CHAPTER

5

Earth's Surface Modeling

T.X. Yue[*,§,1], N. Zhao[*,§], Z.P. Du[*]

[*]Institute of Geographical Science and Natural Resources Research, Beijing, China and
[§]University of Chinese Academy of Sciences, Beijing, China
[1]Corresponding author: E-mail: yue@lreis.ac.cn

OUTLINE

5.1 INTRODUCTION

The Earth's surface environment is an active and complex place, at the interface of the lithosphere, the hydrosphere, the atmosphere, and the biosphere (Phillips, 1999). An earth surface system is a set of interconnected components of the earth surface environment that function together as a complex whole. Earth's surface modeling is generally defined as a spatially explicitly digital description of an earth surface system or a component of the earth surface environment (Yue, 2011).

Surface modeling began to be used in the 1960s, with the general availability of computers (Lo and Yeung, 2002), but because it requires powerful software and a large amount of spatially explicit data, its development was limited before the 1990s. The major advances that enabled the use of surface modeling included trend surface analysis (Ahlberg et al., 1967; Schroeder and Sjoquist, 1976; Legendre and Legendre, 1983), the digital terrain model (Stott, 1977), surface approximation (Long, 1980), spatial simulation of wetland habitats (Sklar et al., 1985), spatial pattern matching (Costanza, 1989), spatial prediction (Turner et al., 1989), and

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modeling costal landscape dynamics (Costanza et al., 1990). Surface modeling has greatly progressed since the early 1990s, with rapid development of remote sensing (RS) and a geographical information system (GIS), as well as the accumulation of spatially explicit data.

It was learnt that slope and curvature are significant variables of Earth's surface analysis in the early 1980s (Evans, 1980). In fact, a plane curve is uniquely determined by its curvature and a space curve is uniquely determined by its curvature and torsion if a translation followed by a rotation is allowed in terms of curve theorems in the plane and in the space (Spivak, 1979). Following this consideration, two equivalent indexes (EQIs) of curves were developed respectively for plane curves and space curves to simulate surfaces (Yue and Ai, 1990) and detect surface changes (Yue et al., 2002) by fitting section lines of a surface and combining them together.

In the early 2000s, it was found that Earth's surface systems are controlled by a combination of global factors and local factors, which cannot be understood without accounting for both the local and global components. The system dynamic cannot be recovered from the global or local controls alone (Phillips, 2002). In fact, in terms of the fundamental theorem of surfaces (FTS), a surface is uniquely defined by the first and the second fundamental coefficients (Somasundaram, 2005). The first fundamental coefficients express the information about the details of the surface, which are observed when we stay on the surface. The second fundamental coefficients express the change of the surface observed from outside the surface (Yue et al., 2015a).

To significantly reduce uncertainty of simulating Earth's surfaces, we suggest an alternative method, high accuracy surface modeling (HASM), which takes global approximate information (e.g., RS images or model simulation results) as its driving field and local accurate information (e.g., ground observation data and/or sampling data) as its optimum control constraints (Yue et al., 2007). HASM completes its operation when its output satisfies the iteration stopping criterion which is determined by application requirement for accuracy (Zhao and Yue, 2014a). A fundamental theorem of earth surface modeling (FTESM) is abstracted on the basis of applying HASM to simulating surfaces of elevation, soil properties, changes of ecosystem services, and driving forces of ecosystem changes on multiscales for about 20 years (Yue et al., 2016).

5.2 EQUIVALENT INDEXES

For curves in space, the EQI of curves can be formulated as follows in terms of curve theorem if a translation and a rotation are not allowed (Yue and Ai, 1990):

$$Eq = \frac{1}{S - S_0} \int_{S_0}^{S} \left((l_1(S_0) - l_2(S_0))^2 + (\tau_1(s) - \tau_2(s))^2 + (k_1(s) - k_2(s))^2 + |\vec{n}_1(s) - \vec{n}_2(s)|^2 \right)^{\frac{1}{2}} ds \tag{5.1}$$

where $k_i(s)$, $\tau_i(s)$, and $\vec{n}_i(s)$ are respectively curvature, torsion, direction of curve l_i ($i = 1,2$); s is arc length; $l_i(S_0)$ is the initial of curve l_i; $L = S - S_0$.

