Mechanisms of Thermal Dark Matter Production: Freeze-Out in the Real Singlet Scalar Higgs-Portal Model

Rui Santos

FCUL

2026

Outline

Thermal dark matter

Thermal dark matter assumes that the dark matter particle χ was once in thermal and chemical equilibrium with the Standard Model plasma.

At early times,

$$\chi\chi \leftrightarrow \mathrm{SM\ SM}$$

keeps the number density close to its equilibrium value $n_\chi \simeq n_{\chi,\mathrm{eq}}$. As the Universe expands and cools, the annihilation rate becomes inefficient:

$$\Gamma_\chi \equiv n_\chi \langle \sigma v \rangle \lesssim H.$$

Then the comoving abundance freezes out:

$$Y_\infty \equiv \frac{n_\chi}{s} \simeq \mathrm{constant}.$$

Expansion versus interactions

The relevant competition is between the interaction rate

$$\Gamma_\chi = n_\chi \langle \sigma v \rangle$$

and the Hubble expansion rate

$$H = \frac{\dot{a}}{a}.$$

If

$$\Gamma_\chi \gg H,$$

interactions are rapid and

$$n_\chi \simeq n_{\chi,\mathrm{eq}}.$$

If

$$\Gamma_\chi \ll H,$$

interactions are inefficient and

$$n_\chi a^3 \simeq \text{constant}.$$

Freeze-out occurs approximately when

$$\boxed{\Gamma_\chi(T_f) \sim H(T_f)}.$$

Dilution from cosmic expansion

The Hubble parameter is $H(t) = \frac{\dot{a}(t)}{a(t)}$. If particle number is conserved inside a comoving volume,

$$\frac{d}{dt}\left[n(t)a^3(t)\right] = 0.$$

Expanding,

$$a^3\,\dot{n} + 3a^2\,\dot{a}\,n = 0.$$

Dividing by a^3,

$$\boxed{\dot{n} + 3Hn = 0}.$$

The term $3Hn$ describes dilution due to the expansion of the Universe.

Boltzmann equation with annihilation

Including annihilation and inverse production processes,

$$\boxed{\dot{n}_\chi + 3Hn_\chi = -\langle \sigma v \rangle \left(n_\chi^2 - n_{\chi,\mathrm{eq}}^2 \right)}.$$

Here:

- n_χ is the dark matter number density;
- $n_{\chi,\mathrm{eq}}$ is the equilibrium number density;
- $\langle \sigma v \rangle$ is the thermally averaged annihilation cross section.

If $n_\chi > n_{\chi,\mathrm{eq}}$, annihilation dominates.
If $n_\chi < n_{\chi,\mathrm{eq}}$, inverse production dominates.

Thermal average and dimensions

Schematically,

$$\langle \sigma v \rangle = \frac{\int d^3p_1\, d^3p_2\, e^{-(E_1+E_2)/T}\, \sigma v_{\rm rel}}{\int d^3p_1\, d^3p_2\, e^{-(E_1+E_2)/T}}.$$

For non-relativistic dark matter,

$$\sigma v_{\rm rel} = a + b v_{\rm rel}^2 + \cdots .$$

The leading term a is the s-wave contribution.

In natural units, $\hbar = c = k_B = 1$. Thus

$$[\sigma] = {\rm GeV}^{-2}, \qquad [v_{\rm rel}] = 1, \qquad [n] = {\rm GeV}^3.$$

Therefore,

$$[\langle \sigma v \rangle n] = {\rm GeV} \sim {\rm time}^{-1},$$

as required for a reaction rate.

Entropy density and yield

The entropy density is

$$s(T) = \frac{2\pi^2}{45} g_{*s}(T) T^3.$$

Entropy conservation gives

$$\frac{d}{dt}\left(sa^3\right) = 0,$$

so $\dot{s} + 3Hs = 0$. Define the yield:

$$\boxed{Y \equiv \frac{n}{s}}.$$

Using the Boltzmann equation,

$$\boxed{\dot{Y} = -\langle \sigma v \rangle s \left(Y^2 - Y_{\mathrm{eq}}^2\right)}.$$

The yield removes the trivial dilution due to cosmic expansion.

