# WIMP: Weakly Interactive Massive Particles

Let us assume a Dark matter candidate WIMP that is in equilibrium with the fluid through this interaction:

$$X + \bar{X} \rightleftharpoons \ell + \bar{\ell}$$

$X$ → The candidate particle

$\ell, \bar{\ell}$ → leptons

Where $\ell$ and $\bar{\ell}$ are essencially massless and tightly bound to the fluid (from the beggining to the end of decoupling of particle X)

- $n_\ell = n_\ell^{eq}$
- $n_{\bar{\ell}} = n_{\bar{\ell}}^{eq}$

Let's also assume that there is no initial assymetry between X and $\bar{X}$ (same amount of particles and antiparticles)

- $n_X = n_{\bar{X}}$

So the Boltzmann equation ($1+2 \rightleftharpoons 3+4$):

$$\frac{1}{a^3}\frac{d(n_1 a^3)}{dt} = -\langle \sigma v \rangle \left[ n_1 n_2 - \left( \frac{n_1 n_2}{n_3 n_4} \right)_{eq} n_3 n_4 \right] =$$

($1 \to X$, $2 \to \bar{X}$, $3 \to \ell$, $4 \to \bar{\ell}$)

$$= -\langle \sigma v \rangle \left[ n_X n_{\bar{X}} - \left( \frac{n_X n_{\bar{X}}}{n_\ell n_{\bar{\ell}}} \right)_{eq} n_\ell n_{\bar{\ell}} \right] =$$

$$= -\langle \sigma v \rangle \left[ n_X^2 - \left( n_X^{eq} \right)^2 \right]$$

We can convert densities into particle numbers:

$$N_X = \frac{n_X}{s} \qquad \text{where} \qquad (s a^3 = S)$$

$$\frac{1}{a^3}\frac{d}{dt}\left[ N_X s a^3 \right] = -\langle \sigma v \rangle \left[ N_X^2 s^2 - \left( N_X^{eq} \right)^2 s^2 \right] \Leftrightarrow$$

→ $S$ = constant

$$\Leftrightarrow \frac{1}{a^3}(s a^3)\frac{d}{dt} N_X = -\langle \sigma v \rangle \left[ N_X^2 - \left( N_X^{eq} \right)^2 \right] s^2 \Leftrightarrow$$

$$\Leftrightarrow \frac{dN_X}{dt} = -\langle \sigma v \rangle s \left[ N_X^2 - \left( N_X^{eq} \right)^2 \right]$$

So let us make a change of variable:

$$t \to \frac{M_X}{T} = x$$

$$N(x(t)) \longrightarrow \frac{dN}{dt} = \frac{dN}{dx}\frac{dx}{dt}$$

So: $\frac{dx}{dt} = \frac{d}{dt}\left(\frac{M_X}{T}\right) = \frac{d}{dt}\left(M_X T^{-1}\right) = M_X \frac{d}{dt}\left(T^{-1}\right)$.

From conservation of entropy: $\boxed{T \propto g_{*s}^{-1/3}(T)\, a^{-1}}$

$$T = A\, g_{*s}^{-1/3}(T)\, a^{-1}$$

$$\frac{dx}{dt} = -\frac{M_X}{T}\frac{1}{T}\frac{dT}{dt} = -\frac{x}{T}\frac{dT}{dt}$$

$$\frac{dx}{dt} = -x\frac{1}{T}\frac{dT}{dt} = -x\frac{1}{\cancel{A}\, g_{*s}^{-1/3}(T)\, a^{-1}}\frac{d}{dt}\left[\cancel{A}\, g_{*s}^{-1/3}\, a^{-1}\right] =$$

$$= -x\frac{1}{g_{*s}^{-1/3}(T)\, a^{-1}}\frac{d}{dt}\left[g_{*s}^{-1/3}(T)\, a^{-1}\right]$$

aproximating to step function, $g_*^{-1/3}(T)$ ~~$g_{*s}$~~ is constant

$$\frac{dx}{dt} \simeq -x\frac{1}{\cancel{g_{*s}^{-1/3}}\, a^{-1}}\cancel{g_{*s}^{-1/3}}\frac{d}{dt}(a^{-1})$$

$$\frac{dN_X}{dt} = \frac{dN_X}{dx}\frac{dT}{dt} \simeq \frac{dN_X}{dx}\left(-x a\left(-\dot{a}\, a^{-2}\right)\right) =$$

$$= \frac{dN_X}{dx}\left(x\frac{\dot{a}}{a}\right) = \frac{dN_X}{dx}\, x H$$

Then the Boltzmann equation reads:

$$\frac{dN_X}{dx}\, x H = -\langle \sigma v\rangle\, s\left[N_X^2 - (N_X^{eq})^2\right] \Leftrightarrow$$

$$\Leftrightarrow \frac{dN_X}{dx} = -\frac{\langle \sigma v\rangle}{x}\frac{s}{H}\left[N_X^2 - \left(N_{X}^{eq}\right)^2\right]$$