If a Euclidean motion of a translation and a rotation is allowed, the EQI can be simplified as,

$$Eq = \frac{1}{L} \int_{S_0}^{S} \left((k_1(s) - k_2(s))^2 + (\tau_1(s) - \tau_2(s))^2 \right)^{\frac{1}{2}} ds \tag{5.2}$$

For curves in plane, if the Euclidean motion is not allowed, Eq. (5.1) can be expressed as,

$$Eq = \frac{1}{S - S_0} \int_{S_0}^{S} \left((l_1(S_0) - l_2(S_0))^2 + (\alpha_1(s) - \alpha_2(s))^2 + (k_1(s) - k_2(s))^2 \right)^{\frac{1}{2}} ds \tag{5.3}$$

where $k_i(s)$ and $\alpha_i(s)$ are, respectively, the slope and curvature of curve l_i ($i = 1,2$) at an arc length of s; $l_i(S_0)$ is the initial value.

If the Euclidean motion is allowed, Eq. (5.3) can be simplified as,

$$Eq = \frac{1}{L} \int_{S_0}^{S} |k_1(s) - k_2(s)| ds \tag{5.4}$$

It can be proven (Yue et al., 1999; Yue and Zhou 1999) that $Eq(L_1,L_2)$ has the following three properties: (a) $Eq(L_1,L_2) \geq 0$; $Eq(L_1,L_2) = 0$ if and only if $L_1 = L_2$; (b) $Eq(L_1,L_2) = Eq(L_2,L_1)$; (c) $Eq(L_1,L_3) \leq Eq(L_1,L_2) + Eq(L_2,L_3)$. In functional analysis, $Eq(L_1,L_2)$ is a kind of distance in metric space of curves (Taylor, 1958).

If curves L_i could be simulated as

$$y = f_i(x) \tag{5.5}$$

then, α_i and k_i can be respectively formulated as

$$\alpha_i(x) = \frac{df_i(x)}{dx} \tag{5.6}$$

$$k_i(x) = \frac{d\alpha_i(x)}{dx} \cdot \left(1 + \alpha_i^2(x)\right)^{-\frac{3}{2}} \tag{5.7}$$

$$ds = \left(1 + \alpha^2(x)\right)^{\frac{1}{2}} dx \tag{5.8}$$

where x is abscissa and s is arc length.

Although the EQIs of curves are very useful for curve fitting and comparison, it is incomplete to use them to simulate surfaces. In fact, a surface is uniquely defined by the first fundamental coefficients, about the details of the surface observed when we stay on the surface, and the second fundamental coefficients, the change of the surface observed from outside the surface, in terms of FTS (Somasundaram, 2005; Yue et al., 2015a).

5.3 HIGH ACCURACY SURFACE MODELING

If $\{(x_i,y_j)\}$ is an orthogonal division of a computational domain and h represents the simulation step length, the central point of lattice (x_i,y_j) could be expressed as $(0.5h + (i-1)h, 0.5h + (j-1)h)$, in which $i = 0,1,2, \ldots, I, I+1$ and $j = 0,1,2, \ldots, J, J+1$. If $f_{i,j}^{(n)}$ ($n \geq 0$) represents the iterants of $f(x,y)$ at (x_i,y_j) in the nth iterative step, in which $\{f_{i,j}^{(0)}\}$ are interpolations based on sampling values $\{\bar{f}_{i,j}\}$, in terms of numerical mathematics (Quarteroni et al., 2000), the iterative formulation of the HASM master equation set can be expressed as (Yue et al., 2013a,b; Zhao and Yue, 2014b),

