METFOG **2023/24**

Final Exam - 2^a Fase **11/07/2024** 9h-12h

1. [**7.0 val**] Consider the Standard Model extended by a complex doublet H_D and with a portal Lagrangian given by

$$\mathcal{L}_{portal} = \lambda_D H_D^\dagger H_D H^\dagger H$$

where $H^\dagger = (0 \quad (v+h)/\sqrt{2})$ is the Higgs doublet in the unitary gauge and the dark doublet is written as $H_D^\dagger = (H_D^+ \quad (h_D - iA_D)/\sqrt{2})$. Consider that h_D is the lightest dark scalar and therefore the DM particle.

a) [**1.0 val**] The dark doublet has isospin 1/2 and hypercharge 1. Write the kinetic term for H_D and write explicitly the corresponding covariant derivative.

b) [**1.0 val**] What are the neutral particles from the dark doublet that can be dark matter candidates? Assume that h_D is the dark matter candidate and write the Feynman rule for the triple coupling $h_D h_D h$.

c) [**3.0 val**] Draw the Feynman diagram and calculate the amplitude for the scattering $h_D q \to h_D q$ where q is a generic quark. Extract the corresponding Wilson coefficient. (The Feynman rule for $\bar{q}qh$ is $-im_q/v$ and $v = 246$ GeV). Write the complete expression for the direct detection cross section.

d) [**0.5 val**] Is this point in parameter space excluded by the Lux-Zeplin experiment?

e) [**1.5 val**] Explain the main differences between direct and indirect detection.

2. [**7.5 val**] Consider the Standard Model extended by a complex singlet S with a Lagrangian given by

$$\mathcal{L} = \mathcal{L}_{SM} + (\partial_\mu S)^\dagger(\partial^\mu S) + \mu_S^2 |S|^2 - \lambda_S |S|^4 - \kappa |S|^2 H^\dagger H + \mu^2(S^2 + S^{*2})$$

with $S = \frac{1}{\sqrt{2}}(v_s + s + iv_a + ia)$ and $H^\dagger = (0 \quad (v+h)/\sqrt{2})$ is the Higgs doublet in the unitary gauge.

a) [**2.5 val**] Discuss the particle spectrum of this extension of the SM. In particular show that there are DM candidates when $v_s = 0$ and when $v_a = 0$, or when they are both equal to zero. Find the Feynman rules for the s DM candidate (in the couplings with the SM Higgs) in the case where $v_s = v_a = 0$.

b) [**2.0 val**] In the scenario $v_s = v_a = 0$, draw the Feynman diagrams and write the amplitude for $ss \to hh$ (note that the SM triple Higgs coupling is $-3im_h^2/v$).

b) [**3.0 val**] In the same scenario calculate the width $\Gamma(h \to ss)$ as a function

κ. Assuming that the total width of the Higgs is 4.6 MeV and that the present measurement on the Higgs invisible branching ratio is 0.11 find a bound for κ for a DM mass of 20 GeV.

3. [**5.5 val**] Consider a temperature dependent potential of the form

$$V(\phi, T) = \lambda(T^2 - T_0^2)\phi^2 - \frac{\lambda}{4}T\phi^3 + \frac{\lambda}{4}\phi^4$$

where T_0 and λ are constants.

a) [**2.0 val**] Calculate the critical temperature and explain your reasoning.

b) [**1.5 val**] Show that this transition is weak independently of the value of λ.

c) [**1.0 val**] Explain the role of the gauge bosons in having a first order phase transition in the SM.

d) [**1.0 val**] What are the variables that characterise the strength of gravitational waves and why?

$$1\,\text{barn} = 0.00257\,\text{MeV}^{-2} = 10^{-24}\,\text{cm}^2$$