Characteristic Declination—A Useful Concept for Accelerating 3D Solar Potential Calculations

Miguel Centeno Brito, Rodrigo Amaro e Silva, and Sara Freitas*

The characteristic declination is the declination for the day on which the daily extraterrestrial irradiation on a horizontal surface is identical to its monthly average value. It was introduced as a means to determine monthly average values of irradiation. Herein, its potential usefulness to reduce computing time when mapping solar potential in complex urban areas is explored. This simplification reduces computing demand by a factor of 30× while introducing a +5% to +8% error in the annual monthly irradiation on a typical urban neighborhood for low and midlatitudes. Errors are larger (+10% to +12%) for high latitudes. The magnitude of the errors is comparable to other relevant uncertainties in solar mapping tools, associated with solar radiation modeling, the layout and details of the buildings, or the photovoltaic (PV) energy yield models.

1. Introduction

First introduced by Hay (1979) and then popularized by Iqbal (1983), the characteristic declination is the declination for the day on which the daily extraterrestrial irradiation on a horizontal surface is identical to its monthly average value.[1] As the declination does not change substantially over 24 h, it is customary to compute the characteristic declination for each day at solar noon. The characteristic declination changes slightly with latitude.

The definition of the characteristic declination makes it a useful concept to determine monthly average values of irradiation and its relationships with other monthly averaged parameters because it enables considering a representative day of the month instead of repeating the calculations for the whole month. For example, in Hay (1979) the characteristic declination was used to determine monthly mean values of solar irradiation on horizontal and inclined surfaces from bright sunshine hours and ground albedo.

In the past, this concept was convenient to reduce the computational cost associated with the calculation of daily irradiation when only monthly analysis was common practice. This application is no longer relevant as computer and data analytics tools allow for much higher time resolutions. However, the same sort of issues arises today when addressing the mapping of urban solar potential, with complex 3D urban structures. In this work, we explore the possibility of using hourly solar assessment for characteristic days as representative of hourly irradiation for the full year.

This issue has been addressed in related areas such as daylight and building performance modeling. A significant reduction in computing time can be achieved using the cumulative sky approach, which allows for the calculation of annual irradiation in a single simulation using the cumulative sky radiance distribution for all hours of the year (or a shorter period).[2] Reported results show more than 1000× decrease in computing time at the expense of 2.2% (root mean square) error. However, by construction, this approach disables assessing hourly solar irradiance profiles, or photovoltaic (PV) generation time series, limiting the analysis of solar potential in urban environments where the match between demand and supply is very relevant. Other approaches include using the default average sun position for 20 days in the EnergyPlus Shadow calculation or the monthly hourly view factors for one day per month,[3] corrected by a 2D interpolation to account for solar-to-clock time conversion and days between months, in simplified radiosity algorithm.[4] These approaches lead to a reduction in computing time of the order of 20 or 30 times, respectively, as in the approach discussed in this work. It should be pointed out that increasing computing capacity, using larger clusters with parallel computing and/or GPU computing, will open opportunities for, until now, untreatable problems such as application of solar radiation models to larger metropolitan areas, subhourly analysis, and real-time PV generation calculations (using satellite solar radiation estimation) for energy balance in renewable energy communities or its integration into urban energy system, thermal comfort, or urban mobility models.[5] These demanding applications, even if running on high-performance computers, can benefit from simplified methods such as those described in this work.

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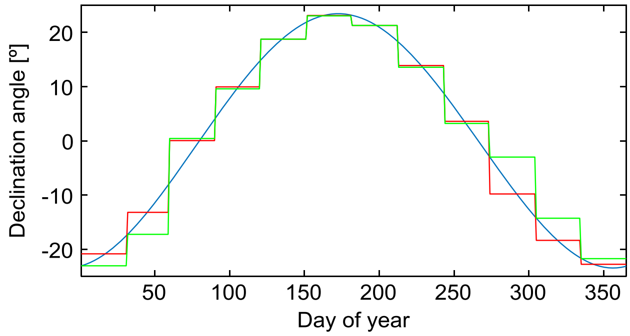

Figure 1. Declination angle for all days of the year: daily value (blue line), characteristic declination for the equator (red line), and characteristic declination for the pole (green line).

2. Characteristic Declination and Irradiation on Surfaces

The characteristic declination will, therefore, depend slightly on the latitude of the location. Supplementary material A presents the characteristic days for each month and latitude. It should be noted that for the two months of the solstice, June and December, there are two values for the characteristic day, 20 days apart.

