in the study of several galaxies dynamics and their rotation curves. Showing how to calculate the velocities of these curves and how to infer the mass contained in the galaxy using these. The presentation of some of the results obtain in the 1970's, and the development of the idea on how these flat rotation curves were an observational evidence for a type of dark and invisible matter.

Keywords: Dark Matter; Rotation Curves; Spiral Galaxies;

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1. Introduction

Rotation curves of spiral galaxies is just one of the many studies that presented astronomers with data that suggested that our understanding of the amount of matter in the universe was lacking a great deal of reality. With this work I intend to share the first considerations made regarding dark matter and bring to light the efforts that were made to describe galaxies in the 1970's and how those studies led to the consideration, of the existence of invisible matter in the outermost parts of galaxies.

1.1. The First Considerations on Dark Matter

During the 1920's, astronomers were still trying to comprehend our own home galaxy: how it rotated, and our place in it. During these measurements, signs of "missing" matter started to appear. By then, astronomers were getting comfortable with the idea of the existence of many galaxies in different shapes, and sizes. But the knowledge of our own galaxy had plenty to grow still. The discovery that our galaxy rotated was developed between 1925 and 1930, and Jan Oort, was one of the big names involved in it. After studying the motions of the stars in our Galaxy, Oort, in 1927, revealed that the Galaxy has a differential rotation, this means that the disk-shaped Galaxy doesn't rotate rigidly. Inner parts of the Galaxy spin faster than the outer regions, making every point in the Galaxy rotate with a different angular velocity.

In 1932, still trying to characterize our Galaxy, Oort conducted a detailed study of the amount of matter present in its' disk. Besides measuring the average and random motions of stars around the Galaxy, he also measured their perpendicular motion to its' plane. This got him an amount of dark, invisible matter, that was twice the amount of visible one. Years later, it was discovered that the study conducted by Oort had major defects, inducing incorrectly the amount of dark matter present in the Galaxy disk. However, this study motivated many other astronomers at the time to take a closer look at this invisible matter that might populate our Galaxy and the Universe.

In 1933, Fritz Zwicky , while measuring radial velocities of members galaxies in the Coma Cluster, was the first to use the Virial Theorem to infer the existence of invisible matter. The greater the spread in velocities (the greater its' kinetic energy), the greater the gravitational energy used to keep the galaxies connected. Since gravity implies matter, Zwicky inferred the gravitational mass of the Coma Cluster, and then estimated the amount of mass for each individual galaxy in it. He then measured the brightness of each galaxy in the cluster, producing a realistic estimate of the total luminosity of the cluster. After deducing the luminous mass of the cluster, Zwicky was left with a much lower value than with the gravitational mass. In 1937, Zwicky published another paper where the purpose was to determine the mass of galaxies, in which he refined and extended his analysis of the Coma Cluster. According to his results, the Coma Cluster was saturated with dark matter. However, the scientific community didn't take Zwicky's result seriously, leaving it unaccredited for a long time.

It wasn't until the 1970's that the first explicit statements defending the existence of additional mass in the outer parts of some galaxies started to appear, and that new observational data came corroborate Zwicky's astonishing amount of invisible matter around galaxies across our known Universe.

1.2. The 1970's and Flat Rotation Curves of Spiral Galaxies

During this decade there were plenty of studies and observations conducted to try to characterize the different types of galaxies found in the sky. Besides the visible observations, with the evolution of the radio astronomy during these years, it was possible to look into the darkest parts of these galaxies, and have a more profound insight of the dynamics in these massive clusters.

The retrieved data from both visible and radio observations showed that the outermost parts of the galaxies rotated much faster that it was expected from the amount of luminous matter present in these parts. This behaviour was seen through representations of the rotational curves of galaxies. The expected behaviour of these curves, considering the amount and mass of visible matter, would be an increase of the rotational velocities from the center of the galaxy, then the curve would peak and start to decrease.Instead of presenting a peak of velocity and then decreasing, these curves peaked and then stayed flat, showing that the outer parts of the galaxies rotated with much higher velocities than anticipated. To allow such velocities, according to Kepler's third law, there had to be a much higher mass there to originate such dynamics.

