# Cosmologia Física

**Homework 3** due 9 April 2026 (iatereno@fc.ul.pt)

---

**Exercise 1:** Galaxy clustering

1.1) Consider a galaxy survey that observed a 3-dimensional spherical volume of radius R and found that all the galaxies were lying on the equatorial disk inside the sphere (and not randomly distributed throughout the volume)

a) Show that the correlation function in that volume is

$$\xi(r) = \frac{2}{3}\frac{R}{r} - 1,$$

where $r$ is the radial coordinate, i.e., the 3D separation from the center of the sphere.
Hint: Remember that $1 + \xi(r)$ can be measured as the ratio between the number of galaxies $N_c(r)$ at separation r from the origin in the clustered sample and the number of galaxies $N_u(r)$ at separation r from the origin in a uncorrelated sample.

**Exercise 2:** Two-point functions

2.1) Consider the ΛCDM universe. At early times, right after inflation, the primordial matter density contrast field has small values at all scales, and its power spectrum is given by a power-law: $P(k) \propto k^{\alpha}$.

a) Derive the corresponding correlation function, i.e. its Fourier transform,

$$\xi(r) = \frac{1}{(2\pi)^3}\int P(k)\, e^{-ik.x}\, d^3k\,,$$

showing it is given by

$$\xi(r) \propto r^{-(3+\alpha)}\,.$$

Note: The following integral will be needed in the derivation:

$$\int_0^{\infty} x^{n+1}\sin(x)\,dx = \Gamma[n+2]\,\sin(\pi(n+2)/2)\,.$$

b) The power spectrum of the matter density contrast field today has evolved from the primordial one. Its shape is now a continuous sequence of power-laws $k^{\alpha(k)}$, with $\alpha$ ranging from 1 on the largest scales to $-3$ on the smallest scales. Measurements of the spatial distribution of astrophysical objects at $z \approx 0$ show that the correlation function for a certain type of objects (let us call it type A) is $\xi \propto r^{-1.7}$, while for another type of objects (type B) is $\xi \propto r^{-2.1}$, where

one of the types is galaxies and the other is galaxy clusters. From these measurements and the result of exercise 2.1a, are galaxy clusters the type A or type B objects? Why?

2.2) Consider an alternate universe where the matter correlation function is a Gaussian function centered in $r = 0$ and with variance $\sigma^2$.

a) Compute the power spectrum of this universe.
Note: The following integral will be needed:

$$\int_0^\infty x \sin(x) \exp(-x^2/(2c^2))dx = (\pi/2)^{1/2}c^3 \exp(-c^2/2).$$

b) Does this universe verify the cosmological principle? Why (or why not) ?
Hint: Consider at which scale is the peak of this power spectrum.