Let us assume that the decoupling of X is during the radiation phase domination:

$$H = \frac{\pi}{3}\left(\frac{g_*}{10}\right)^{1/2}\frac{T^2}{M_p}$$

Since $x = \frac{M_x}{T} \Leftrightarrow T = \frac{M_x}{x}$

$$H = \underbrace{\frac{\pi}{3}\left(\frac{g_*}{10}\right)^{1/2}\left(\frac{M_x}{x}\right)^2\frac{1}{M_{PL}}}_{H(T\sim M_x)} = H(T\sim M_x)\cdot\frac{1}{x^2}$$

$\hookrightarrow$ since we compute in step function

Now for the specific entropy: $s = \frac{2\pi^2}{45} g_{*s} T^3$

$$s = \frac{2\pi^2}{45} g_{*s}\left(\frac{M_x}{x}\right)^3 = \frac{2\pi^2}{45} g_{*s}(T\sim M_x)\left(\frac{M_x}{x}\right)^3$$

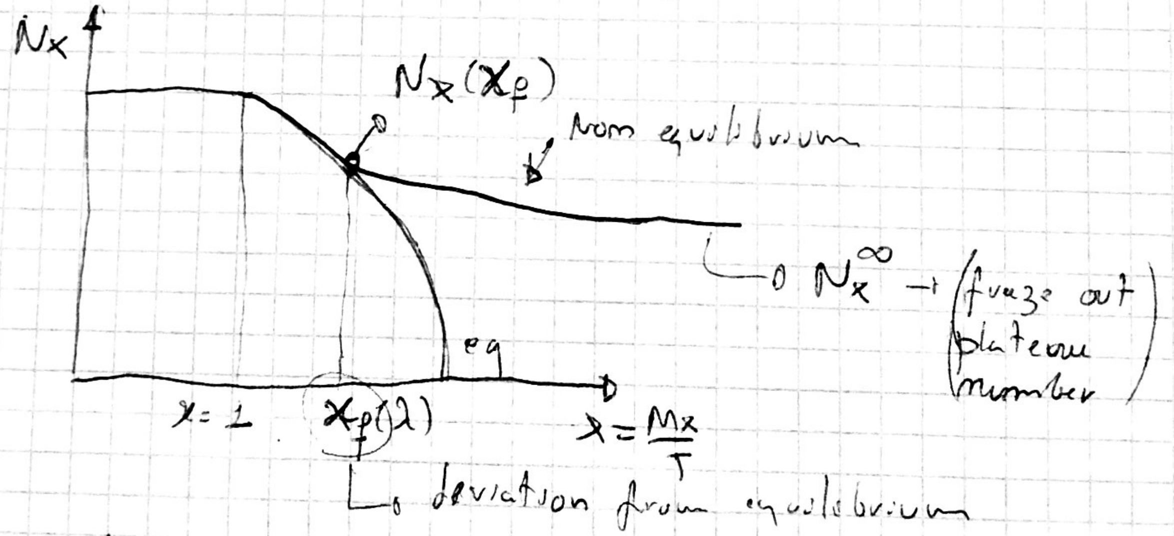
$$\frac{dN_x}{dx} = -\frac{\langle\sigma v\rangle}{x}\,\frac{\frac{2\pi^2}{45} g_{*s}(T\sim M_x)\frac{M_x^3}{x^3}}{H(T\sim M_x)\frac{1}{x^2}}\left[N_x^2 - (N_x^{eq})^2\right] =$$

$$= -\underbrace{\frac{2\pi^2}{45}\,\frac{g_{*s}(T\sim M_x)}{H(T\sim M_x)}\langle\sigma v\rangle\, M_x^3}_{\lambda}\frac{1}{x^2}\left[N_x^2 - (N_x^{eq})^2\right]$$