This introduction has as sources: Gianfranco Bertone and Dan Hooper [1], Matts Roos [2], and Freeman and Mcnamara [3].

2. Velocity Measurements and Mass Calculations

In the paper *Rotation of the Andromeda Nebula from a Spectroscopic Survey of Emission Regions*, published in 1970, Vera C. Rubin and W. Kent Ford,Jr [4], showed results of observations of sixty-seven emission HII regions from 3 to 24 kpc from the nucleus of Andromeda. Their main goal was to obtain the field velocity and determine its mass.

I will use this paper to present some methods, and calculations used to obtain the velocities and mass of galaxies.

The observations were conducted retrieving the spectra of these emission regions. With well-known emission lines (such as the H_α line), the authors calculated the Doppler shift present in the observations to obtain the radial velocity of these emission regions. In order to obtain the rotational velocity , the authors relied on the model that considers the galaxy a thin disk (Burbidge et al. [5])

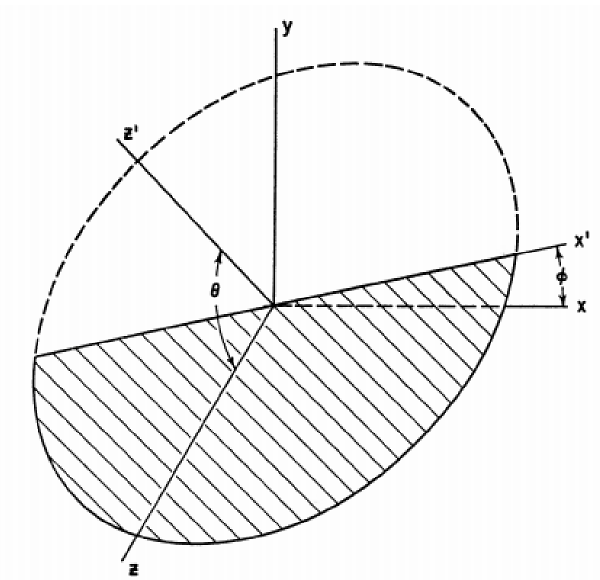

Figure 1. Model where the galaxy is replaced by a thin disk in differential rotation. The z axis is along the line of sight, z′ is perpendicular to the plane of the galaxy, x′ is the intersection of the (x,y) and (x′,y′) planes.

In this model one must consider the orientation of the galaxy with respect to the line of sight. Considering (x', y', z') the co-ordinate system with origin at the center of the galaxy, and z' perpendicular to its plane, and (x, y, z) another co-ordinate system with the same origin, but the z axis is in the line of sight of the observer. The angle between z and z' is θ, and ϕ is the angle between the x and x' axes. Since the observed velocity-curves are linear, $\mathbf{V} = w\mathbf{lz'} \times \mathbf{r}$, with w constant, is the space velocity of a mass element at distance $\mathbf{r}$ from the center. Where $\mathbf{lz}$ and $\mathbf{lz'}$ are the unit vectors along the z and z' axes. The observed quantities are the line of sight velocity, V_z and the projection of $\mathbf{r}$ on the (x,y) plane. Thus we can obtain V_z from the observations as a function of x and y for the points lying along the slit of the spectrograph. With this we can write:

$$V_z(x, y) = w \sin\theta(x \cos\phi + y \sin\phi) \tag{1}$$

For an emission region with these coordinates, the observed radial velocity is related to the circular-velocity component $V(r)$ and the expansion component $E(r)$. Re-rewriting equation 1 where $V_z = V_{obs} - V_c$, and V_c is the systemic velocity of the galaxy, in the case of Andromeda, is the velocity that it travels towards us.:

$$V_{obs} - V_c = V(r) \sin\theta \cos\phi + E(r) \sin\theta \sin\phi \tag{2}$$

This assuming that all motions are constrained to the main plane of Andromeda. If there aren't any expansion motions, $E(r) = 0$, then the second term of equation 2 disappears. Knowing V_{obs},V_c, θ, and $\cos\phi = x/r$ from the observations, one can calculate the circular velocity, $V(r)$.

