Cosmologia Física

Homework 2 due 1 April 2026 (iatereno@fc.ul.pt)

Exercise 1: Redshift drift

1.1) The redshift of a source may vary with time as the universe expands (for example, a comoving galaxy with z=1 today will have a different z tomorrow). The change of redshift, i.e., $\Delta z_s = z_s(t_0 + \Delta t_0) - z_s(t_0)$, is known as the redshift drift. The redshift drift tests cosmology in "real-time", being a direct detection of the expansion. In the above expression, $z_s(t_0)$ is the redshift of a source that emitted at t_s and was observed at t_0, i.e. the usual

$$z_s(t_0) = \frac{a(t_0)}{a(t_s)} - 1$$

and $z_s(t_0+\Delta t_0)$ is the redshift of the same source observed at a later time $t_0+\Delta t_0$ (corresponding to an emission made at $t_s + \Delta t_s$):

$$z_s(t_0 + \Delta t_0) = \frac{a(t_0 + \Delta t_0)}{a(t_s + \Delta t_s)} - 1,$$

a) Derive a necessary condition that the expansion must verify to allow for the existence of the redshift drift effect.

b) Taylor expanding the scale factor to first order in time, both around t_0 and t_s, show that the redshift drift is given by,

$$\frac{\Delta z_s}{1 + z_s} = H_0 \Delta t_0 \left[1 - \frac{H(z_s)}{(1 + z_s)H_0}\right].$$

c) Consider a quasar at $z = 6$ in the concordance model. What fraction of the Hubble time do we need to wait in order to observe a variation of 50% in the quasar redshift?

d) The redshift drift of the quasar can be detected by measuring the absorption lines it produces in the spectrum of its neighbouring gas clouds (so-called Lyman-alpha forest). This spectrum must be observed twice (with a time separation) to detect the shift in the wavelength λ of the lines due to the redshift drift. The result of 'c' shows that the redshift changes very slowly. How long do we need to wait for the redshift drift of the quasar be detectable by the high-resolution ESPRESSO spectrograph, knowing its resolving power is $\lambda/\Delta\lambda = 200000$?

e) In 'c' and 'd' you found that the redshift of a source decreases with time. Derive the condition the scale factor must verify for the redshift drift to be negative.

f) Show that for a power law decelerating universe, the condition derived in 'e' holds at all times (and so the redshift drift is negative for all sources).

g) In the concordance model the redshift of some sources increases with time. Compute the source redshift threshold below which the redshift drift is positive.
Hint: use Friedmann's equation