Changing variables to $x = m_\chi / T$

Define

$$x \equiv \frac{m_\chi}{T}.$$

During radiation domination, for slowly varying g_* and g_{*s}, $T \propto a^{-1}$. Therefore,

$$\frac{\dot{T}}{T} = -H.$$

Since $x = m_\chi / T$,

$$\dot{x} = Hx.$$

Thus,

$$\frac{d}{dt} = Hx \frac{d}{dx}.$$

The yield equation becomes

$$\boxed{\frac{dY}{dx} = -\frac{s\langle \sigma v \rangle}{Hx} \left(Y^2 - Y_{\rm eq}^2 \right)}.$$

Radiation domination

The radiation energy density is

$$\rho(T) = \frac{\pi^2}{30} g_*(T)\, T^4 .$$

The Friedmann equation is

$$H^2 = \frac{8\pi G}{3}\rho .$$

Using $G = \frac{1}{M_{\rm Pl}^2}$, one obtains

$$\boxed{H(T) = 1.66\, g_*^{1/2}\, \frac{T^2}{M_{\rm Pl}}} .$$

Here

$$\boxed{M_{\rm Pl} = 1.22 \times 10^{19}\ \text{GeV}}$$

is the usual Planck mass.

Final yield equation

Using

$$s = \frac{2\pi^2}{45} g_{*s} T^3$$

and

$$H = 1.66\, g_*^{1/2} \frac{T^2}{M_{\rm Pl}},$$

we find

$$\frac{s}{Hx} = \frac{\frac{2\pi^2}{45} g_{*s} T^3}{1.66\, g_*^{1/2} \frac{T^2}{M_{\rm Pl}} x}.$$

Since

$$T = \frac{m_\chi}{x},$$

this gives

$$\boxed{\frac{s}{Hx} = 0.264 \frac{g_{*s}}{g_*^{1/2}} M_{\rm Pl} \frac{m_\chi}{x^2}}.$$

Therefore,

$$\boxed{\frac{dY}{dx} = -\lambda \frac{1}{x^2} \left(Y^2 - Y_{\rm eq}^2\right)}$$

with

$$\boxed{\lambda = 0.264 \frac{g_{*s}}{g_*^{1/2}} M_{\rm Pl} m_\chi \langle \sigma v \rangle}.$$

Equilibrium number density

For non-relativistic dark matter obeying Maxwell-Boltzmann statistics,

$$n_{\rm eq} = g_\chi \left(\frac{m_\chi T}{2\pi}\right)^{3/2} e^{-m_\chi/T}.$$

Using

$$x = \frac{m_\chi}{T}, \qquad T = \frac{m_\chi}{x},$$

we obtain

$$n_{\rm eq} = g_\chi \left(\frac{m_\chi^2}{2\pi x}\right)^{3/2} e^{-x}.$$

Therefore,

$$\boxed{n_{\mathrm{eq}} = g_\chi \frac{m_\chi^3}{(2\pi)^{3/2}} x^{-3/2} e^{-x}}.$$

This has the correct dimension:

$$[n_{\mathrm{eq}}] = \mathrm{GeV}^3.$$

Equilibrium yield

The entropy density is

$$s = \frac{2\pi^2}{45} g_{*s} T^3 = \frac{2\pi^2}{45} g_{*s} \frac{m_\chi^3}{x^3}.$$

Thus,

$$Y_{\mathrm{eq}} = \frac{n_{\mathrm{eq}}}{s}.$$

Substituting,

$$Y_{\mathrm{eq}} = \frac{g_\chi \frac{m_\chi^3}{(2\pi)^{3/2}} x^{-3/2} e^{-x}}{\frac{2\pi^2}{45} g_{*s} \frac{m_\chi^3}{x^3}}.$$

Therefore,

$$\boxed{Y_{\rm eq}(x) \simeq 0.145\,\frac{g_\chi}{g_{*s}}\,x^{3/2}\,e^{-x}}.$$

For a real scalar dark matter particle,

$$\boxed{g_\chi = 1}.$$

Freeze-out condition

The interaction rate per particle is $\Gamma(T) = n_{\rm eq}(T)\langle\sigma v\rangle$. Freeze-out occurs approximately when

$$\boxed{\Gamma(T_f) \simeq H(T_f)}.$$

Using the non-relativistic equilibrium density gives

$$\boxed{x_f = \ln\left[\frac{0.038\, g_\chi\, M_{\rm Pl}\, m_\chi\, \langle\sigma v\rangle}{g_*^{1/2} x_f^{1/2}}\right]}.$$

his equation is implicit because x_f appears on both sides. It is solved iteratively, for example with

$$x_f^{(0)} = 20.$$

Typical WIMP values are

$$\boxed{x_f \sim 20 - 30}.$$

Post-freeze-out solution

After freeze-out, $Y \gg Y_{\rm eq}$. The yield equation simplifies to

$$\frac{dY}{dx} \simeq -\lambda \frac{Y^2}{x^2}.$$

Separating variables,

$$\frac{dY}{Y^2} = -\lambda \frac{dx}{x^2}.$$

Integrating from x_f to ∞,

$$\int_{Y_f}^{Y_\infty} \frac{dY}{Y^2} = -\lambda \int_{x_f}^{\infty} \frac{dx}{x^2}.$$