Taking in count the observations of a distance to the center bigger than 3.2kpc, the circular velocities of these emission regions were calculated. From these sixty-seven emission regions, fifty-three are large associations of emission regions and OB stars that are distributed in thirty-four associations. Forming a mean rotational velocity for each association, the scatter in the data considering the sixty-seven regions is considerably reduced. This data is used to determine the rotation curve.

In Rubin and Ford [4], the rotation curve for Andromeda is the one presented in figure 2. For this work we have only interest in the data regarding the velocities of the OB associations in the outer parts of the galaxy.

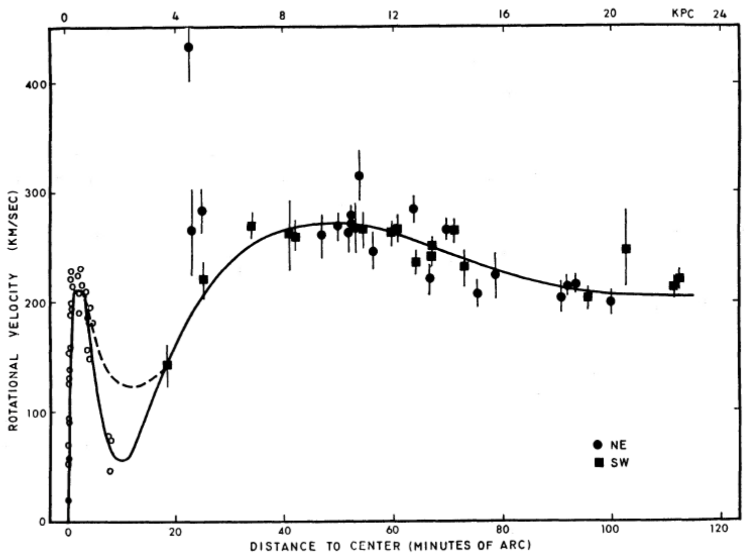

Figure 2

In order to estimate the mass of a galaxy one can consider that the distribution of the luminosity in galaxies is roughly elliptical in outline if we disregard the irregularities induced by the spiral structure. For the space distribution of the luminosity one is therefore led to an ellipsoidal or spheroidal distribution. A spheroid is an ellipsoid in which two axes are identical, [6] The following expressions are valid for a homogeneous spheroid with radius a, an excentricity $e = \sqrt{1 - c^2/a^2}$, where c is the polar radius, if we designate the gravitational field strength or force acting on a unit mass point by K_r and K_z respectfully. The following deduction can be found in more detail in Lecture Notes in Physics [6].

Defining β as

$$r^2 \sin\beta^2 + z^2 \tan\beta = a^2 e^2 \tag{3}$$

for a point (r, θ, z) exterior to the spheroid, while

$$\sin\beta = e \tag{4}$$

is valid for an interior point. With this, the force functions and gravitational potential for a homogeneous spheroid with constant density ρ are given by:

$$K_r = 2\pi G e^{-3}\sqrt{1-e^2}\rho r(\beta - \sin\beta\cos\beta) \tag{5}$$

$$K_z = 4\pi G e^{-3}\sqrt{1-e^2}\rho z(\tan\beta - \beta) \tag{6}$$

$$\Phi = 2\pi G e^{-1}\sqrt{1-e^2}\rho a^2\beta - \frac{1}{2}(rK_r + zK_z) \tag{7}$$

For the equatorial plane, z=0:

$$\Phi(r; z=0, a, e) = 2\pi G e^{-2}\sqrt{1-e^2}\rho. \left[\frac{a}{r}(r^2 - e^2a^2)^{1/2} - \frac{r}{e}\arcsin\left(\frac{ea}{r}\right)\right] \tag{8}$$

Thus, one obtains an integral equation for the gravitational field of an inhomogeneous spheroid in the equatorial plane

$$K_r = -\frac{\partial\Phi}{\partial r} = -4\pi G\sqrt{1-e^2}.\frac{1}{r}\int_0^r \frac{\rho(a)a^2da}{\sqrt{r^2 - e^2a^2}} \tag{9}$$

Due to the gravitational attraction of the matter in the spheroid, this force, in a stationary model, must be compensated by other forces such as centrifugal force or pressure terms.