Figure 1 shows the values of declination determined using the daily equation as a reference, and the characteristic declination defined for two extreme latitudes (the equator and pole). One can observe the step-like function, with larger differences at the time of the equinoxes.

The effect of using the characteristic declination on the irradiation on different surfaces is shown in **Figure 2**, again for two different latitudes. It is clear that for midlatitudes this is a reasonable approximation, whereas for latitudes above the Arctic Circle there will be large errors, with an up to 2 weeks' delay/anticipation of the arrival/end of the winter night.

It is interesting to note that the error introduced by the approximation of the characteristic declination is much less relevant for optimal tilted surfaces than for horizontal or vertical surfaces. This is because the monthly irradiation on an optimally tilted surface is much more uniform across the year, and therefore less dependent on the declination. This effect can be observed in **Figure 3** showing the normalized root mean square error (NRMSE) in annual extraterrestrial irradiation for different surfaces. It shows that the characteristic declination is a suitable method for tilted surfaces for latitudes below 50°. For vertical surfaces, errors can reach 10% when irradiation is lower (at lower latitudes, in the summer months). For high latitudes, results feature high deviations regardless of the surface tilt. The monthly values of NRMSE for the three tilts for all latitudes are presented for reference in Supplementary Material B.

It should be underlined that all these results refer to extraterrestrial irradiation, thus neglecting the effect of the atmosphere. In more realistic applications, for ground-level irradiance one has to take into account the absorption and scattering effects introduced by the atmosphere, which are usually described by considering a direct and a diffuse component of irradiation. On an

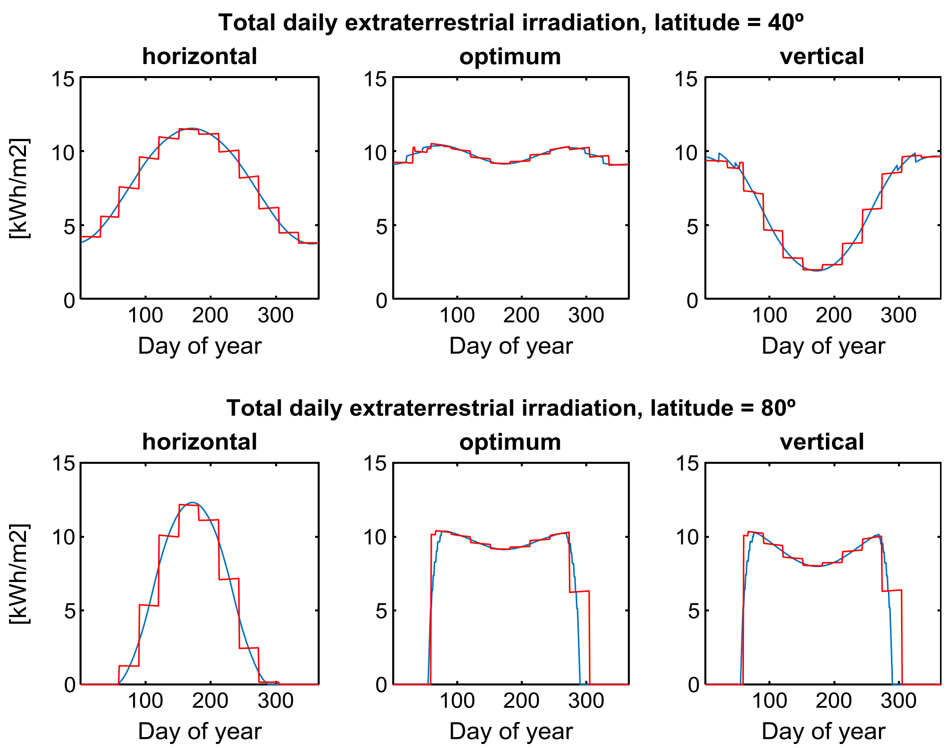

Figure 2. Extraterrestrial daily irradiation for surfaces on the horizontal (left), with optimal inclination (middle), and vertical (right) for mid latitude (40°, top line) and high latitude (80°, bottom line). The blue and red lines show irradiation calculated using the daily declination and the characteristic declination, respectively.