This gives

$$\boxed{\frac{1}{Y_\infty} = \frac{1}{Y_f} + \frac{\lambda}{x_f}}.$$

Usually,

$$\frac{1}{Y_f} \ll \frac{\lambda}{x_f}.$$

Therefore,

$$\boxed{Y_\infty \simeq \frac{x_f}{\lambda}}.$$

Relic abundance scaling

Using

$$\lambda = 0.264 \frac{g_{*s}}{g_*^{1/2}} M_{\rm Pl} m_\chi \langle \sigma v \rangle,$$

we get

$$\boxed{Y_\infty \simeq \frac{x_f}{0.264 \frac{g_{*s}}{g_*^{1/2}} M_{\rm Pl} m_\chi \langle \sigma v \rangle}}.$$

If $g_{*s} \simeq g_*$,

$$\boxed{Y_\infty \simeq \frac{x_f}{0.264\, g_*^{1/2} M_{\rm Pl} m_\chi \langle \sigma v \rangle}}.$$

The main scaling is

$$\boxed{Y_\infty \propto \frac{1}{\langle \sigma v \rangle}}.$$

Therefore,

$$\boxed{\Omega_\chi h^2 \propto \frac{1}{\langle \sigma v \rangle}}.$$

Relic density today

The present dark matter energy density is $\rho_{\chi,0} = m_\chi n_{\chi,0}$. Since the yield is conserved after freeze-out,

$$n_{\chi,0} = s_0 Y_\infty.$$

Therefore,

$$\rho_{\chi,0} = m_\chi s_0 Y_\infty.$$

The density parameter is

$$\Omega_\chi = \frac{\rho_{\chi,0}}{\rho_c}.$$

The standard relation is

$$\boxed{\Omega_\chi h^2 = 2.742 \times 10^8 \left(\frac{m_\chi}{\mathrm{GeV}}\right) Y_\infty}.$$

The observed value is approximately

$$\boxed{\Omega_{\rm DM} h^2 \simeq 0.120 \pm 0.001}.$$

Approximate relic-density formula

Using the freeze-out result

$$Y_\infty \simeq \frac{x_f}{0.264\, g_*^{1/2}\, M_{\rm Pl}\, \langle \sigma v \rangle}$$

and

$$\Omega_\chi h^2 = 2.742 \times 10^8 \left(\frac{m_\chi}{\rm GeV} \right) Y_\infty ,$$

the dark matter mass cancels and we obtain

$$\Omega_\chi h^2 \simeq \frac{1.04 \times 10^9\, x_f}{g_*^{1/2}\, M_{\rm Pl}\, \langle \sigma v \rangle} \frac{1}{\rm GeV} .$$

Using

$$M_{\mathrm{Pl}} = 1.22 \times 10^{19}\ \mathrm{GeV},$$

the final approximate expression becomes

$$\boxed{\Omega_\chi h^2 \simeq 8.5 \times 10^{-11} \frac{x_f}{g_*^{1/2} \langle \sigma v \rangle}}$$

with $\langle \sigma v \rangle$ in GeV^{-2}.
For electroweak-scale WIMPs:

$$g_* \simeq g_{*s} \simeq 106.75$$

is usually an excellent approximation.

Thermal target cross section

For a standard thermal relic, the observed abundance corresponds roughly to

$$\boxed{\langle \sigma v \rangle \simeq 2.6 \times 10^{-9}\ \mathrm{GeV}^{-2}}.$$

Equivalently,

$$\boxed{\langle \sigma v \rangle \simeq 3 \times 10^{-26}\ \mathrm{cm}^3\,\mathrm{s}^{-1}}.$$

The important relation is

$$\boxed{\Omega_\chi h^2 \propto \frac{1}{\langle \sigma v \rangle}}.$$

Thus:

- larger $\langle \sigma v \rangle$ gives smaller relic density;
- smaller $\langle \sigma v \rangle$ gives larger relic density.