$$\begin{cases}\dfrac{f_{i+1,j}^{(n+1)}-2f_{i,j}^{(n+1)}+f_{i-1,j}^{(n+1)}}{h^2}=\left(\Gamma_{11}^1\right)_{i,j}^{(n)}\dfrac{f_{i+1,j}^{(n)}-f_{i-1,j}^{(n)}}{2h}+\left(\Gamma_{11}^2\right)_{i,j}^{(n)}\dfrac{f_{i,j+1}^{(n)}-f_{i,j-1}^{(n)}}{2h}+\dfrac{L_{i,j}^{(n)}}{\sqrt{E_{i,j}^{(n)}+G_{i,j}^{(n)}-1}}\\ \dfrac{f_{i,j+1}^{(n+1)}-2f_{i,j}^{(n+1)}+f_{i,j-1}^{(n+1)}}{h^2}=\left(\Gamma_{22}^1\right)_{i,j}^{(n)}\dfrac{f_{i+1,j}^{(n)}-f_{i-1,j}^{(n)}}{2h}+\left(\Gamma_{22}^2\right)_{i,j}^{(n)}\dfrac{f_{i,j+1}^{(n)}-f_{i,j-1}^{(n)}}{2h}+\dfrac{N_{i,j}^{(n)}}{\sqrt{E_{i,j}^{(n)}+G_{i,j}^{(n)}-1}}\\ \dfrac{f_{i+1,j+1}^{(n+1)}-f_{i+1,j}^{(n+1)}-f_{i,j+1}^{(n+1)}+2f_{i,j}^{(n+1)}-f_{i-1,j}^{(n+1)}-f_{i,j-1}^{(n+1)}+f_{i-1,j-1}^{(n+1)}}{2h^2}\\ \quad=\left(\Gamma_{12}^1\right)_{i,j}^{(n)}\dfrac{f_{i+1,j}^{(n)}-f_{i-1,j}^{(n)}}{2h}+\left(\Gamma_{12}^2\right)_{i,j}^{(n)}\dfrac{f_{i,j+1}^{(n)}-f_{i,j-1}^{(n)}}{2h}+\dfrac{M_{i,j}^{(n)}}{\sqrt{E_{i,j}^{(n)}+G_{i,j}^{(n)}-1}}\end{cases} \tag{5.9}$$

where,

$$E_{i,j}^{(n)}=1+\left(\frac{f_{i+1,j}^{(n)}-f_{i-1,j}^{(n)}}{2h}\right)^2$$

$$F_{i,j}^{(n)}=\left(\frac{f_{i+1,j}^{(n)}-f_{i-1,j}^{(n)}}{2h}\right)\left(\frac{f_{i,j+1}^{(n)}-f_{i,j-1}^{(n)}}{2h}\right)$$

$$G_{i,j}^{(n)}=1+\left(\frac{f_{i,j+1}^{(n)}-f_{i,j-1}^{(n)}}{2h}\right)^2$$

$$L_{i,j}^{(n)}=\frac{\frac{f_{i+1,j}^{(n)}-2f_{i,j}^{(n)}+f_{i-1,j}^{(n)}}{h^2}}{\sqrt{1+\left(\frac{f_{i+1,j}^{(n)}-f_{i-1,j}^{(n)}}{2h}\right)^2+\left(\frac{f_{i,j+1}^{(n)}-f_{i,j-1}^{(n)}}{2h}\right)^2}}$$

$$M_{i,j}^{(n)}=\frac{\left(\frac{f_{i+1,j+1}^{(n)}-f_{i+1,j-1}^{(n)}}{4h^2}\right)-\left(\frac{f_{i-1,j+1}^{(n)}-f_{i-1,j-1}^{(n)}}{4h^2}\right)}{\sqrt{1+\left(\frac{f_{i+1,j}^{(n)}-f_{i-1,j}^{(n)}}{2h^2}\right)^2+\left(\frac{f_{i,j+1}^{(n)}-f_{i,j-1}^{(n)}}{2h^2}\right)^2}}$$

$$N_{i,j}^{(n)}=\frac{\frac{f_{i,j+1}^{(n)}-2f_{i,j}^{(n)}+f_{i,j-1}^{(n)}}{h^2}}{\sqrt{1+\left(\frac{f_{i+1,j}^{(n)}-f_{i-1,j}^{(n)}}{2h}\right)^2+\left(\frac{f_{i,j+1}^{(n)}-f_{i,j-1}^{(n)}}{2h}\right)^2}}$$

$$(\Gamma_{11}^1)_{i,j}^{(n)} = \frac{G_{i,j}^{(n)}\left(E_{i+1,j}^{(n)} - E_{i-1,j}^{(n)}\right) - 2F_{i,j}^{(n)}\left(F_{i+1,j}^{(n)} - F_{i-1,j}^{(n)}\right) + F_{i,j}^{(n)}\left(E_{i,j+1}^{(n)} - E_{i,j-1}^{(n)}\right)}{4\left(E_{i,j}^{(n)}G_{i,j}^{(n)} - \left(F_{i,j}^{(n)}\right)^2\right)h}$$