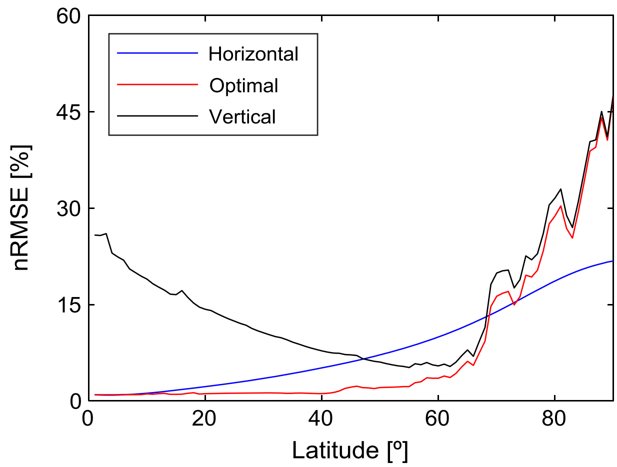

Figure 3. Normalized error (NRMSE) due to the assumption of characteristic declination for extraterrestrial irradiation for three different tilts as a function of latitude.

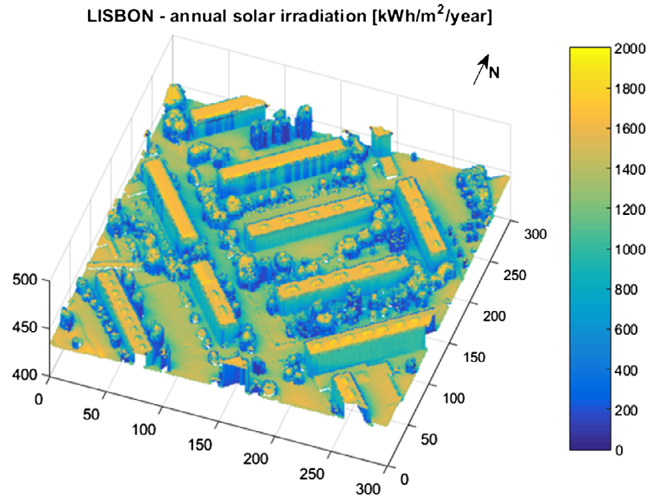

Figure 4. Total annual solar irradiation on rooftops and facades in the DSM, for the Lisbon case study.

overcast day, irradiation is mostly diffuse and, therefore, at least in a first approach, isotropic. Thus, in those conditions, the incidence angle is not relevant and therefore the previous discussion of the effect of the characteristic declination, which is only applicable to the direct component of the irradiation, ought to be considered an upper limit.

3. Application to City Models

Assessing the solar potential in urban areas is a relevant tool for public policy, for example, as a means to identify priority locations for the deployment of photovoltaics.[7] It is a complex and computing-intensive exercise as one needs to calculate the irradiance incident on all available areas on rooftops and facades of all buildings in the urban area while considering the mutual shadow cast by neighboring buildings and trees.[8]

The application of the concept of the characteristic declination to solar city models has the potential to significantly reduce the computation demand by a factor of 30 as it would allow estimating the incoming irradiation for a whole month by calculating its hourly values for the characteristic day only. Its impact on model accuracy is expected to be lower than the estimation for extraterrestrial irradiation discussed in the previous section due to the fraction of isotropic diffuse irradiation.

To test the applicability of the characteristic declination to the urban context, we have run the solar city model SOL for different urban areas with a wide range of latitudes:[8] Nairobi (1.3° S), Mumbai (19.1° N), Lisbon (38.7° N), Geneva (42.2° N), and Oslo (59.9° N). For each city, hourly irradiation data for a full year were taken from Meteonorm software (Meteotest). To identify the impact of the latitude on the error introduced by the characteristic declination approximation, the digital surface model (DSM) of the urban area was assumed constant for all modeled areas (**Figure 4**) as if the buildings' heights and layouts were exactly the same in all cities. The DSM, with a 1 m^2 resolution, has an area of 300 × 300 m^2, where 20% of the ground is occupied by buildings with an average height of 18 m and a maximum of 34 m. Building surfaces' slopes are predominantly vertical and horizontal, whereas the azimuths are mostly southeast, southwest, northwest, and northeast.

Two approaches were tested: one that considers the monthly characteristic declination and another, more extreme, that calculates the hourly irradiation for only 4 days in the year (two solstice and equinox days). This approximation reduces the computation time by an additional factor of 3. For the simulations, the hourly solar irradiation for the characteristic days was considered to be the average solar irradiation during the corresponding month.