This model gets simpler if one assumes that the matter in the equatorial plane moves in concentric circular orbits, and therefore neglects the pressure terms. For pure circular motion, and neglecting the pressure and its variations

$$\frac{< V(r) >^2}{r} = \frac{\partial\Phi}{\partial r} \tag{10}$$

Therefore,

$$V(r)^2 = 4\pi G\sqrt{1-e^2}\int_0^r \frac{\rho(a)a^2da}{\sqrt{r^2 - e^2a^2}} \tag{11}$$

This is the fundamental equation for the purely gravitational interpretation of galactic rotational curves. From this equation, one deduces the mass contained in the spheroids interior, as in Brandt [7].

Considering the density $\rho = \rho(a)$, a being the semimajor axisand, e, eccentricity. Knowing that the mass of the spheroid is

$$dm(a,e) = 4\pi a^2(1-e^2)^{1/2}\rho(a,e)da \tag{12}$$

Thus one can re-write the previous equation as

$$V^2(r) = G\int_0^r \frac{dm(a,e)}{\sqrt{r^2 - e^2a^2}} = G\int_0^r \left[\frac{dm(a,e)}{ada}\right]\frac{ada}{\sqrt{r^2 - e^2a^2}} \tag{13}$$

The term in brackets in the previous equation 13 has dimensions of a surface density. Requiring that the eccentricity $e \to 1$ tends to a sensible limit, one can rewrite 13 equation as

$$V^2(r) = G\int_0^r \sigma(a)\frac{ada}{\sqrt{r^2 - a^2}} \tag{14}$$

With a small substitution of $a^2 = u$ this transforms the equation 14 into Abel's integral equation. In this context this allows one to write

$$\sigma(r) = \frac{2}{G\pi r}\frac{d}{dr}\int_0^r \frac{V^2(a)ada}{\sqrt{r^2 - a^2}} \tag{15}$$

Recalling that the pseudo-surface density is defined as

$$\frac{dm(a, e \to 1)}{ada} = \sigma(a) \tag{16}$$

to calculate the mass contained in the interior of the spheroid to a point on the major axis a distance r from the center of the galaxy. The quantity becomes

$$M(r) = \int_0^r \sigma(a)ada = \frac{2}{G\pi}\int_0^r \frac{V^2(a)ada}{\sqrt{r^2 - a^2}} \tag{17}$$

With the substitution $a = r\sin\theta$ equation 17 is in a form suitable for numerical integration. The calculations of the mass distributions for Andromeda are presented in figure 3.

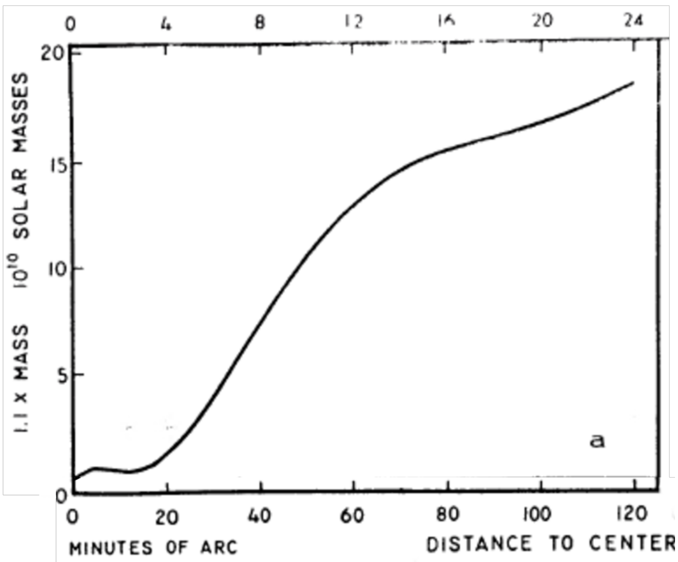

Figure 3. Total mass of Andromeda out to r=24kpc determined from the rotation curve (figure 2), as a function from distance to the center.