$$(\Gamma_{12}^1)_{i,j}^{(n)} = \frac{G_{i,j}^{(n)}\left(E_{i,j+1}^{(n)} - E_{i,j-1}^{(n)}\right) - 2F_{i,j}^{(n)}\left(G_{i+1,j}^{(n)} - G_{i-1,j}^{(n)}\right)}{4\left(E_{i,j}^{(n)}G_{i,j}^{(n)} - \left(F_{i,j}^{(n)}\right)^2\right)h}$$

$$(\Gamma_{22}^1)_{i,j}^{(n)} = \frac{2G_{i,j}^{(n)}\left(F_{i,j+1}^{(n)} - F_{i,j-1}^{(n)}\right) - G_{i,j}^{(n)}\left(G_{i+1,j}^{(n)} - G_{i-1,j}^{(n)}\right) - F_{i,j}^{(n)}\left(G_{i,j+1}^{(n)} - G_{i,j-1}^{(n)}\right)}{4\left(E_{i,j}^{(n)}G_{i,j}^{(n)} - \left(F_{i,j}^{(n)}\right)^2\right)h}$$

$$(\Gamma_{11}^2)_{i,j}^{(n)} = \frac{2E_{i,j}^{(n)}\left(F_{i+1,j}^{(n)} - F_{i-1,j}^{(n)}\right) - E_{i,j}^{(n)}\left(E_{i,j+1}^{(n)} - E_{i,j-1}^{(n)}\right) - F_{i,j}^{(n)}\left(E_{i,j+1}^{(n)} - E_{i,j-1}^{(n)}\right)}{4\left(E_{i,j}^{(n)}G_{i,j}^{(n)} - \left(F_{i,j}^{(n)}\right)^2\right)h}$$

$$(\Gamma_{12}^2)_{i,j}^{(n)} = \frac{E_{i,j}^{(n)}\left(G_{i+1,j}^{(n)} - G_{i-1,j}^{(n)}\right) - F_{i,j}^{(n)}\left(E_{i,j+1}^{(n)} - E_{i,j-1}^{(n)}\right)}{4\left(E_{i,j}^{(n)}G_{i,j}^{(n)} - \left(F_{i,j}^{(n)}\right)^2\right)h}$$

$$(\Gamma_{22}^2)_{i,j}^{(n)} = \frac{E_{i,j}^{(n)}\left(G_{i,j+1}^{(n)} - G_{i,j-1}^{(n)}\right) - 2F_{i,j}^{(n)}\left(F_{i,j+1}^{(n)} - F_{i,j-1}^{(n)}\right) + F_{i,j}^{(n)}\left(G_{i+1,j}^{(n)} - G_{i-1,j}^{(n)}\right)}{4\left(E_{i,j}^{(n)}G_{i,j}^{(n)} - \left(F_{i,j}^{(n)}\right)^2\right)h}$$

where $E_{i,j}^{(n)}$, $F_{i,j}^{(n)}$, and $G_{i,j}^{(n)}$ are the iterants of the first fundamental coefficients at the nth iterative step; $L_{i,j}^{(n)}$, $M_{i,j}^{(n)}$, and $N_{i,j}^{(n)}$ represent the iterants of the second fundamental coefficients at the nth iterative step; $(\Gamma_{11}^1)_{i,j}^{(n)}$, $(\Gamma_{11}^2)_{i,j}^{(n)}$, $(\Gamma_{22}^1)_{i,j}^{(n)}$, and $(\Gamma_{22}^2)_{i,j}^{(n)}$ the iterants of the Christoffel symbols of the second kind at the nth iterative step, which depend only upon the first fundamental coefficients and their derivatives.

The matrix formulation of HASM master equations can be respectively expressed as,