The model

Add a real scalar singlet S to the Standard Model and impose

$$Z_2 : \qquad S \to -S.$$

The Lagrangian is

$$\mathcal{L} = \mathcal{L}_{\rm SM} + \frac{1}{2}\partial_\mu S \partial^\mu S - \frac{1}{2}\mu_S^2 S^2 - \frac{\lambda_S}{4} S^4 - \frac{\lambda_{HS}}{2} S^2 H^\dagger H.$$

After electroweak symmetry breaking,

$$H = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v + h \end{pmatrix},$$

with

$$v = 246\ {\rm GeV}.$$

Mass and interactions

We have

$$H^\dagger H = \frac{(v+h)^2}{2} = \frac{v^2}{2} + vh + \frac{h^2}{2}.$$

Therefore,

$$-\frac{\lambda_{HS}}{2} S^2 H^\dagger H = -\frac{\lambda_{HS}}{4} v^2 S^2 - \frac{\lambda_{HS}}{2} vhS^2 - \frac{\lambda_{HS}}{4} h^2 S^2.$$

The physical scalar mass is

$$\boxed{m_S^2 = \mu_S^2 + \frac{\lambda_{HS}}{2} v^2}.$$

The relevant vertices are

$$\boxed{hSS: \quad -i\lambda_{HS} v}$$

and

$$\boxed{hhSS: \quad -i\lambda_{HS}}.$$

Annihilation channels

The Higgs portal allows annihilation through an intermediate Higgs:

$$SS \to h^* \to \mathrm{SM\ SM}.$$

For light dark matter, an important final state is $SS \to b\bar{b}$, because the Higgs coupling to fermions is proportional to the fermion mass.

For heavier dark matter, additional channels become important:

$$SS \to W^+W^-, \qquad SS \to ZZ,$$

$$SS \to hh, \qquad SS \to t\bar{t}.$$

Near the Higgs resonance,

$$\boxed{m_S \simeq \frac{m_h}{2}},$$

annihilation can be strongly enhanced.

Matrix element for $SS \to b\bar{b}$

The Higgs-bottom interaction is

$$\mathcal{L} \supset -\frac{m_b}{v} h\bar{b}b.$$

Thus,

$$h\bar{b}b : \qquad -i\frac{m_b}{v}.$$

The portal vertex is

$$hSS : \qquad -i\lambda_{HS} v.$$

For

$$SS \to h^* \to b\bar{b},$$

the amplitude is

$$i\mathcal{M} = (-i\lambda_{HS} v)\frac{i}{s - m_h^2 + i m_h \Gamma_h}\left(-i\frac{m_b}{v}\right)\bar{u}(k_1)v(k_2).$$

Therefore,

$$\boxed{\mathcal{M} = \lambda_{HS}\, m_b \frac{\bar{u}(k_1)v(k_2)}{s - m_h^2 + i m_h \Gamma_h}}.$$

Spin and color summed amplitude

The spin-summed bilinear is

$$\sum_{\text{spins}} |\bar{u}(k_1)v(k_2)|^2 = \mathrm{Tr}\left[(\not{k}_1 + m_b)(\not{k}_2 - m_b)\right].$$

This gives

$$\sum_{\text{spins}} |\bar{u}(k_1)v(k_2)|^2 = 2(s - 4m_b^2).$$

Including color, $N_c = 3$. Therefore,

$$\boxed{\overline{|\mathcal{M}|^2} = N_c \lambda_{HS}^2 m_b^2 \frac{2(s-4m_b^2)}{(s-m_h^2)^2 + m_h^2\Gamma_h^2}}.$$

There is no initial spin average because S is a scalar.

Cross section

The general two-body cross section is

$$\sigma = \frac{1}{16\pi s} \frac{|\vec{p}_f|}{|\vec{p}_i|} \overline{|\mathcal{M}|^2}.$$

Define

$$\beta_S = \sqrt{1 - \frac{4m_S^2}{s}}, \qquad \beta_b = \sqrt{1 - \frac{4m_b^2}{s}}.$$

Then

$$\frac{|\vec{p}_f|}{|\vec{p}_i|} = \frac{\beta_b}{\beta_S}.$$

Thus,

$$\boxed{\sigma(SS \to b\bar{b}) = \frac{N_c \lambda_{HS}^2 m_b^2}{8\pi} \frac{\beta_b^3}{\beta_S} \frac{1}{(s - m_h^2)^2 + m_h^2 \Gamma_h^2}}.$$

