**METFOG** **2024/25**

**Final Exam - 2$^a$ Fase** **11/07/2025** 9h-12h

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**1.** [**7.0 val**] Consider the following Lagrangian, where $S_1$ and $S_2$ are real spin zero fields,

$$\mathcal{L} = \mathcal{L}_{SM} + \frac{1}{2}(\partial_\mu S_1)(\partial^\mu S_1) - \frac{\mu_1^2}{2}S_1^2 - \lambda_1 S_1^4 + \frac{1}{2}(\partial_\mu S_2)(\partial^\mu S_2) - \frac{\mu_2^2}{2}S_2^2 - \lambda_2 S_2^4 - \frac{\kappa_1}{2}S_1^2 H^\dagger H - \frac{\kappa_2}{2}S_2^2 H^\dagger H - \frac{\kappa_{12}}{4}S_1^2 S_2^2 \quad (1)$$

where $\mathcal{L}_{SM}$ is the Standard Model Lagrangian and the Higgs field can be written in the unitary gauge as

$$H = \begin{pmatrix} 0 \\ \frac{1}{\sqrt{2}}(v_H + h) \end{pmatrix} . \quad (2)$$

**a)** [**1.5 val**] Show the Lagrangian is invariant under the independent $Z_2$ symmetries $S_1 \to -S_1$ and $S_2 \to -S_2$. What does this mean for the possible dark matter candidates in the model?

**b)** [**2.5 val**] Find the Feynman rules for $hS_1S_1$ and $hS_2S_2$. Calculate the branching ratio $BR(h \to S_1S_1)$ and assuming the branching ratio to invisible is 0.11, find a bound on $\kappa_1$ for a DM mass of 30 GeV.

**c)** [**1.5 val**] Draw the Feynman diagrams for mono-Higgs and mono-jet events for a final state with two $S_1$ particles.

**d)** [**1.5 val**] Extend the model to three singlets with three independent $Z_2$ symmetries by writing the new terms in the Lagrangian, and discuss the particle content in terms of DM candidates.

**2.** [**6.5 val**] Consider the Standard Model extended by a singlet $S$ with a Lagrangian given by

$$\mathcal{L} = \mathcal{L}_{SM} + \frac{1}{2}(\partial_\mu S)(\partial^\mu S) - V_N + V_{SM}$$

with

$$V_N = bS^2 + dS^4 + \kappa_1 S^2 H^\dagger H + \mu^2 H^\dagger H + \lambda(H^\dagger H)^2$$

where $H^\dagger = (0 \quad (v+h)/\sqrt{2})$ is the Higgs doublet in the unitary gauge and S is the singlet field with a zero vacuum expectation value.

**a)** [**1.5 val**] What is the symmetry imposed to the Lagrangian to have a DM candidate? How would the potential change if this symmetry was not present? What are the mass dimensions of the new parameters?

**b)** [**3.0 val**] Draw the Feynman diagrams and calculate the thermal averaged cross section for the scattering $SS \to hh$ (note that there are three diagrams and that the Feynman rule for the SM triple Higgs coupling is $-3im_h^2/v$). Use the following approximations: $< \sigma v >_{SS\to hh} = v\sigma_{SS\to hh}$ with $v = 2\sqrt{\frac{s}{4m_S^2} - 1}$ and after that also $\sqrt{s} = 2m_S$ and neglect the SM Higgs width. Remember the cross section can be written as

$$\sigma(SS \to hh) = \frac{1}{16\pi s}\sqrt{\frac{s - 4m_h^2}{s - 4m_S^2}}|T|^2,$$

where $T$ is the amplitude.

**c) [2.0 val]** Using the formulae below, calculate the value of $\kappa_1$ that reproduces the correct value for the relic density for a DM mass of 1 TeV.

$$\Omega_\chi h^2 = m_\chi s_0 Y_0 \frac{8\pi G}{3H^2} \approx 2.742 \times 10^8 \frac{m_S}{GeV} Y_0 \qquad (\Omega h^2)_{DM}^{obs} = 0.1186 \pm 0.0020$$

$$Y_0 = 10/\lambda \qquad \lambda = N < \sigma v >_{SS\to hh} \qquad N = 1.6 \times 10^{20}$$

**3. [6.5 val]** Consider a temperature dependent potential of the form

$$V(\phi, T) = \lambda(T^2 - T_0^2)\phi^2 - \frac{\lambda}{4}T\phi^3 + \frac{\lambda}{4}\phi^4$$

where $T_0$ and $\lambda$ are constants.

**a) [2.0 val]** Calculate the critical temperature and explain your reasoning.

**b) [1.5 val]** Show that this transition is weak independently of the value of $\lambda$.

**c) [1.5 val]** Explain the role of the gauge bosons in having a first order phase transition in the SM. Why is the SM phase transition weak?

**d) [1.5 val]** What would happen if there was no cubic term in the potential? What would be the critical temperature in that scenario?

$$1\,\text{barn} = 0.00257\,\text{MeV}^{-2} = 10^{-24}\,\text{cm}^2$$