The effect of the characteristic declination on the estimation of hourly irradiation on rooftops and facades for one of the modeled areas is shown in **Figure 5**. One can observe that weather variability and the intricate shadows cast in this complex environment lead to daily variations of the total irradiation. Using the characteristic declination, one only has 12 values for the full year, which seemingly follow the seasonal changes.

The results are summarized in **Table 1** and plotted in **Figure 6**. One can observe that the normalized errors range from 5% to 8%, well below the NRMSE for the extraterrestrial irradiation

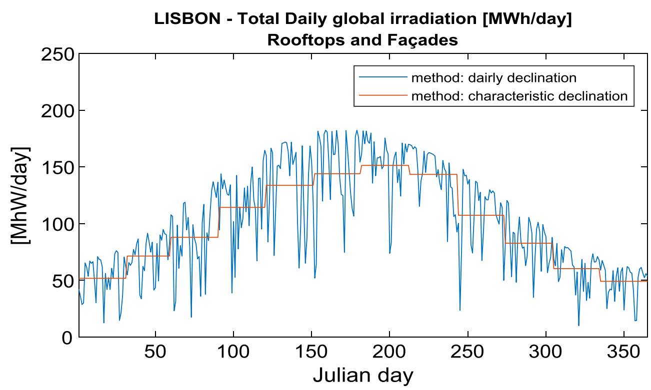

Figure 5. Total daily solar irradiation on rooftops and facades using the hourly declination (blue line) and characteristic declination (orange line) methods. Case study for Lisbon.

Table 1. Normalized errors (NRMSE) for total annual irradiation for all case studies considering the characteristic declination approximation (12 days) and the solstice/equinox approximation (4 days).

City		Nairobi	Mumbai	Lisbon	Geneva	Oslo
Latitude		1.3° S	19.1° N	38.7° N	46.2° N	59.9° N
Diffuse fraction		0.45	0.49	0.38	0.49	0.51
Rooftops	NRMSE (12 days)	7.25	5	5.7	8.04	9.99
	NRMSE (4 days)	8.01	6.31	6.8	10.55	16.29
Facades	NRMSE (12 days)	4.12	6.33	3.51	8.37	6.01
	NRMSE (4 days)	4.82	6.66	3.91	10.8	12.8
Rooftops and facades	NRMSE (12 days)	5.69	5.3	4.87	7.79	7.99
	NRMSE (4 days)	6.48	6.26	5.78	10.35	14.63

presented in Section 3. This is due to the high level of diffuse irradiation because other buildings obstruct direct irradiation, which is not dependent on the position of the Sun in the sky and therefore does not change with the declination. This effect is particularly significant for facades, justifying the lower NRMSE values achieved for facades, compared to rooftops when using the declination approximation (cf. Table 1).

Because a complex 3D urban structure features surfaces with a wide diversity of tilts and orientations, the overall error is expected to (qualitatively) follow an average of the three tilts shown in Figure 3. This is precisely what one can observe in Figure 6, with higher errors for higher latitudes (Geneva and Oslo) and lower errors for midlatitudes (Mumbai and Lisbon) and low latitudes (Nairobi). It is also noteworthy to observe that, except for higher latitudes, the seasonal approach (4 days to represent the year) does not considerably increase the error (15–20% relative error).

Figure 7 shows the monthly NRMSE for total solar irradiation on rooftops and facades for all case studies. One can observe that the monthly errors are mostly within the range of 1% to 4%, slightly higher for the high-latitude locations during the winter months as it would be expected, due to lower solar elevation.

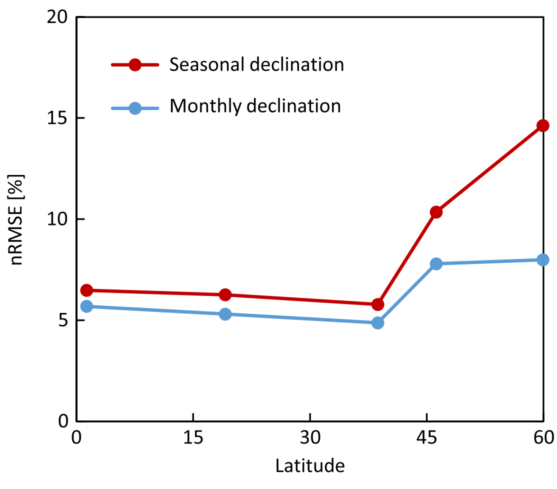

Figure 6. Annual normalized errors (NRMSE) for total solar irradiation on rooftops and facades on all case studies as a function of latitude, considering the monthly characteristic declination (blue line) and the seasonal declination (red line).