$\lambda \rightarrow$ we can treat it like a constant for each branch of step function where $g_{*s}$ varies

$$\boxed{\frac{dN_x}{dx} = -\frac{\lambda}{x^2}\left[N_x^2 - (N_x^{eq})^2\right]} \rightarrow \text{Riccati Equation}$$

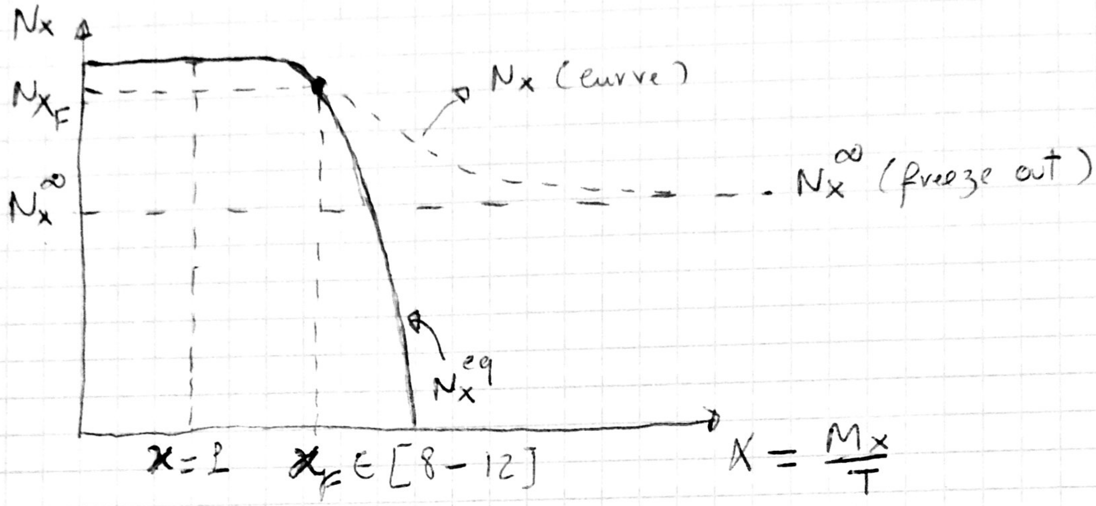
$$\lambda = \frac{2\pi^2}{45}\,\frac{g_{*s}(M_x)\,M_x^3\langle\sigma v\rangle}{\frac{\pi}{3}\left(\frac{g_*(M_x)}{10}\right)^{1/2}\frac{M_x^2}{M_{PL}}} = \frac{2\pi}{15}\left(\frac{10\,g_{*s}^2(M_x)}{g_*(M_x)}\right)^{1/2}\langle\sigma v\rangle M_x M_{PL}$$




- The Riccati equation has no analytical solution, it needs to be solved numerically
- $N_x^\infty < N_x(x_f)$ by a factor of at least 10
- $N_x$ and $N_x^\infty \gg N_x^{eq}$

19ª Aula 09/12/2021

WIMP: As an explamation for D.M
($\Omega_{DM} \simeq 0,27$) $\rightarrow$ what we want to find!




Things we know: $N_x^\infty >> N_x^{eq}$

$N_{x_F} > N_x^\infty \sim 10$

We want to solve the Ricatti equation for $x >> x_F \Rightarrow N_x >> N_x^{eq}$.

$$\frac{dN_x}{dx} = -\frac{\lambda}{x^2}\left[N_x^2 - (N_x^{eq})^2\right] \simeq -\frac{\lambda}{x^2} N_x^2$$

$$dN_x = \frac{dN_x}{dx}\,dx = -\frac{\lambda}{x^2} N_x^2\,dx \Leftrightarrow \frac{dN_x}{N_x^2} = -\lambda\frac{dx}{x^2}$$

Integrating:

$$\int_{N_{x_F}}^{N_x^\infty} N_x^{-2}\,dN_x = -\lambda\int_{x_F}^{\infty} x^{-2}\,dx \Leftrightarrow$$

$$\Leftrightarrow \left[\frac{N_x^{-1}}{-1}\right]_{N_{x_F}}^{N_x^\infty} = \lambda\left[\frac{x^{-1}}{-1}\right]_{x_F}^{\infty} \Leftrightarrow$$

$$\Leftrightarrow \frac{1}{N_x^\infty} - \frac{1}{N_{x_F}} = -\lambda\left[0 - \frac{1}{x_F}\right]$$