$$\mathbf{A}\cdot\mathbf{z}^{(n+1)} = \mathbf{d}^{(n)} \tag{5.10}$$

$$\mathbf{B}\cdot\mathbf{z}^{(n+1)} = \mathbf{q}^{(n)} \tag{5.11}$$

$$\mathbf{C}\cdot\mathbf{z}^{(n+1)} = \mathbf{p}^{(n)} \tag{5.12}$$

where **A**, **B**, and **C** represent coefficient matrixes of the first equation, the second equation, and the third equation; $\mathbf{d}^{(n)}$, $\mathbf{q}^{(n)}$, and $\mathbf{p}^{(n)}$ are right-hand side vectors of the three equations respectively; $\mathbf{z}^{(n+1)} = (f_{1,1}^{(n+1)}, \ldots, f_{1,J}^{(n+1)}, \ldots\ldots, f_{I,1}^{(n+1)}, \ldots, f_{I,J}^{(n+1)})^T = (z_1^{(n+1)}, \ldots, z_J^{(n+1)}, \ldots, z_{(I-1)\cdot J+1}^{(n+1)}, \ldots, z_{I\cdot J}^{(n+1)})^T$, $f_{i,j}^{(n)}$ is the value of the nth iteration of $f(x,y)$ at grid cell (x_i,y_i), and $z_{(i-1)\cdot J+j}^{(n+1)} = f_{i,j}^{(n+1)}$ for $1 \le i \le I$, $1 \le j \le J$.

If $\bar{f}_{i,j}$ is the value of $z = f(x,y)$ at the pth sampled point (x_i, y_j), $s_{p,(i-1)\times J+j} = 1$, and $k_p = \bar{f}_{i,j}$.There is only one nonzero element, 1, in every row of the coefficient matrix, $\mathbf{S}$, making it a sparse matrix. The solution procedure of HASMabc, taking the sampled points as its constraints, can be transformed into solving the following linear equation set in terms of the least squares principle:

$$\begin{bmatrix} \mathbf{A}^T & \mathbf{B}^T & \mathbf{C}^T & \lambda\cdot\mathbf{S}^T \end{bmatrix} \begin{bmatrix} \mathbf{A} \\ \mathbf{B} \\ \mathbf{C} \\ \lambda\cdot\mathbf{S} \end{bmatrix} \mathbf{z}^{(n+1)} = \begin{bmatrix} \mathbf{A}^T & \mathbf{B}^T & \mathbf{C}^T & \lambda\cdot\mathbf{S}^T \end{bmatrix} \begin{bmatrix} \mathbf{d}^{(n)} \\ \mathbf{q}^{(n)} \\ \mathbf{p}^{(n)} \\ \lambda\cdot\mathbf{k} \end{bmatrix} \tag{5.13}$$

The parameter λ is the weight of the sampling points and determines the contribution of the sampling points to the simulated surface. λ could be a real number, which means all sampling points have the same weight, or a sector, which means every sampling point has its own weight. An area affected by a sampling point in a complex region is smaller than in a flat region. Therefore, a smaller value of λ is selected in a complex region and a larger value of λ is selected in a flat region.

Let $\mathbf{W} = \begin{bmatrix} \mathbf{A}^T & \mathbf{B}^T & \mathbf{C}^T & \lambda\cdot\mathbf{S}^T \end{bmatrix} \begin{bmatrix} \mathbf{A} \\ \mathbf{B} \\ \mathbf{C} \\ \lambda\cdot\mathbf{S} \end{bmatrix}$ and $\mathbf{v}^{(n)} = \begin{bmatrix} \mathbf{A}^T & \mathbf{B}^T & \mathbf{C}^T & \lambda\cdot\mathbf{S}^T \end{bmatrix} \begin{bmatrix} \mathbf{d}^{(n)} \\ \mathbf{q}^{(n)} \\ \mathbf{p}^{(n)} \\ \lambda\cdot\mathbf{k} \end{bmatrix}$, then HASMabc has the following formulation:

$$\mathbf{W}\cdot\mathbf{z}^{(n+1)} = \mathbf{v}^{(n)} \tag{5.14}$$

If the third equation is deleted in the Eq. (5.9), we can get an equation set as follows:

$$\begin{cases} \dfrac{f_{i+1,j}^{(n+1)} - 2f_{i,j}^{(n+1)} + f_{i-1,j}^{(n+1)}}{h^2} = \left(\Gamma_{11}^1\right)_{i,j}^{(n)} \dfrac{f_{i+1,j}^{(n)} - f_{i-1,j}^{(n)}}{2h} + \left(\Gamma_{11}^2\right)_{i,j}^{(n)} \dfrac{f_{i,j+1}^{(n)} - f_{i,j-1}^{(n)}}{2h} + \dfrac{L_{i,j}^{(n)}}{\sqrt{E_{i,j}^{n} + G_{i,j}^{n} - 1}} \\ \dfrac{f_{i,j+1}^{(n+1)} - 2f_{i,j}^{(n+1)} + f_{i,j-1}^{(n+1)}}{h^2} = \left(\Gamma_{22}^1\right)_{i,j}^{(n)} \dfrac{f_{i+1,j}^{(n)} - f_{i-1,j}^{(n)}}{2h} + \left(\Gamma_{22}^2\right)_{i,j}^{(n)} \dfrac{f_{i,j+1}^{(n)} - f_{i,j-1}^{(n)}}{2h} + \dfrac{N_{i,j}^{(n)}}{\sqrt{E_{i,j}^{(n)} + G_{i,j}^{(n)} - 1}} \end{cases} \tag{5.15}$$

Eq. (5.15) has the following matrix formulation:

$$\begin{cases} \mathbf{A}\cdot\mathbf{z}^{(n+1)} = \mathbf{d}^{(n)} \\ \mathbf{B}\cdot\mathbf{z}^{(n+1)} = \mathbf{q}^{(n)} \end{cases} \tag{5.16}$$

If optimum control constraints are considered, a method for high accuracy surface modeling, termed HASMab, can be formulated as (Yue, 2011),

$$\begin{cases} \min \left\| \begin{bmatrix} \mathbf{A} \\ \mathbf{B} \end{bmatrix} \cdot z^{(n+1)} - \begin{bmatrix} \mathbf{d}^{(n)} \\ \mathbf{q}^{(n)} \end{bmatrix} \right\| \\ s.t. \quad \mathbf{S} \cdot \mathbf{z}^{(n+1)} = \mathbf{k} \end{cases} \tag{5.17}$$

According to the Method of Lagrange Multipliers (Kolman and Trend, 1971), Eq. (5.17) can be transferred into

$$\mathbf{z}^{(n+1)} = \left(\mathbf{A}^T \cdot \mathbf{A} + \mathbf{B}^T \cdot \mathbf{B} + \lambda^2 \cdot \mathbf{S}^T \cdot \mathbf{S}\right)^{-1} \left(\mathbf{A}^T \cdot \mathbf{d}^{(n)} + \mathbf{B}^T \cdot \mathbf{q}^{(n)} + \lambda^2 \cdot \mathbf{S}^T \cdot \mathbf{k}\right) \tag{5.18}$$

Accuracy of HASMabc is much higher than the one of HASMab when the driving field has a bigger error, while HASMabc is little more accurate than HASMab when the driving field is relatively accurate (Yue, 2016). HASMabc has a much more complex computation because it includes the third equation, which makes it need about two times more memory and has much slower computational-speed compared to HASMab, especially with increase in computational size. In other words, Eq. (5.12) can be ignored to reduce the calculated amount and improve computational-speed.

5.4 THE FUNDAMENTAL THEOREM OF EARTH'S SURFACE MODELING AND ITS COROLLARIES

HASM has been successfully applied to constructing digital elevation models (Yue et al., 2007; Yue and Wang, 2010; Yue et al., 2010a, b; Chen and Yue, 2010; Chen et al., 2012, 2013a, b), modeling surface soil properties (Shi et al., 2011) and soil pollution (Shi et al., 2009) as well as soil antibiotics (Shi et al., 2016), filling voids in the Shuttle Radar Topography Mission dataset (Yue et al., 2012), simulating climate change (Yue et al., 2013a, b; Zhao and Yue, 2014a, b), filling voids on remotely sensed XCO_2 surfaces (Yue et al., 2015b), analyzing ecosystem responses to climatic change (Yue et al., 2015c), and mapping superresolution land cover (Chen et al., 2015). In all these applications, HASM produced more accurate results than the classical methods.

A FTESM and its Corollaries have been developed on the basis of summarizing the successful applications of HASM (Yue et al., 2016): "an Earth's surface system or a component surface of the Earth's surface environment can be simulated with HASM when its spatial resolution is fine enough, which is uniquely defined by both extrinsic and intrinsic invariants of the surface."

The approaches to Earth surface modeling can be classified into five categories: (1) spatial interpolation, (2) data fusion, (3) data assimilation, (4) upscaling, and (5) downscaling. Spatial interpolation is defined as predicting the values of a primary variable at points within the same region of sampled locations in terms of spatial data in the form of discrete points or in the form of data partition (Wang and Wang, 2012; Li and Heap, 2014). Data fusion is the process of integration of multiple data and knowledge streams representing the same real-world object into a consistent, accurate, and useful representation (Mitchell, 2012). Data assimilation is the process by which measured observations are incorporated into a system model (Nichols, 2010).