It may also be noted that if one uses the hourly solar irradiation for the characteristic days, instead of using the monthly averages, results will be affected by the natural variability of solar irradiation, leading to errors ≈20% (relative) higher, in particular for higher latitudes. These are shown in Supplementary Material C.

4. Discussion

The uncertainty of the estimation of the solar energy yield introduced by the characteristic declination ought to be compared with the other relevant uncertainty sources. The uncertainty in the estimation of urban solar potential is constrained by three major factors: the uncertainty on the solar irradiation modeling, due to the separation model (identifying the diffuse irradiance component of the global irradiance) and the transposition model (from global horizontal irradiation to irradiation on a tilted surface); the level of detail (LOD) of the buildings and the accuracy of the DSM; and the PV system energy yield model, which computes the PV power from the incident irradiance.

For Golden, Colorado, Gueymard (2009) estimated an NRMSE of 0.7% to 1.2% for a 40° tilted surface and 1.8% to 3.1% for a vertical surface oriented toward the south, depending on the transposition model considered.[9] These errors increase to 1.4–1.9% and 3.5–4.7% if the diffuse irradiation is not measured but estimated using standard separation models.

The LOD in 3D city models defined by vector polygons is also a relevant source of error.[10] For example, Strzalka et al. found a maximum difference in the heating demand from an LOD1 to LOD3 model of ≈12%, and Peronato et al. observed that using LOD1 or LOD2 can lead to significant overestimation or underestimation of the solar potential by neglecting shadow-casting features on rooftops or overhangs, respectively.[11] Biljecki et al. estimated a 14% error in the assessment of solar potential using LOD1, and 3% for LOD2, highlighting that positional errors may reach comparable impacts on solar estimation.[12] A related concept, for raster 2.5D models such as the one discussed in Section 4, is the spatial granularity. For a

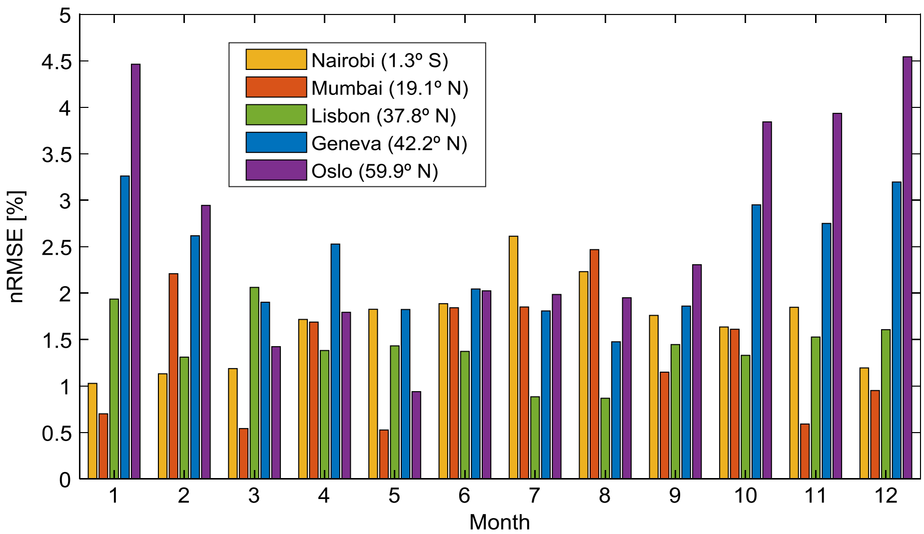

Figure 7. Monthly NRMSE for total solar irradiation on rooftops and facades in all case studies.

neighborhood in Geneva, Switzerland, Peronato et al. (2018) have explored grids with spacings 1 to 4 m, showing that the error increases linearly with spacing, with maximum errors of the order 7%.[13]

The third factor of uncertainty is associated with the conversion of solar irradiance to PV power. The common approach for 3D solar potential assessment models is to consider a temperature-corrected single-point efficiency model, assuming that the PV power is proportional to the incoming irradiance on the surface, considering an effective performance ratio (PV system conversion efficiency including the effect of temperature). Makrides et al. published a benchmark analysis that identified typical errors of the order 5–6% for PV modules of various technologies and manufacturers. They also point out that using the one-diode model to estimate energy yield, a well-established yet seldom used approach for solar city models, would halve this uncertainty.[14] One should, however, note that the solar potential in complex urban settings is prone to be strongly affected by partial shading events, which can be very detrimental to PV generation and therefore this 5–6% error ought to be considered an underestimation of the uncertainty introduced by PV energy yield models.