Non-relativistic limit

For non-relativistic identical scalar dark matter,

$$v_{\mathrm{rel}} \simeq 2\beta_S .$$

Using $s \simeq 4m_S^2$, we obtain

$$\boxed{\sigma v_{\mathrm{rel}} \simeq \frac{N_c \lambda_{HS}^2 m_b^2}{4\pi} \frac{\beta_b^3}{(4m_S^2 - m_h^2)^2 + m_h^2 \Gamma_h^2}}.$$

Since

$$\beta_b = \sqrt{1 - \frac{m_b^2}{m_S^2}},$$

we get

$$\boxed{\sigma v_{\rm rel} \simeq \frac{N_c \lambda_{HS}^2 m_b^2}{4\pi} \frac{\left(1 - \frac{m_b^2}{m_S^2}\right)^{3/2}}{(4m_S^2 - m_h^2)^2 + m_h^2 \Gamma_h^2}}.$$

Caveat near the Higgs resonance

The approximation

$$s \simeq 4m_S^2$$

is not reliable near the Higgs resonance:

$$m_S \simeq \frac{m_h}{2}.$$

In that region, the thermal distribution samples values of s near

$$s \simeq m_h^2.$$

Therefore, an exact thermal average over the Breit-Wigner propagator is required.

Close to $m_S \simeq 62.5\ \mathrm{GeV}$, the simple non-relativistic expression can give an inaccurate relic-density estimate.

Freeze-out values

The freeze-out equation is

$$x_f = \ln\left[\frac{0.038\, g_\chi\, M_{\rm Pl}\, m_S\, \langle \sigma v\rangle}{g_*^{1/2} x_f^{1/2}}\right].$$

Take

$$g_\chi = 1, \qquad g_* = 106.75, \qquad \langle \sigma v\rangle = 2.6 \times 10^{-9}\ \mathrm{GeV}^{-2}.$$

For

$$m_S = 50\ \mathrm{GeV},$$

iterating gives

$$\boxed{x_f \simeq 21.0}.$$

For

$$m_S = 200\ \mathrm{GeV},$$

iterating gives

$$\boxed{x_f \simeq 22.3}.$$

Thus the standard estimate

$$x_f \sim 20 - 30$$

is justified.

Relic-density target

The observed relic abundance is

$$\Omega_{\rm DM} h^2 \simeq 0.12.$$

For a standard thermal relic this corresponds approximately to

$$\boxed{\langle \sigma v \rangle \simeq 2.6 \times 10^{-9}\ \mathrm{GeV}^{-2}}.$$

We use this value as a target for the annihilation cross section.

Input parameters:

$$m_b = 4.18\ \mathrm{GeV}, \qquad m_h = 125\ \mathrm{GeV}, \qquad \Gamma_h = 4.1\ \mathrm{MeV}.$$

Since

$$\Gamma_h = 4.1 \times 10^{-3}\ \mathrm{GeV},$$

we have

$$m_h^2 \Gamma_h^2 \simeq 0.26\ \mathrm{GeV}^4.$$

Away from the Higgs resonance, this is usually negligible compared with

$$(4m_S^2 - m_h^2)^2.$$

Example: $m_S = 50$ GeV

For

$$m_S = 50 \text{ GeV},$$

we have

$$4m_S^2 - m_h^2 = 4(50)^2 - 125^2.$$

Therefore,

$$4m_S^2 - m_h^2 = 10000 - 15625 = -5625 \text{ GeV}^2.$$

Thus,

$$(4m_S^2 - m_h^2)^2 \simeq 3.16 \times 10^7 \text{ GeV}^4.$$

Using the $b\bar{b}$ approximation,

$$\sigma v_{\rm rel} \simeq 1.3 \times 10^{-7} \lambda_{HS}^2 \ {\rm GeV}^{-2}.$$

Imposing

$$\sigma v_{\rm rel} \simeq 2.6 \times 10^{-9} \ {\rm GeV}^{-2},$$

gives

$$\boxed{\lambda_{HS} \simeq 0.14}.$$

Invisible Higgs decay

For

$$m_S < \frac{m_h}{2},$$

the decay $h \to SS$ is kinematically allowed. The partial width is

$$\boxed{\Gamma(h \to SS) = \frac{\lambda_{HS}^2 v^2}{32\pi m_h}\sqrt{1 - \frac{4m_S^2}{m_h^2}}}.$$

For $m_S = 50~\mathrm{GeV}$ and $\lambda_{HS} \simeq 0.14$, this width is much larger than the Standard Model Higgs width,

$$\Gamma_h^{\mathrm{SM}} \simeq 4.1~\mathrm{MeV}.$$

Thus this region is strongly constrained by invisible Higgs decay limits. The coupling is perturbative, but this point is not automatically phenomenologically viable.

