METFOG **2023/24**

Final Exam - 1^a Fase **12/06/2024** 9h-12h

1. [6.0 val] Consider the following Lagrangian, where X_μ is a real spin one field, S is a complex spin zero field and $X_{\mu\nu}$ is the field strength tensor for X_μ,

$$\mathcal{L} = \mathcal{L}_{SM} - \frac{1}{4} X_{\mu\nu} X^{\mu\nu} + (D_\mu S)^\dagger (D^\mu S) - \mu_S^2 |S|^2 - \lambda_S |S|^4 - \kappa |S|^2 H^\dagger H \quad (1)$$

where $\mathcal{L}_{SM}$ is the Standard Model Lagrangian and $D_\mu = \partial_\mu + i g_X X_\mu$ and the fields can be written in the unitary gauge as

$$H = \begin{pmatrix} 0 \\ \frac{1}{\sqrt{2}}(v_H + h) \end{pmatrix} \qquad S = \frac{1}{\sqrt{2}}(v_s + s + ia) \quad (2)$$

a) [1.0 val] Show the Lagrangian is invariant under the symmetry $X_\mu \to -X_\mu$, $S \to S^*$.

b) [2.0 val] Expanding the covariant derivative, find the Feynman rules involving X_μ and the scalar fields s and a and show that X_μ is a good dark matter candidate, if a and X_μ have the same dark quantum number.

c) [1.5 val] The field s mixes with the doublet scalar h, such that there are couplings of the form $X^2 h_1$, $X^2 h_2$, $X^2 h_1^2$ and $X^2 h_2^2$. h_1 and h_2 are the mass eigenstates and couple exactly like the SM Higgs to the fermions (modulo constant factors). Draw the Feynman diagrams for the production of DM in the process $q\bar{q} \to X_\mu X_\nu$ at tree-level. What would one see at a collider with such a final state?

d) [1.5 val] Draw the Feynman diagrams for the production of DM with a jet in the process $q\bar{q} \to g X_\mu X_\nu$ at tree-level, where g is a gluon.What is the signal seen at a collider?

2. [7.5 val] Consider the Standard Model extended by a complex singlet S and with a portal Lagrangian given by

$$\mathcal{L}_{portal} = \lambda_D S^* S H^\dagger H$$

where $H^\dagger = (0 \quad (v+h)/\sqrt{2})$ is the Higgs doublet in the unitary gauge and the complex singlet is written as $S = \frac{1}{\sqrt{2}}(s + ia)$. Consider that a is the lightest dark scalar and therefore the DM particle.

a) [1.0 val] Write the Feynman rules for the vertices ssh, $sshh$, aah and $aahh$.

b) [2.0 val] Draw the Feynman diagram and calculate the amplitude for the scattering $aq \to aq$ where q is a generic quark. Extract the corresponding Wilson coefficient. (The Feynman rule for $\bar{q}qh$ is $-im_q/v$ and $v = 246$ GeV).

c) [2.5 val] Using the leading order form factor, and assuming zero exchange momentum, calculate the direct detection cross section (choose a DM mass of 100 GeV and $\lambda_D = 0.1$). Consider that DM collides with a proton and that $m_p = 1$ GeV.

d) [0.5 val] Is this point in parameter space excluded by the Lux-Zeplin experiment?

e) [1.5 val] Suppose now that the scalar acquires a vacuum expectation value such that $S = \frac{1}{\sqrt{2}}(v_s + s + iv_a + ia)$. Is there still a dark matter candidate? And what if $v_a = 0$? And what if $v_s = 0$?

3. [6.5 val] Consider again the Lagrangian in equation (1) and focus only on the term

$$\mathcal{L} = (D_\mu S)^\dagger (D^\mu S)$$

with $D_\mu = \partial_\mu + ig_X X_\mu$ and $S = \frac{1}{\sqrt{2}}(v_s + s + ia)$. As discussed in problem 1, s and h mix and give rise to two mass eigenstates defined as h_1 and h_2. For this problem we consider that only h_1 exists. The interaction term between h_1 and dark matter is $g_X^2 v_s X_\mu X^\mu h_1 \cos\theta$. The corresponding Feynman rule is $ig_X^2 v_s \cos\theta g^{\mu\nu}$.

a) [2.5 val] Draw the Feynman diagrams and calculate the thermal averaged cross section for the scattering $XX \to W^+W^-$ (consider only the diagram with the exchange of h_1, which has a coupling to W bosons of the form $igm_W \sin\theta g^{\mu\nu}$). Use the following approximations: $< \sigma v >_{XX\to WW} = v\sigma_{XX\to WW}$ with $v = 2\sqrt{\frac{s}{4m_X^2} - 1}$ and after that also $\sqrt{s} = 2m_X$ and neglect the h_1 width. Remember the cross section can be written as

$$\sigma(XX \to WW) = \frac{1}{16\pi s}\sqrt{\frac{s - 4m_W^2}{s - 4m_X^2}}|T|^2,$$

where T is the amplitude.

b) [2.5 val] Using the formulae below, calculate the value of g_X that reproduces the correct value for the relic density for a DM mass of 1 TeV. Take $\sin\theta = 0.9$, $v_S = 100$ GeV, $m_W = 80.37$ GeV, $m_{h_1} = 125$ GeV and $g = 0.641$.

$$\Omega_X h^2 = m_X s_0 Y_0 \frac{8\pi G}{3H^2} \approx 2.742 \times 10^8 \frac{m_X}{GeV} Y_0 \qquad (\Omega h^2)_{DM}^{obs} = 0.1186 \pm 0.0020$$

$$Y_0 = 10/\lambda \qquad \lambda = N < \sigma v >_{XX\to WW} \qquad N = 1.6 \times 10^{20}$$

c) [1.5 val] Explain the main differences between the mechanisms of freeze-out and freeze-in.

$$1\,\text{barn} = 0.00257\,\text{MeV}^{-2} = 10^{-24}\,\text{cm}^2$$