The transfer of knowledge from a finer resolution to a coarser resolution is referred to as upscaling to mostly reduce computational costs (Schlummer et al., 2014). However, spatial resolutions of many models or data are sometimes too coarse to be used for analyses on regional or local scales. To overcome this problem, downscaling approaches are developed to obtain information at finer spatial resolution from the coarser-spatial-resolution models and data (Zhang et al., 2004).

In terms of the **FTESM**, the following seven corollaries can be derived:

Corollary 1 (Interpolation): If only an intrinsic invariant is available, HASM can be used to create the Earth's surface or a component surface of the Earth's surface environment with higher accuracy after the necessary extrinsic invariants have been extracted from the intrinsic invariant by geostatistics.

Corollary 2 (Upscaling): If a surface on a finer spatial resolution is transferred to the one on a coarser-spatial-resolution, ground-based observations are necessary to operate HASM for obtaining higher accuracy.

Corollary 3 (Downscaling): If a surface at a coarse spatial resolution is available, it is necessary to supplement ground-based observations to obtain a corresponding surface at finer spatial resolution with higher accuracy by means of HASM.

Corollary 4 (Data fusion): When remotely sensed data from satellites are available, ground measurements have to be obtained and incorporated before HASM can be used to generate a more accurate surface.

Corollary 5 (Data fusion): When both remotely sensed data from satellites and ground measurements are available, HASM can be used to generate a surface that is more accurate than the one from either the satellite observations or the ground measurements.

Corollary 6 (Data assimilation): When a system model is available, a more accurate surface can be produced when ground observations are incorporated into the system model.

Corollary 7 (Data assimilation): When both a system model and ground observations are available, a surface can be produced by using HASM to incorporate the ground observations into the system model, which is more accurate than the one either from system model or from ground observations.

5.5 CONCLUSIONS

Error problems and slow-computational-speed problems are the two critical challenges currently faced by GIS and Computer-Aided Design Systems (CADS). The method for HASM provides solutions to these problems that have long troubled GIS and CADS (Jorgensen, 2011).

HASM can take advantage of limited observation data to construct a continuous surface by filling missing data, with higher accuracy comparing with the classical methods such as triangulated irregular network (TIN), inverse distance weighting (IDW), ordinary Kriging (OK), and Spline (Yang et al., 2015). TIN calculates the value of each point within a triangle by means of a linear function based on its location, while it ignores nonlinear information and is unable to represent cliffs, caves, or holes. IDW uses an IDW function to determine the interpolation value for any given point within the calculated area, but it fails to incorporate the spatial structure and ignores information beyond the neighborhood. OK tries to have the mean residual or error equal to zero and aims at minimizing the variance of the errors, but the goals are practically unattainable since the mean error and the variance of the errors are always unknown. Spline is approximately used to simulate surfaces, while few types of surfaces fit the formulation of Splines.

HASM can improve the quality of the information so that it is more accurate than would be possible if the data sources were used individually, by means of its function of data fusion. Since data from different sources have varying accuracy and coverage, the benefits of this data fusion include improved system reliability, extended coverage, and reduced uncertainty. HASM can operate data assimilation to use measured observations in combination with a system model to derive accurate estimates of the current and future states of the system, together with estimates of the uncertainty in the estimates.

HASM is able to transfer information from one spatial scale to another with improved errors, which are mainly caused by the spatial heterogeneity of objects and process nonlinearities, the scale dependency of the characteristics of objects and processes, feedbacks associated with process interactions at small and large scales, emergent properties that arise at larger scales through the interaction of small-scale processes, and the time lags of system response to external perturbation.

The further research focuses of HASM include (1) theoretical analyses of convergence and stability of numerical solution procedure of HASM equation set, (2) clarification of physical significance of HASM parameters and variables as well as their effects on solution accuracy and speed, (3) construction of finite element method of HASM under spherical coordinates, (4) development of a faster numerical solver by selecting an optimal preconditioning operator, and (5) parallelization of HASM to find a solution for the problems of large memory requirements and slow computing speed.

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