Example: $m_S = 200$ GeV

For

$$m_S = 200 \ \mathrm{GeV},$$

we obtain

$$4m_S^2 - m_h^2 = 4(200)^2 - 125^2.$$

Thus,

$$4m_S^2 - m_h^2 = 160000 - 15625 = 144375 \ \mathrm{GeV}^2.$$

Hence,

$$(4m_S^2 - m_h^2)^2 \simeq 2.08 \times 10^{10} \ \mathrm{GeV}^4.$$

Using only the $b\bar{b}$ approximation gives roughly

$$\boxed{\lambda_{HS} \simeq 3.6}.$$

However, this is not a realistic relic-density estimate, because other channels are open and usually dominate.

Breakdown of the $b\bar{b}$ approximation

For $m_S = 200\ \mathrm{GeV}$, the following channels are open or important:

$$SS \to W^+W^-, \qquad SS \to ZZ,$$

$$SS \to hh, \qquad SS \to t\bar{t}.$$

These channels generally dominate over $SS \to b\bar{b}$. Therefore, the $b\bar{b}$-only result is useful pedagogically, but not sufficient for a realistic calculation.
A realistic computation must include:

- all kinematically allowed final states;
- exact thermal averages;
- thresholds;
- the Higgs resonance region;
- collider constraints;
- direct-detection constraints.

Direct detection

The Higgs portal induces elastic scattering on nuclei:

$$SN \to SN.$$

This process is mediated by Higgs exchange:

$$SN \to h^* \to SN.$$

The spin-independent cross section scales approximately as

$$\sigma_{\rm SI} \propto \frac{\lambda_{HS}^2 f_N^2 \mu_N^2}{m_h^4},$$

where:

- f_N parametrizes the Higgs-nucleon matrix element;
- μ_N is the reduced mass of the S-nucleon system;
- m_h is the Higgs mass.

Thus large portal couplings are strongly constrained by direct-detection experiments.

Important parameter regions

The real singlet Higgs-portal model has several characteristic regions.

- Low-mass region:

$$m_S < \frac{m_h}{2}.$$

  The decay $h \to SS$ is open.

- Higgs funnel:

$$m_S \simeq \frac{m_h}{2}.$$

  Annihilation is resonantly enhanced.

- Intermediate and high masses:

$$m_S > m_W.$$

  Annihilation into W^+W^-, ZZ, hh, and eventually $t\bar{t}$ becomes important.

- Large coupling region:

$$\lambda_{HS} \gg 1.$$

  Perturbation theory may become unreliable.

Summary of freeze-out

- The Boltzmann equation is

$$\dot{n}_\chi + 3Hn_\chi = -\langle \sigma v \rangle \left(n_\chi^2 - n_{\chi,\mathrm{eq}}^2 \right) .$$

- The yield removes dilution from expansion:

$$Y = \frac{n}{s}.$$

- Freeze-out occurs when

$$\Gamma_\chi \sim H.$$

- The asymptotic yield is approximately

$$Y_\infty \simeq \frac{x_f}{\lambda}.$$

- The relic abundance scales as

$$\Omega_\chi h^2 \propto \frac{1}{\langle \sigma v \rangle}.$$

- The observed abundance corresponds roughly to

$$\langle \sigma v \rangle \simeq 2.6 \times 10^{-9}\ \mathrm{GeV}^{-2}.$$

Final message

Thermal freeze-out provides a simple and predictive mechanism for producing the observed dark matter abundance.

The central result is

$$\boxed{\Omega_\chi h^2 \propto \frac{1}{\langle \sigma v \rangle}}.$$

In the real singlet scalar Higgs-portal model, the same coupling controls:

$$SS \to \mathrm{SM\ SM},$$

$$h \to SS,$$

and

$$SN \to SN.$$

Therefore, relic density, invisible Higgs decays, and direct detection are tightly connected. A realistic study must include all annihilation channels, exact thermal averaging, collider constraints, and direct-detection bounds.