

## UNIVERSO PRIMITIVO

Mestrado em Física Astronomia 2021-2022

### Exercise Sheet 2

1. Convert kilogram (kg), second (s), meter (m) and kelvin (K) to giga-electron volt (GeV), assuming natural units  $c = \hbar = k_B = 1$ . Use your findings to express your weight, age, height and body temperature in GeV.
2. Derive expressions for the number density, energy density and pressure of a gas of ultra-relativistic particles in thermal equilibrium with vanishing chemical potential.
3. Show that the energy density and pressure of non-relativistic particles with vanishing chemical potential is given by  $\rho = (m + \frac{3}{2}T)n$  and  $P = nT$ , respectively. Explain why in these conditions one has  $P \ll \rho$ .
4. Consider a thermal equilibrium distribution of relativistic particles with non-vanishing chemical potential  $\mu$ . Compute the number density, energy and pressure for:
  - 4.1. Degenerate fermions with  $\mu \gg T$ ;
  - 4.2.  $\mu < 0$  and  $|\mu| < T$[Hint: in 4.1 assume that for degenerate fermions all energy states are occupied up to a maximum energy equal to  $\mu$ .]
5. Consider now the case of the non-relativistic limit, with a non-vanishing chemical potential. Prove the expressions below. The overbar denotes densities for anti-particles. Assume that particles and anti-particles are in chemical equilibrium. (Regarding 5.2 and 5.3: note that in general there can be excess of particles over antiparticles):

$$5.1. \quad n = g \left( \frac{mT}{2\pi} \right)^{3/2} \exp\left(-\frac{m-\mu}{T}\right)$$

$$5.2. \quad n - \bar{n} = 2g \left( \frac{mT}{2\pi} \right)^{3/2} \exp\left(-\frac{m}{T}\right) \sinh\left(\frac{\mu}{T}\right)$$

$$5.3. \quad \rho + \bar{\rho} = 2gm \left( \frac{mT}{2\pi} \right)^{3/2} \exp\left(-\frac{m}{T}\right) \cosh\left(\frac{\mu}{T}\right)$$