The observations of Andromeda presented in Rubin and Ford [4] were highly constrained by the capabilities of the observational instruments used at the time, and the authors knew that their insight in Andromeda's composition was still incomplete. In the paper the authors wrote:

It does not appear possible, from the presently available data, to infer anything about the mass beyond 24 kpc in M31. (...) extrapolation beyond that distance is cleary a matter of taste.

From the data retrieved it was possible to see that the rotation curve (Figure 2) decreased slowly with large r, but it wasn't possible to see beyond that.

3. Flat Rotation Curves

3.1. Radio Observations

In 1975, in the paper *The Rotation Curve and Geometry of M31 at Large Galactocentric Distances* by Morton S. Roberts and Robert N. Whitehurst [8], radio (21-cm) observations of Andromeda brought to light the dynamics of this galaxy up to a distance of 30 kpc from its center. The methods used to infer the velocities and mass were the same used in the optical observations. These observations came as a continuation of Rubin and Ford's work. In these observations, neutral hydrogen is detected to a distance of 30 kpc. This hydrogen has an essentially constant radial velocity along the major axis over the outer 10 kpc. From this data the authors derived a rotation curve which has a constant rotational velocity from 20 to 30 kpc. The rotations curve built from data from both papers is shown in figure 4

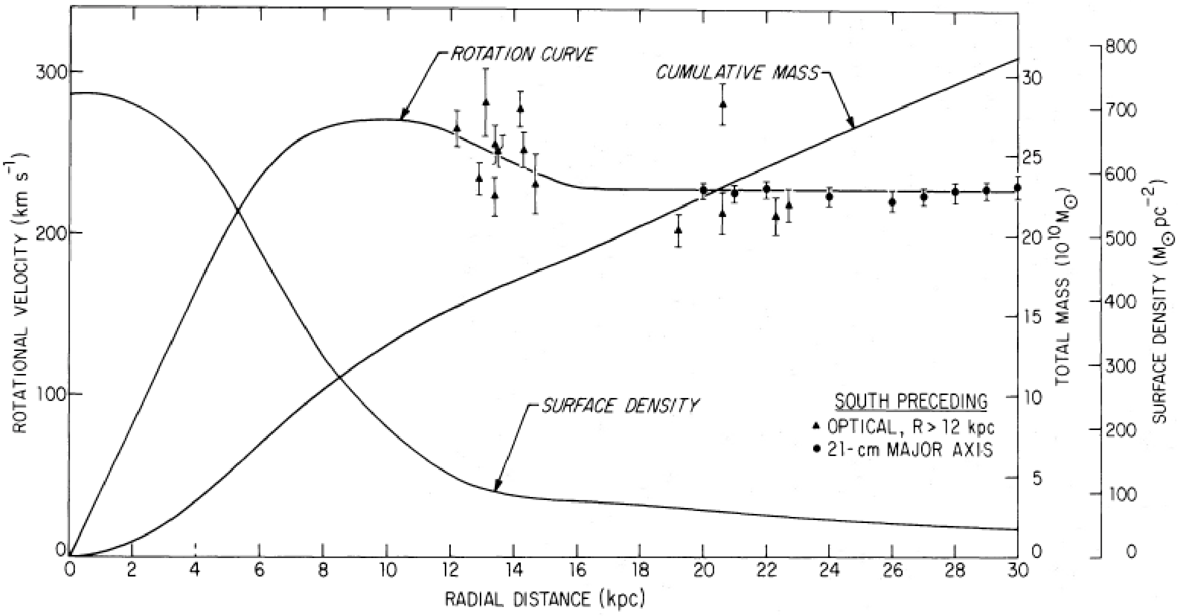

Figure 4. The adopted rotation curve, a composite of optical (Rubin and Ford 1970) data and 21-cm axis measurements. The surface density and cumulative mass are for a highly flattened model.