So we got: $$\boxed{\frac{1}{N_x^\infty} - \frac{1}{N_x(x_F)} = + \frac{\lambda}{x_F}}$$

We can aproximate the result by:

$$\frac{1}{N_x^\infty} = \frac{\lambda}{x_F} \Leftrightarrow N_x^\infty = \frac{x_F}{\lambda}$$

WIMP Miracle:

- Let us relate the ~~...~~ freeze-out abundance of DM particles to the density of today $\Omega_{DM}$

$$\Omega_{x,0} = \frac{\rho_{x,0}}{\rho_{crit,0}}$$

$$\rho_{crit,0} = 3H_0^2/8\pi G$$

$$M_{Pl} = \sqrt{\frac{\hbar c}{8\pi G}} \Leftrightarrow \sqrt{\frac{1}{8\pi G}}$$

$$8\pi G = \frac{1}{M_{Pl}^2}$$

$$\rho_{crit} = 3H_0^2 M_{Pl}^2$$

$$\Omega_{x,0} = \frac{\overbrace{n_{x,0} M_x}^{\rho_{x,0}}}{3H_0^2 M_{Pl}^2} = \frac{M_x (N_{x,0}\, s_0)}{3H_0^2 M_{Pl}^2} = \frac{M_x}{3M_{Pl}^2} N_x^\infty \frac{s_0}{H_0^2}$$

since: $N_x = \frac{n_x}{s}$

$$\boxed{\Omega_{x,0} = M_x \left(\frac{x_F}{\lambda}\right) \frac{s_0}{3M_{Pl}^2 H_0^2}}$$

So using: $$\lambda = \frac{2\pi^2}{45} g_{*s}(T \sim M_x) \frac{M_x^3 \langle \sigma v \rangle}{H(T \sim M_x)}$$

we got this from last lecture

$s_0 = \frac{2\pi^2}{45} g_{*s}(T_0)\, T_0^3$, so we get:

$$\Omega_{x,0} = M_x x_F \left( \frac{45}{2\pi^2} \frac{H(T\sim M_x)}{g_{*s}(M_x) M_x^3 \langle \sigma v \rangle} \right) \cdot \frac{\frac{2\pi^2}{45} g_{*s}(T_0)\, T_0^3}{3M_{Pl}^2 H_0^2}$$

But we saw last lecture:

$$H(T\sim M_x) = \frac{\pi}{3}\left(\frac{g_*(T\sim M_x)}{10}\right)^{1/2}\frac{M_x^2}{M_{Pl}}$$

Where: $H(x) = H(T\sim M_x)\frac{1}{x^2}$

So using this:

$$\Omega_{x_0} = \frac{\frac{\pi}{3}\left(\frac{g_*(M_x)}{10}\right)^{1/2}\frac{M_x^2}{M_{Pl}}}{M_x^2}\,\frac{x_F}{\langle\sigma v\rangle}\,\frac{g_{*s}(T_0)}{g_{*s}(M_x)}\,\frac{T_0^3}{3M_{Pl}^2H_0^2}$$

$$\Omega_{x_0} = \frac{\pi}{3}\frac{x_F}{\langle\sigma v\rangle}\left(\frac{g_*(M_x)}{10}\right)^{1/2}\frac{g_{*s}(T_0)}{g_{*s}(M_x)}\,\frac{T_0^3}{M_{Pl}^3H_0^2}$$

From observations:

$H_0 = 100\ h\ \text{km s}^{-1}\text{Mpc}^{-1}$ $\quad h \in [0{,}65; 0{,}7]$

$\Omega_{DM} = 0{,}27$ $\quad\rightarrow$ converting to energy

$T_0 = T_{CMB} = 2{,}7259\ K$

$g_{*s}(T_0) \simeq 3{,}91$

So:

$$\boxed{\underbrace{\Omega_{x,0}h^2}_{0{,}1225} \simeq 0{,}1\left(\frac{\overset{\sim 10}{x_f}}{10}\right)\left(\frac{10}{g_*(M_x)}\right)^{1/2}\frac{10^{-8}}{\langle\sigma v\rangle}\ \text{GeV}^{-2}}$$

If $\langle\sigma v\rangle$ is mediated by weak interaction
then $\langle\sigma v\rangle \sim 10^{-8}\ \text{GeV}^{-2}$

So everything depends on $g_*(M_x)$