Other factors that introduce uncertainty in solar modeling are the effect of vegetation and the modeling of reflected solar irradiation, whose impact on the modeling accuracy is particularly important when assessing the solar potential of facades. For instance, Fogl and Moundry estimated that the fraction of annual solar irradiation lost to tree shading was between 3% and 11% for solar potential in five different European cities,[15] whereas Peronato estimated about 20% loss when considering the solar potential of facades in Neuchatel, in Switzerland.[16] For a greener city, or at higher latitude, the effect of shadowing due to trees is expected to increase. Modeling reflected irradiation using a new method based on an inverted DSM, Revesz et al. were able to reduce the RMSE by 50% when accurately modeling reflected irradiance on a vertical wall, in Vienna, Austria.[17] The relevance of reflected irradiance increases with higher latitudes and brighter (higher albedo) environments.

Considering all these sources of errors, one can estimate a combined uncertainty of ≈10–25% when estimating the PV yield of complex urban environments, depending on the available data. The upper limit is in line with uncertainty estimates for the tested 3D solar potential model in an urban area in Lisbon, as shown in Brito et al.[8] In this context, the relatively small error introduced by considering the monthly characteristic declination, with a significant saving of computational effort, can thus be considered a valuable compromise, especially at low and midlatitudes.

5. Conclusion

The concept of characteristic declination, for which the daily extraterrestrial irradiation on a horizontal plane is identical to its monthly mean, is revisited as an approximation to reduce computing time in complex urban solar models. The effect of this approximation on total extraterrestrial irradiation is determined for various surfaces and latitudes.

Its effect on solar potential estimation in complex urban areas is tested for different case studies, at different latitudes and different climates. Results show that the approximation leads to an annual NRMSE in the range of 5–8%, higher for high latitudes.

Comparing with other relevant uncertainties in the solar mapping of complex 3D urban structures, such as the solar radiation model, the description of the buildings, and the PV energy yield model, it is shown that the error introduced by the use of the characteristic declination is relatively small and one may, therefore, assume this simplification in most applications while reducing the computing effort by a factor of 30.

It was also shown that an even bolder assumption, assuming a seasonal characteristic declination, penalizes the accuracy of the calculations by ≈15–20% (relative) for lower and midlatitudes, with an additional reduction in computing time by a factor of 3.

6. Experimental Section

To assess the effect of the declination on the direct irradiance on a surface, one ought to notice that the direct irradiance $I_{\gamma\beta}$ on a surface with orientation γ and inclination β is given by Equation (1)

$$I_{\gamma\beta} = I_{\text{d}} \cos\theta \tag{1}$$

where I_{d} is the direct irradiance on the normal plane and θ is the incidence angle, which is given by Equation (2)

$$\begin{aligned}\cos\theta = &(\sin\phi\cos\beta - \cos\phi\sin\beta\cos\gamma)\sin\delta \\ &+ (\cos\phi\cos\beta + \sin\phi\sin\beta\cos\gamma)\cos\delta\cos\omega \\ &+ \cos\delta\sin\beta\sin\gamma\sin\omega\end{aligned} \tag{2}$$

Here, ϕ is the local latitude, ω is the hour angle, and δ is the declination. The latter, corresponding to the angle between a line joining the centers of the Sun and the Earth to the equatorial plane, was calculated using the SG2 function.[6] This fast and accurate solution simplifies a rather complex formulation using approximations with truncated Fourier series for a restricted time coverage ranging from 1980 to 2030, as described in Equation (3)

$$\delta \approx \delta_{\text{SG2}} = \delta^{\text{g}} + (x\cos\omega^{\text{g}}\sin\delta^{\text{g}} - y\cos\delta^{\text{g}})\xi \tag{3}$$

where δ^{g} is the Sun's geocentric declination (in radians), ω^{g} the geocentric hour angle (in radians), x and y two intermediate parameters and ξ the equatorial horizontal parallax of the Sun.

Supporting Information

Supporting Information is available from the Wiley Online Library or from the author.

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Conflict of Interest

The authors declare no conflict of interest.

Data Availability Statement

Research data are not shared.

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