Like previously discussed, the form of the extended rotation curve, constant $V(r)$, implies a significant mass contribution, one that increases approximately linearly with r - a significant mass at large r is required to keep V from decreasing. One interesting hypothesis shared by the authors on how this amount of mass came to be, is that it belongs to undetected dwarf M-type stars that would satisfy the upper limits on brightness required.

3.2. Optical Observations

In 1978, in the paper *Extended Rotation Curves of High-Luminosity Spiral Galaxies, IV. Systematic Dynamical Proterties, Sa→ Sc* by Vera C. Rubic, W. Kent Ford, Jr., and Norbert Thonnard [9], the observations show that, for a sample of 10 high-luminosity spiral galaxies, all rotation curves are approximately flat to distances as great as $r = 50$ kpc.

Prior to this time the constraints on the observations had been principally instrumental. The optical rotation curves gave great insights on the dynamics of galaxies, but mostly in their inner regions, and the radio observations revealed integral properties of these galaxies, but with limited spatial resolution. By this time the available optical instrumentation permitted the detection of emission across a very large portion of the disk of spirals. The rotation curves of the observed galaxies are shown in figure 5.

Like before, this dynamical behaviour induces a crescent amount of mass as one gets to the outer most regions of the galaxies, the linear increase of mass with radius is a consequence of flat rotation curves. The plot of the integral masses of these galaxies are shown in figure 6. The method to calculate the integral mass is the same presented before.

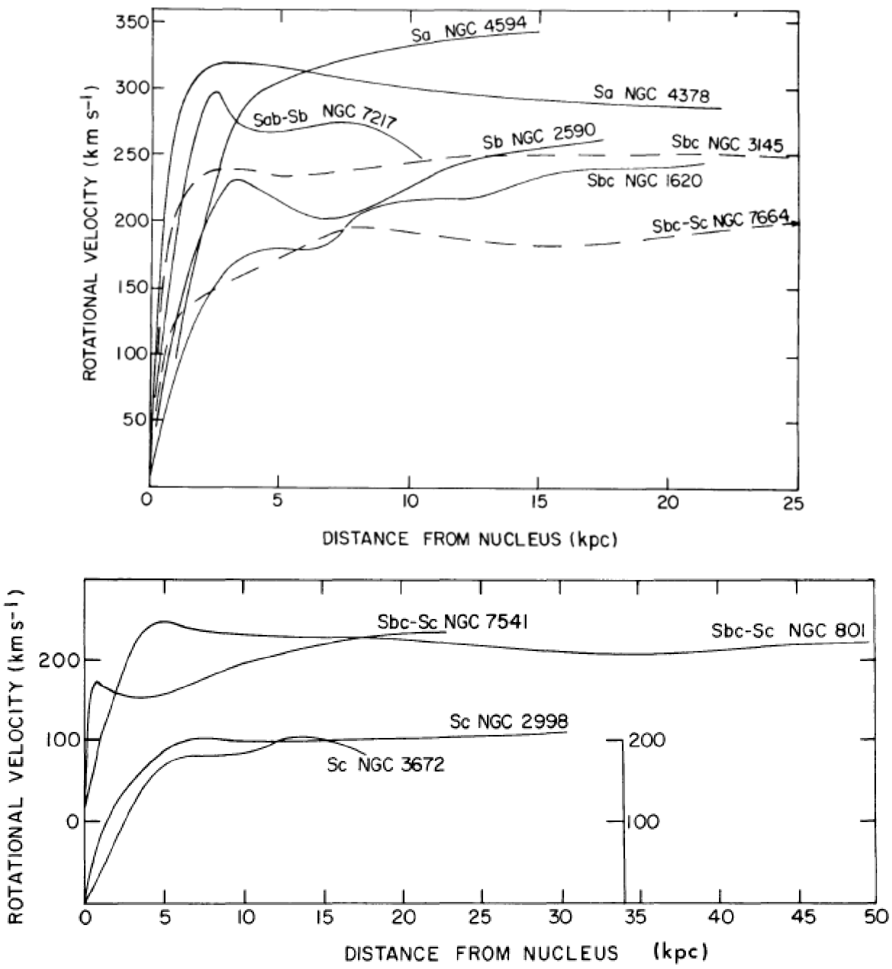

Figure 5. Rotational velocities for ten galaxies, as a function of distance from nucleus.

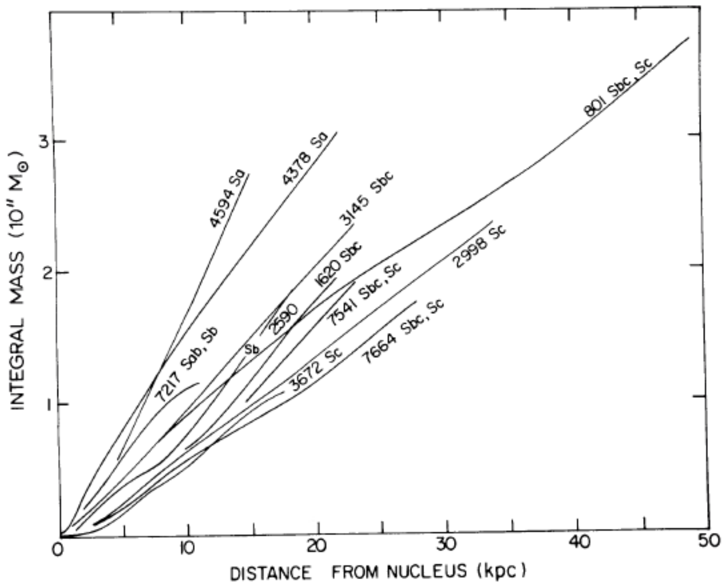

Figure 6

By the end of this paper, the authors write:

These results take on added importance in conjunction with suggestion (...) that galaxies contain massive halos extending to large r.

These models imply that with increasing r, the galaxy mass increases requiring that the rotational velocities remain high. The authors observations are necessary but not sufficient condition to support massive halos models.

4. Observational Evidence of Dark Matter

As previously shown, the dynamical study of galaxies brought to light an unexpected behaviour in the outermost parts of these celestial bodies. The constantly high radial velocities in these parts demand the existence of a great amount of mass to allow such movements, much more than previously theorized. This effect presents itself as an observational evidence for a great amount of matter that astronomers couldn't see nor detect. This discovery came to revolutionize the idea of the composition, not only of galaxies, but the whole universe.

Besides the flat rotation curves presented here, other observational evidences of black matter came to light the more the research continued. One other example comes in the form of gravitational lensing, the way the light curves when it finds massfull bodies on its way. Much like the flat rotation curves, this effect, the way it was observed and registered, in some cases, required the existence of a much bigger amount of mass that the one that could be inferred from the luminous observations. Another example is the dynamics of galaxy clusters. Like Zwicky showed, considering the velocities that these galaxies interact within themselves, they'd have separated and stopped interacting long ago, unless there's a much larger invisible mass keeping their interactions going.

As an alternative to the hypothesis of existence of such a great amount of dark undetectable matter, some theories of modified gravity started to appear. Most of these consider that the idea of dark matter can be replaced by alternatives to general relativity. These models attempt to account for all observations without invoking supplemental non-baryonic matter.

This issue is still a very open question today. Dark matter has yet to be observed directly which shows that if it exists it must barely interact with ordinary baryonic matter and radiation except through gravity.

5. Final Remarks

This journey through history was very interesting to study. It's astonishing how in just almost a decade between Vera Rubin's papers ([4] and [9]) the observational instruments improved so much and allowed the astronomers to look further into the unknown. In observations that were made mostly to characterize and get to know the galaxies in the sky, turned out to be a door of knowledge in a much deeper level of our universe. This particular event shows how much of an open mind a person studying the universe has to have, and also a special taste for surprises.

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